

Algorithms

Basic Coding

Winter 2022: Dan Calderone

Algorithms

Search Algorithms

- Binary search
- Breadth First Search (BFS)
- Depth First Search (DFS)
- Kadane's algorithm
- KMP algorithm
- Quick select algorithm
- Boyer-Moore Majority Vote algorithm
- Euclid's algorithm

Sorting Algorithms

- Insertion Sort
- Selection Sort
- Quick Sort
- Merge Sort
- Heap Sort
- Introsort
- Bubble sort
- Non-comparison sorts
- Counting sorts
- Radix sort
- Bucket sort
- (Kahn's topological sort)

Graph Algorithms

- **Sorting**
- Kahn's topological sort (DAG)

Tree/Cycle algorithms

- Floyd's Cycle Detection Algorithm
- Union-Find Cycle Detection
- Kruskal's Algorithm

Shortest path

- Dijkstra's Algorithm
- Bellman Ford Algorithm
- Floyd Warshall Algorithm
- Lee Algorithm

Compression Algorithms

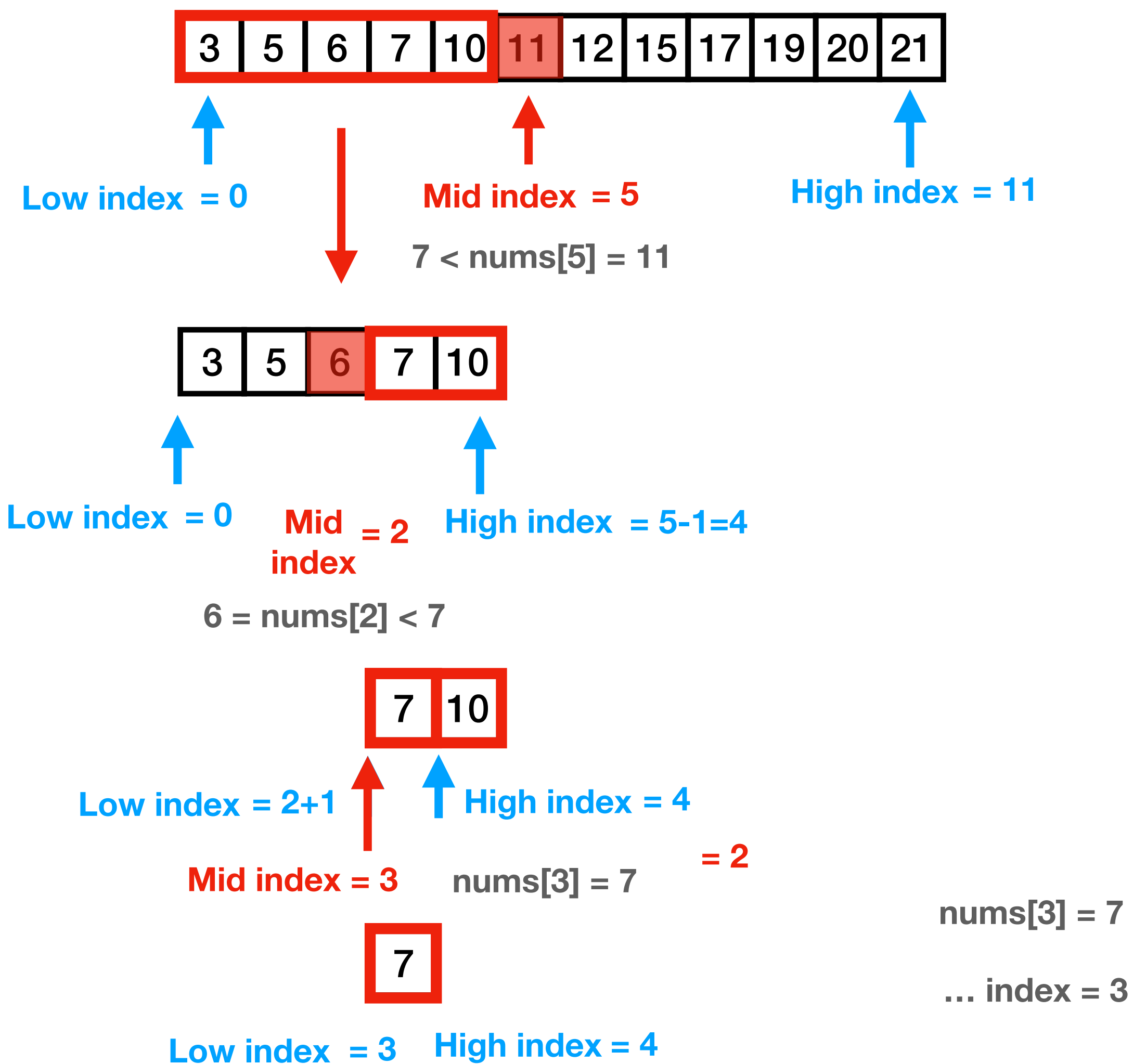
- Huffman Coding

Fill Algorithm

- Flood Fill Algorithm

Binary Search Algorithm: (Divide and Conquer)

nums = [3, 5, 6, 7, 10, 11, 12, 15, 17, 19, 20, 21]; target = 7;



nums[3] = 7

... index = 3

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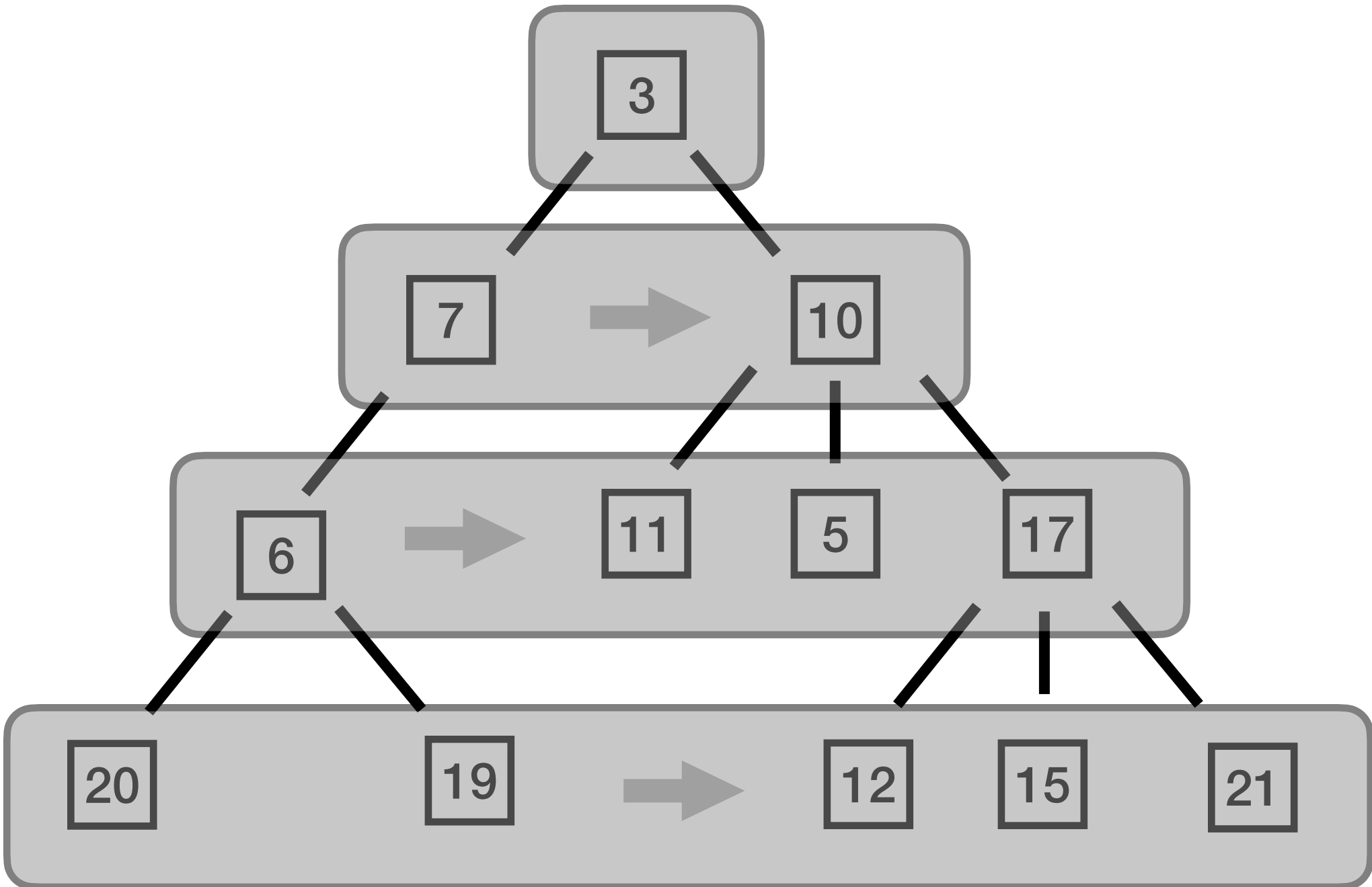
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Breadth First Search (BFS)



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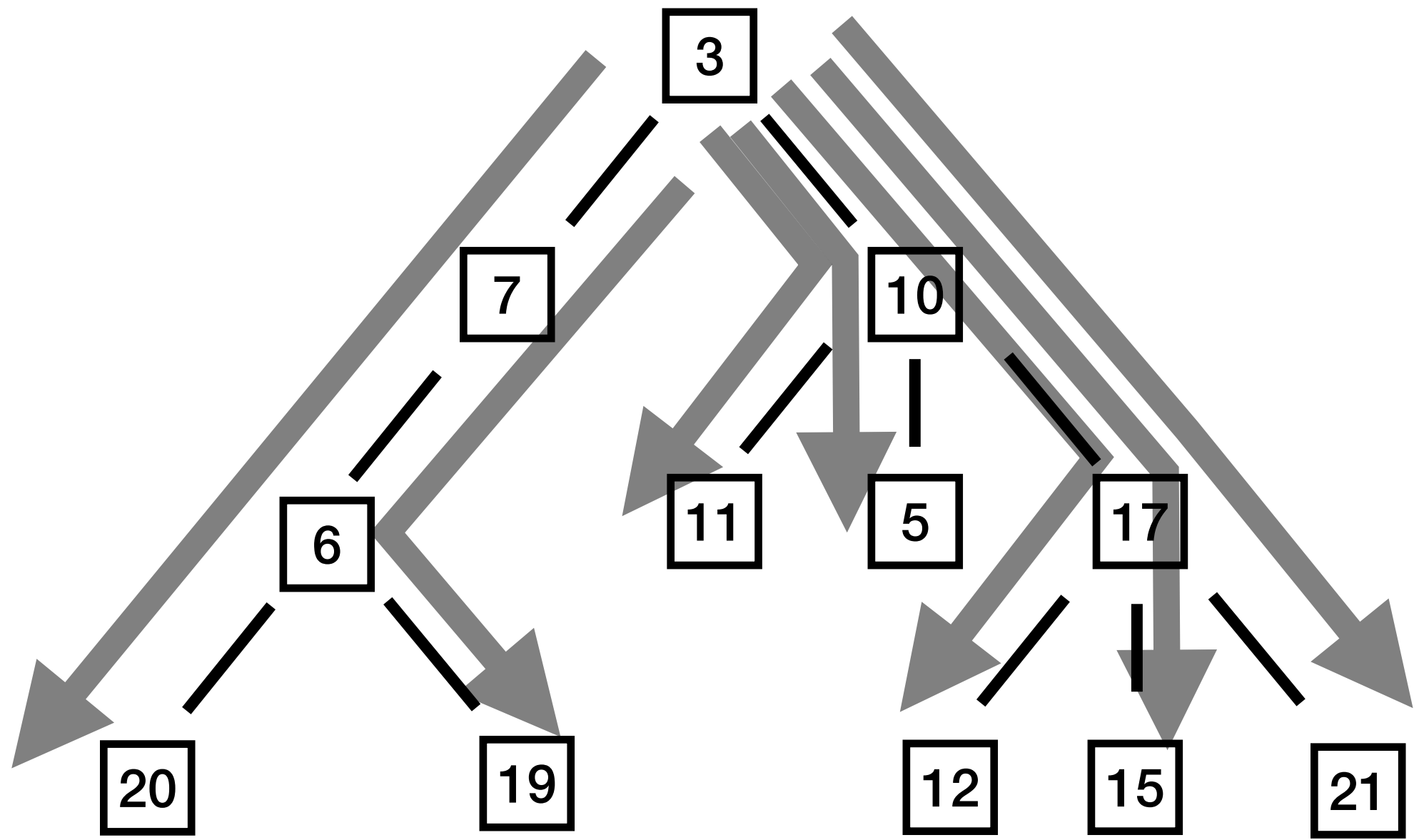
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Depth First Search (DFS)



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Kadane's Algorithm

“find the maximum sum contiguous subarray”

- empty = 0
- all positive: solution is full array
- all negative: solution is empty array

Find the largest sum:

Initialize

```
best_sum = 0
current_sum = 0;
```

```
for each element x (left to right):
    current_sum = max(0, current_sum+x)
    best_sum = max(best_sum, current_sum)
```

...best = 4, current = 4

...best = 4, current = 0

...best = 4, current = 0

...best = 7, current = 7

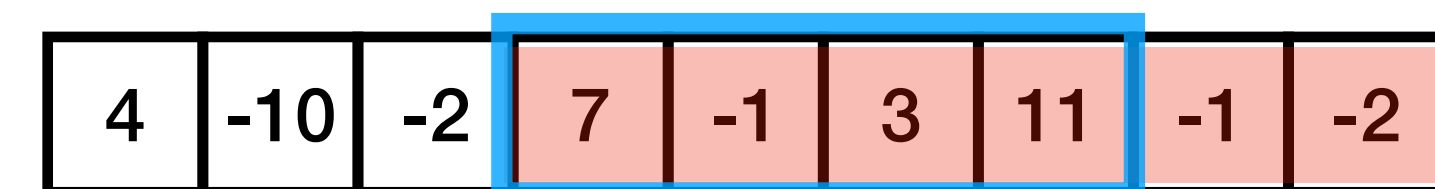
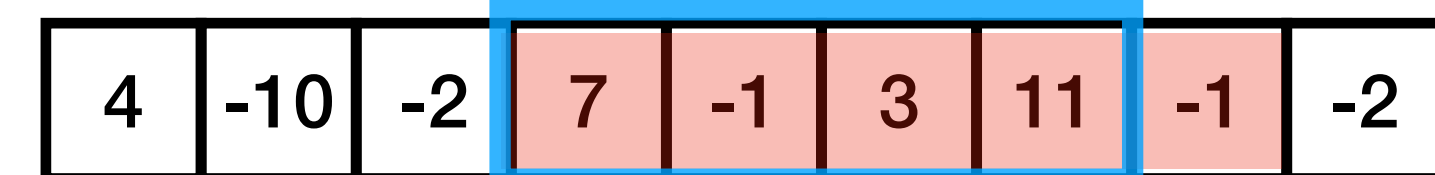
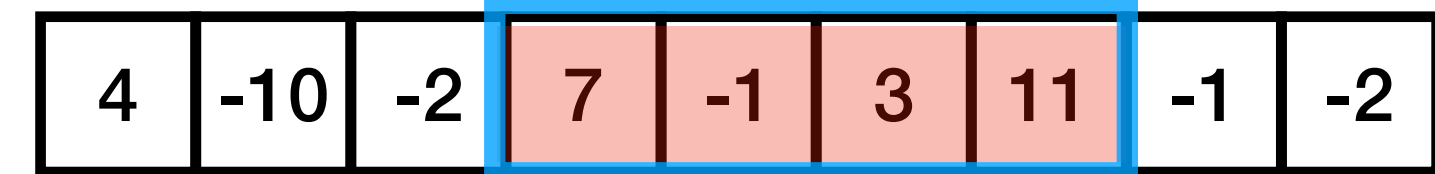
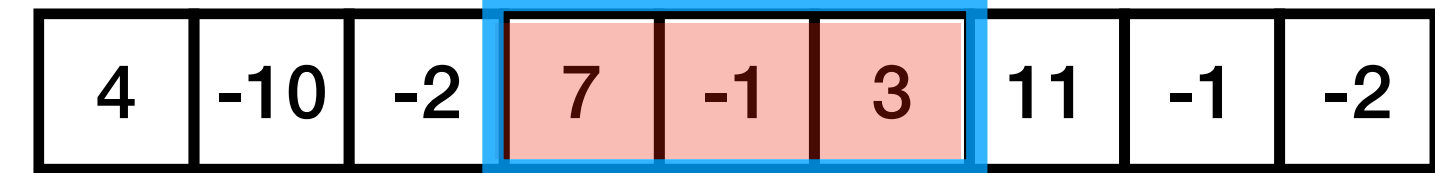
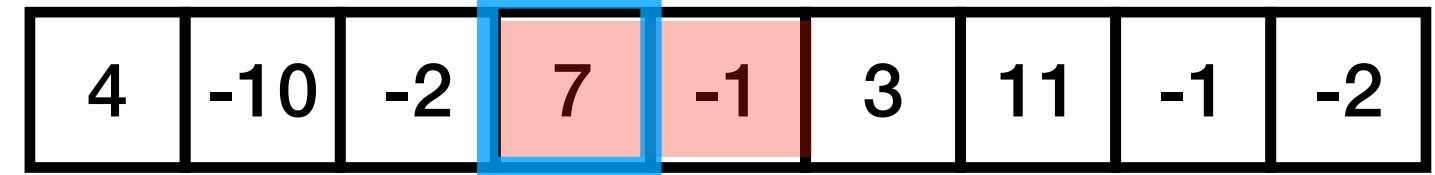
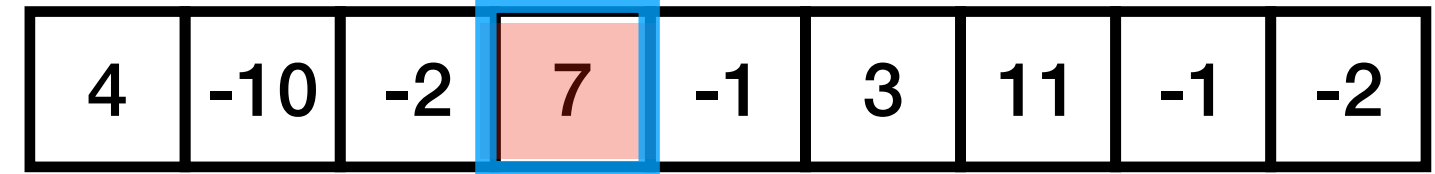
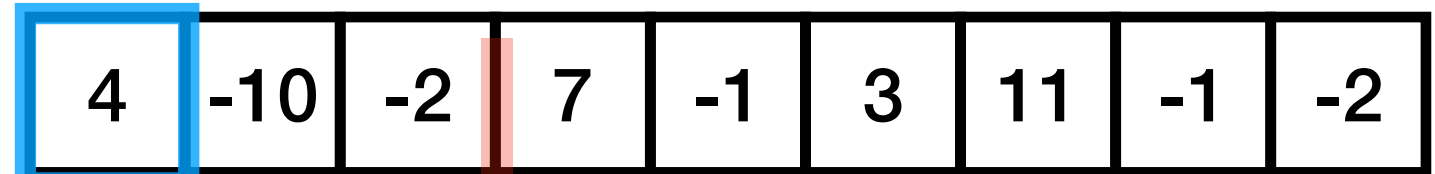
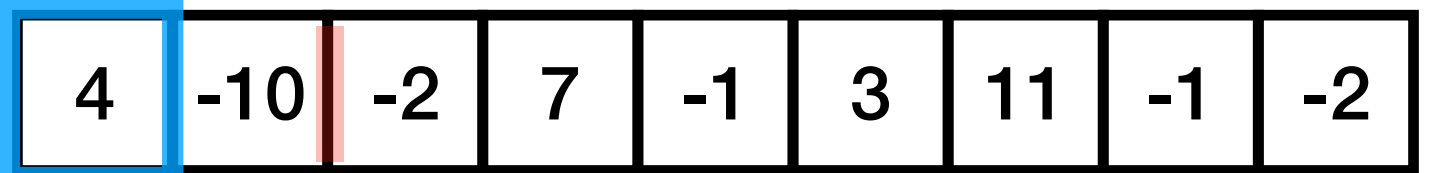
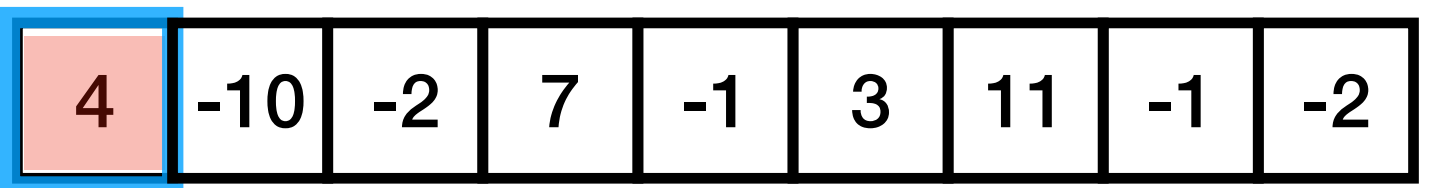
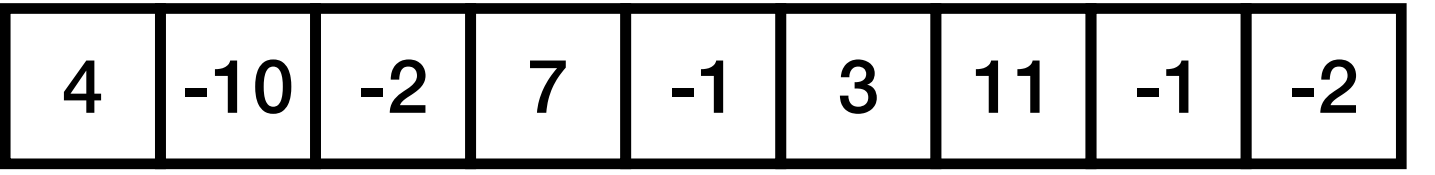
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...best = 9, current = 9

..best = 20, current = 20

..best = 20, current = 19

..best = 20, current = 17



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Kadane's Algorithm

“find the maximum sum contiguous subarray”

- empty = 0
- all positive: solution is full array
- all negative: solution is empty array

Find the largest sum w/ position

Initialize

```
best_sum = 0
current_sum = 0;
best_start = best_end = 0;
current_start = current_end = 0;
```

```
for each element x (left to right):
  If current_sum <= 0:
    current_start = current_end
    current_sum = x
  else:
    current_sum += x
```

current_end++

```
If current_sum > best_sum:
  best_sum = current_sum
  best_start = current_start
  best_end = current_end
```

...best = 4, current = 4

...best = 4, current = 0

...best = 4, current = 0

...best = 7, current = 7

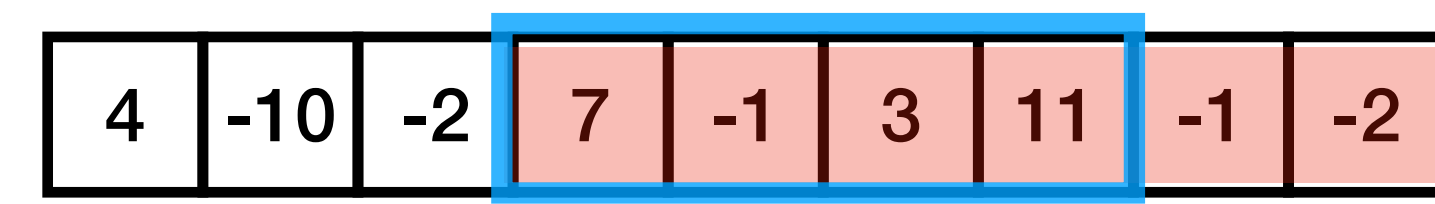
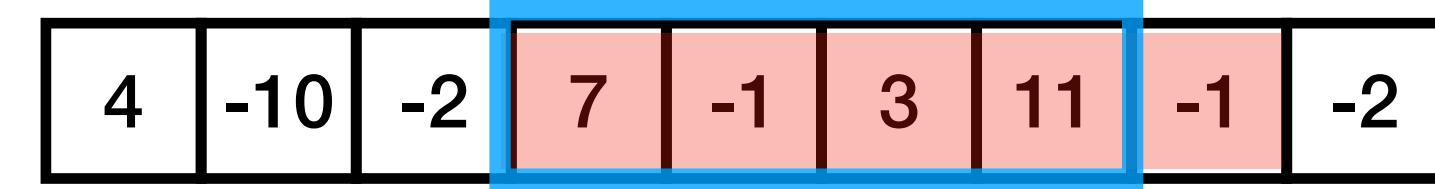
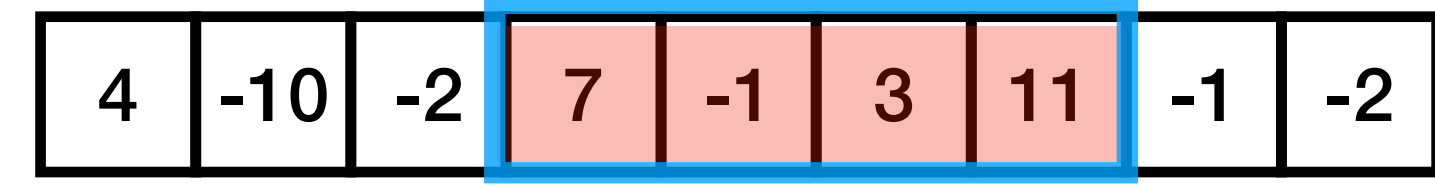
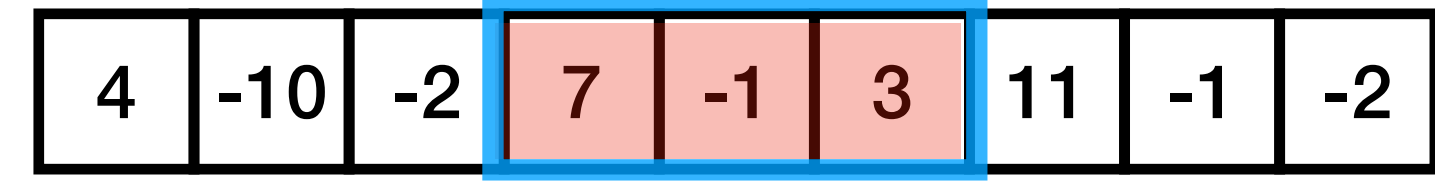
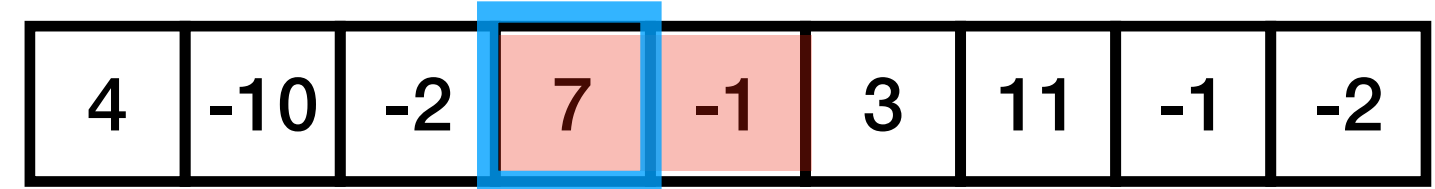
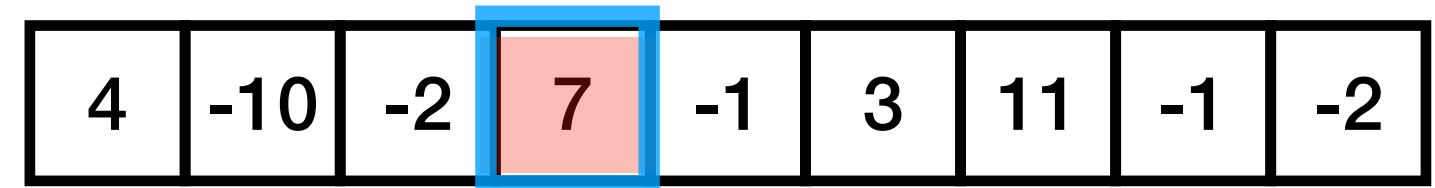
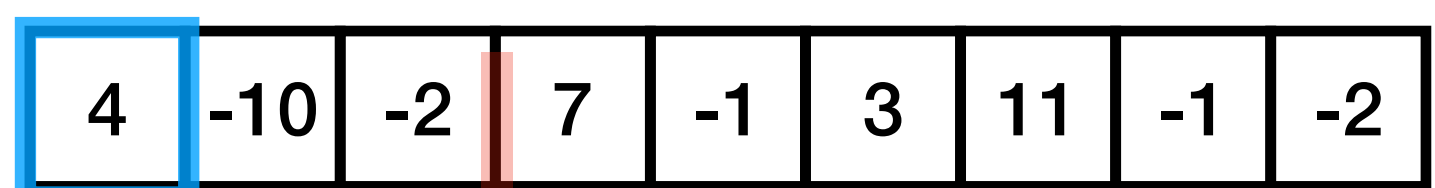
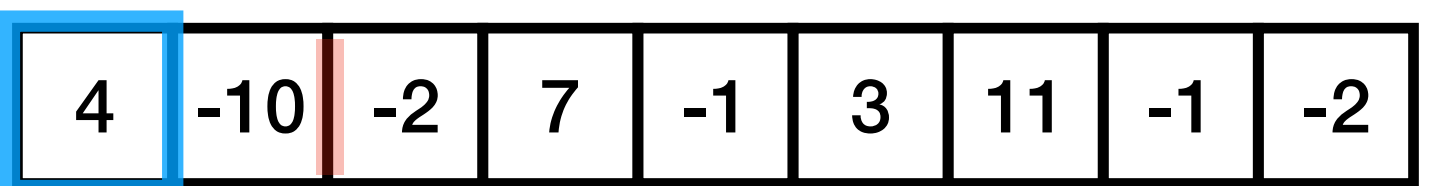
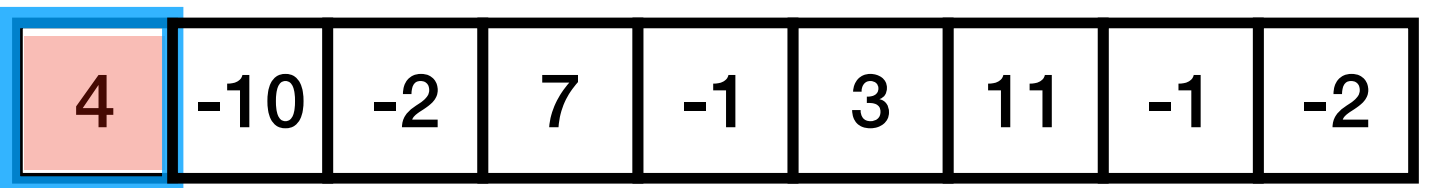
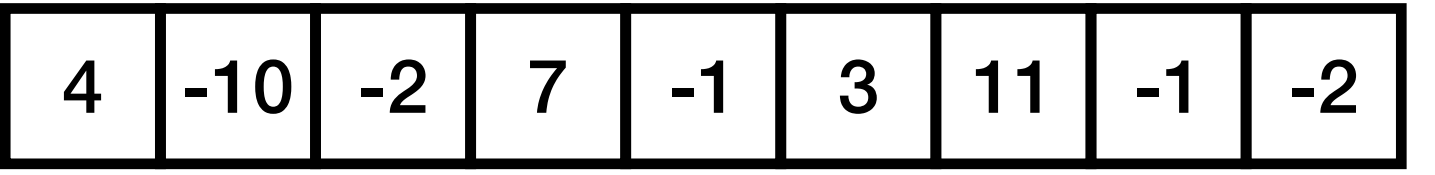
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Algorithms

Computation Cost:
Worst Case: $O(nm)$

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Graph Algorithms

Sorting

- Kahn's topological sort (DAG)

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Shortest path

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Compression Algorithms

- Huffman Coding

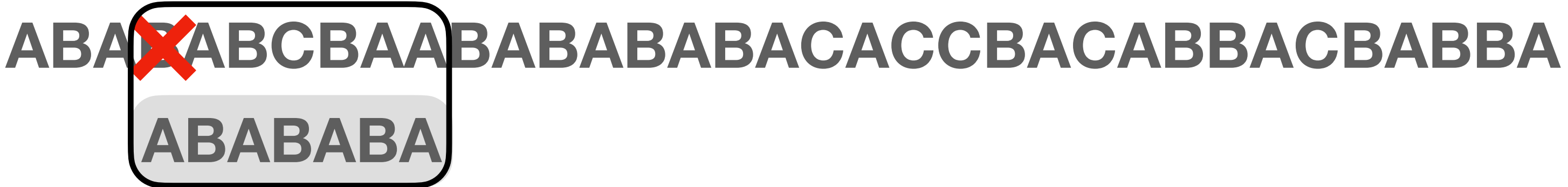
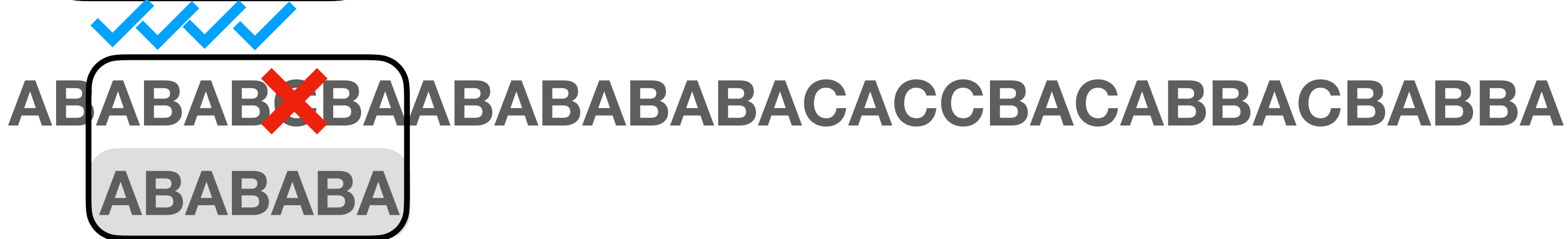
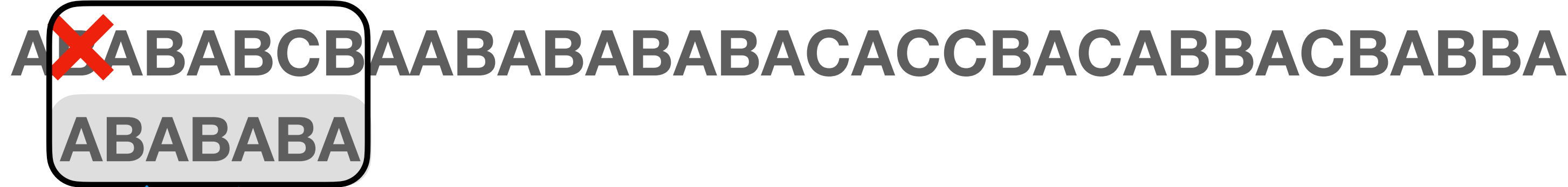
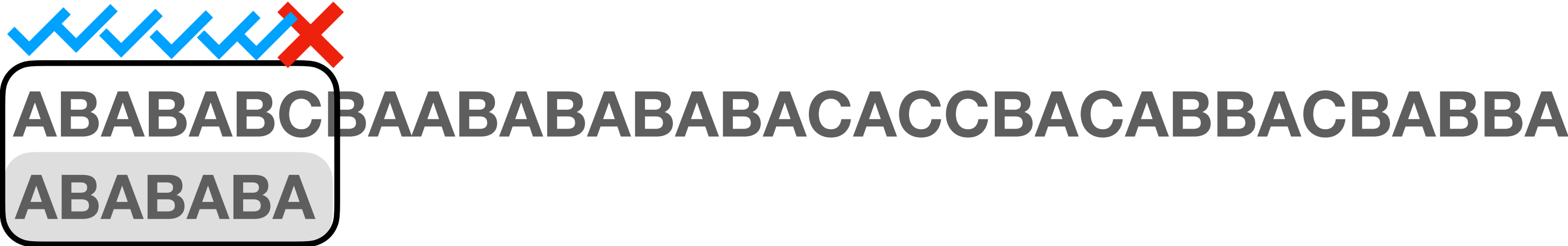
Fill Algorithm

- Flood Fill Algorithm

Naive Pattern Matching

"find a pattern of length m in an array of length n"

Pattern (m): **ABABABA** Array (n): **AABABCBAABABABABACACCCBACABBACBABBA**



⋮

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Computation Cost: $O(n + m)$

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KMP Algorithm (Knuth-Morris-Pratt)

"find a pattern of length m in an array of length n"

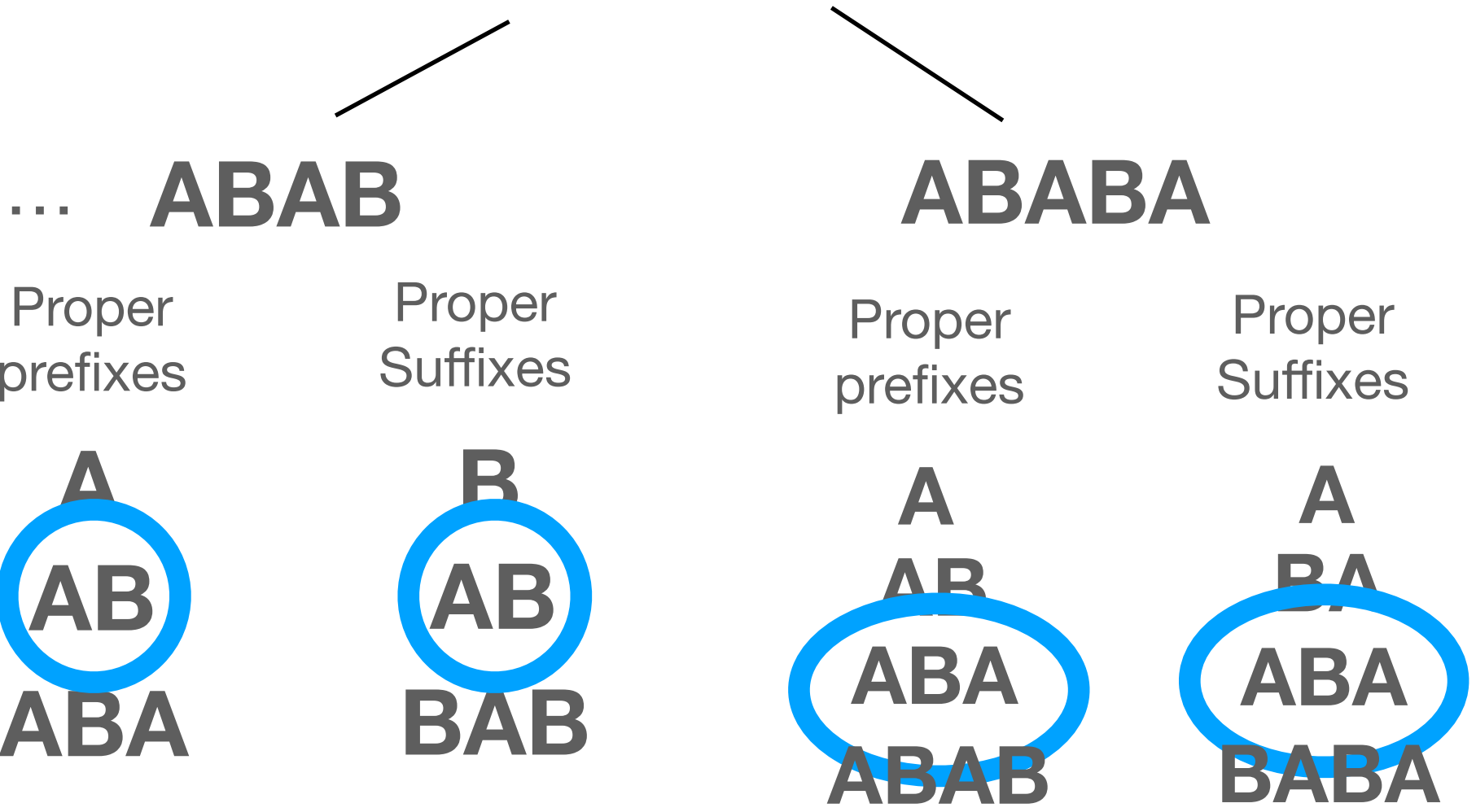
Pattern (m): **ABABABA** Array (n): **AABABCBAABABABABACACCBACABBACBABBA**

preprocess the pattern to save time...



Partial Match Table:

<i>..first 1 char</i>	<i>..first 2 char</i>				<i>..first 6 char</i>	<i>..first 7 char</i>
A	AB	ABA	ABAB	ABABA	ABABAB	ABABABA
0	0	1	2	3	4	5



longest proper prefix that is also a suffix"

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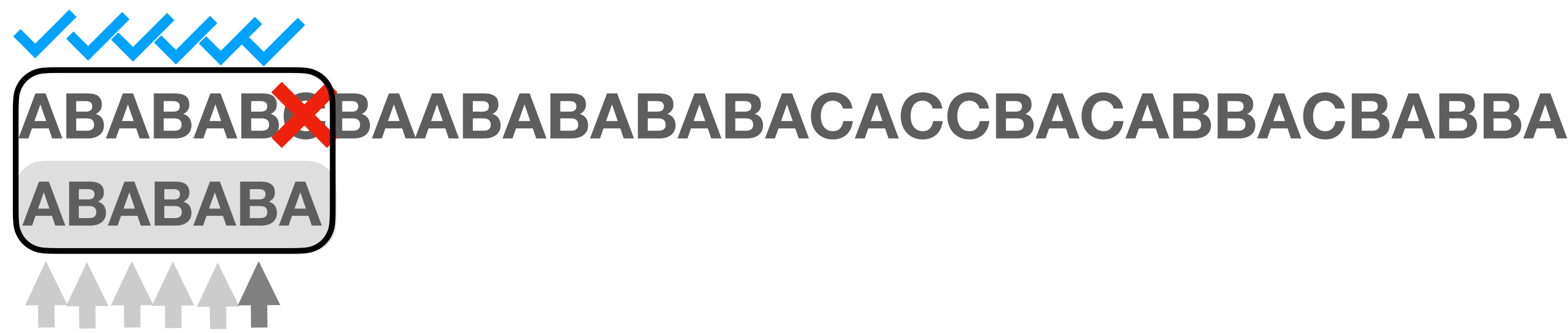
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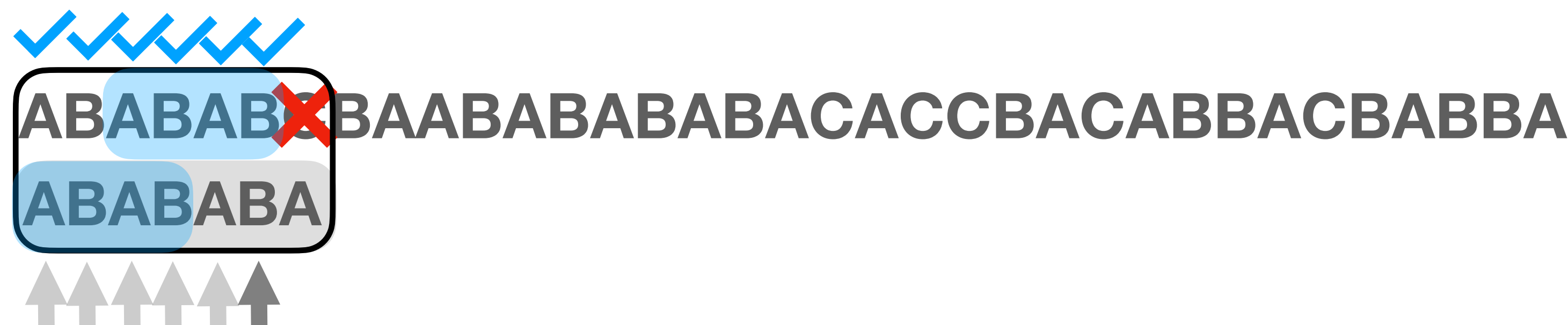
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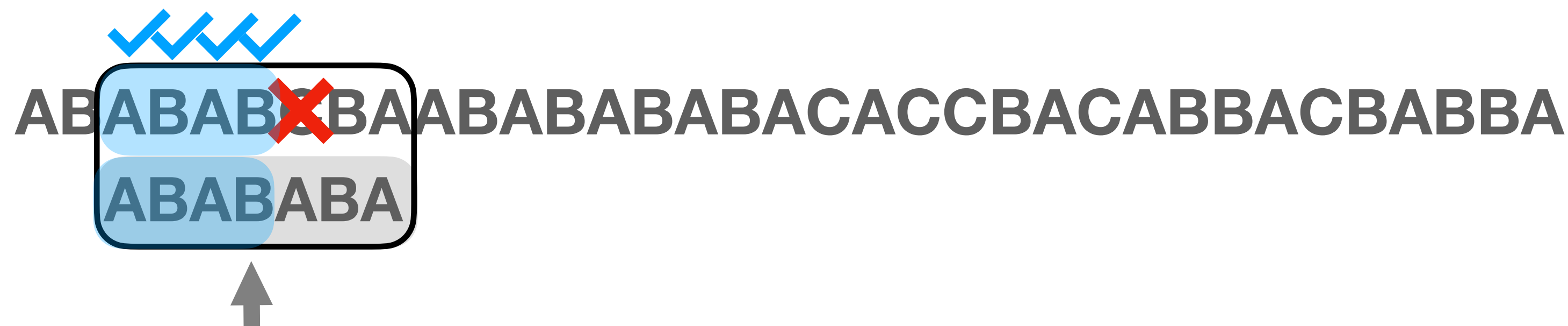
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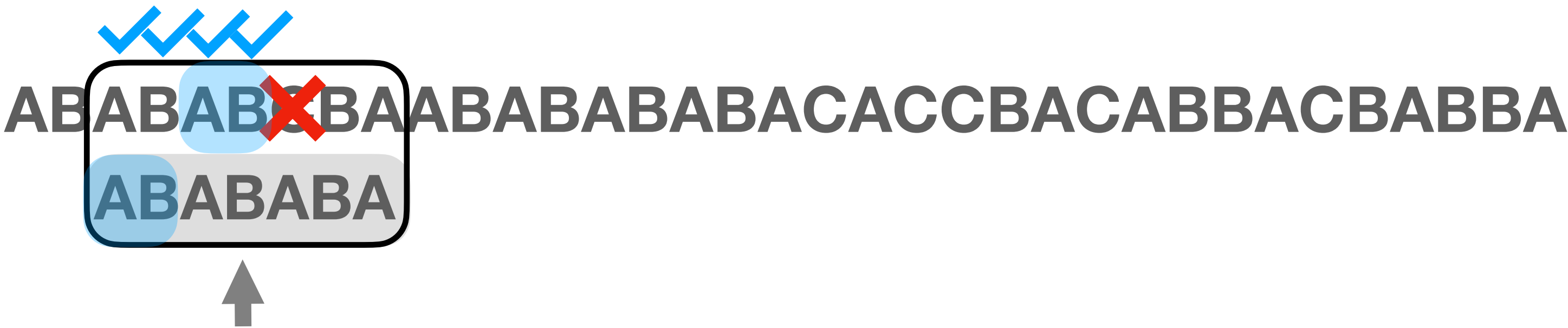
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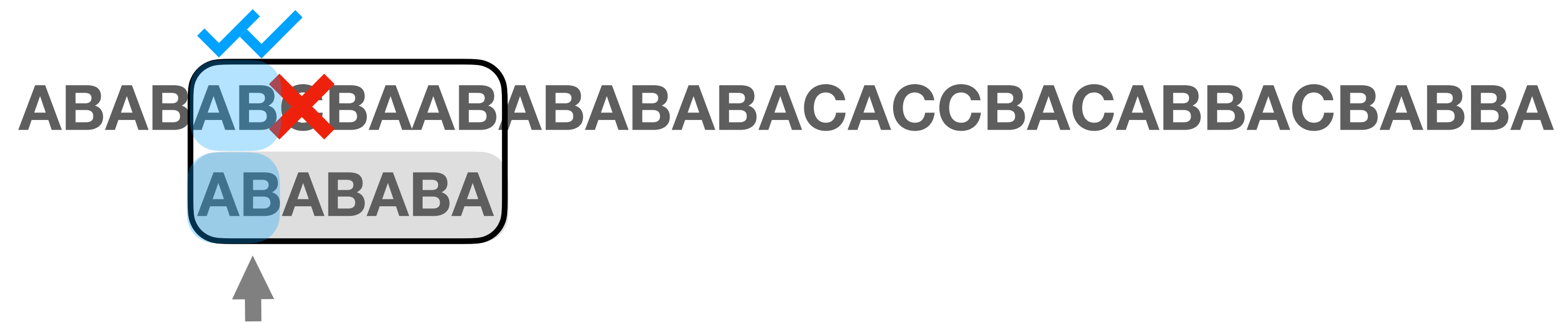
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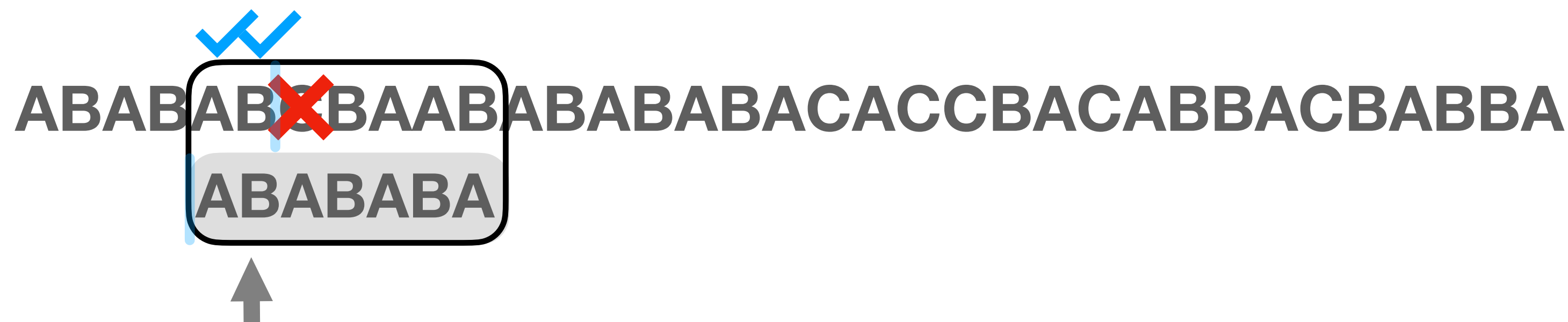
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Computation Cost: $O(n + m)$

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Shortest path

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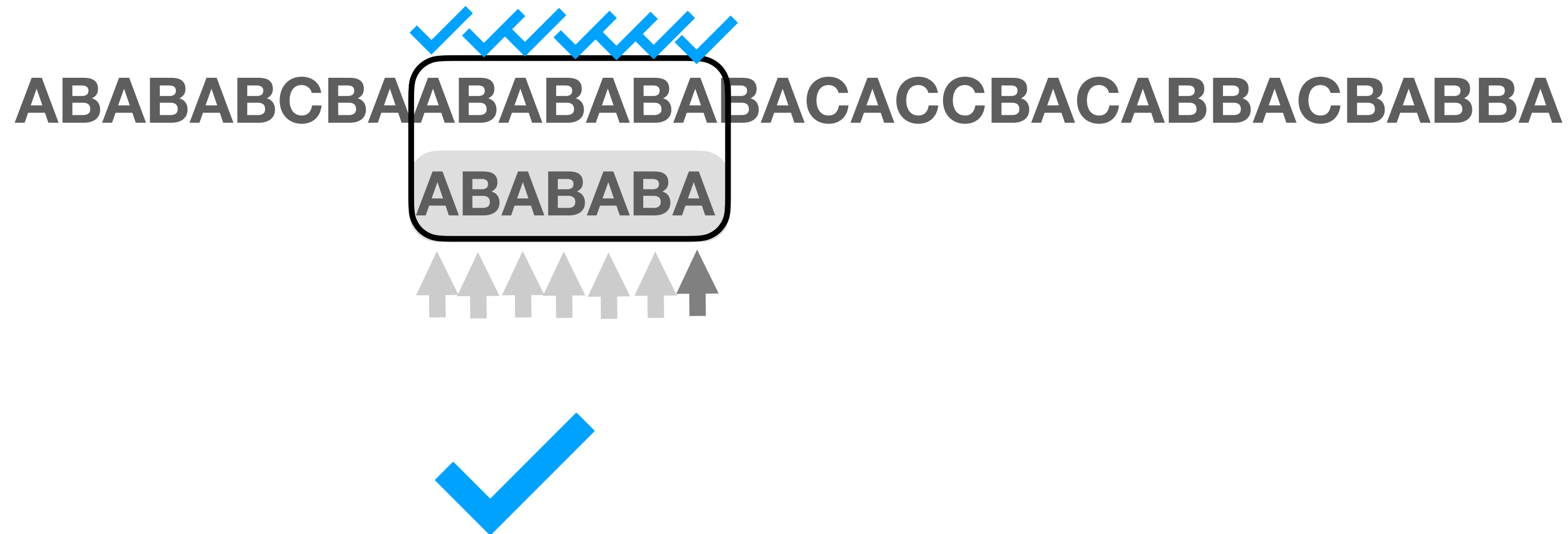
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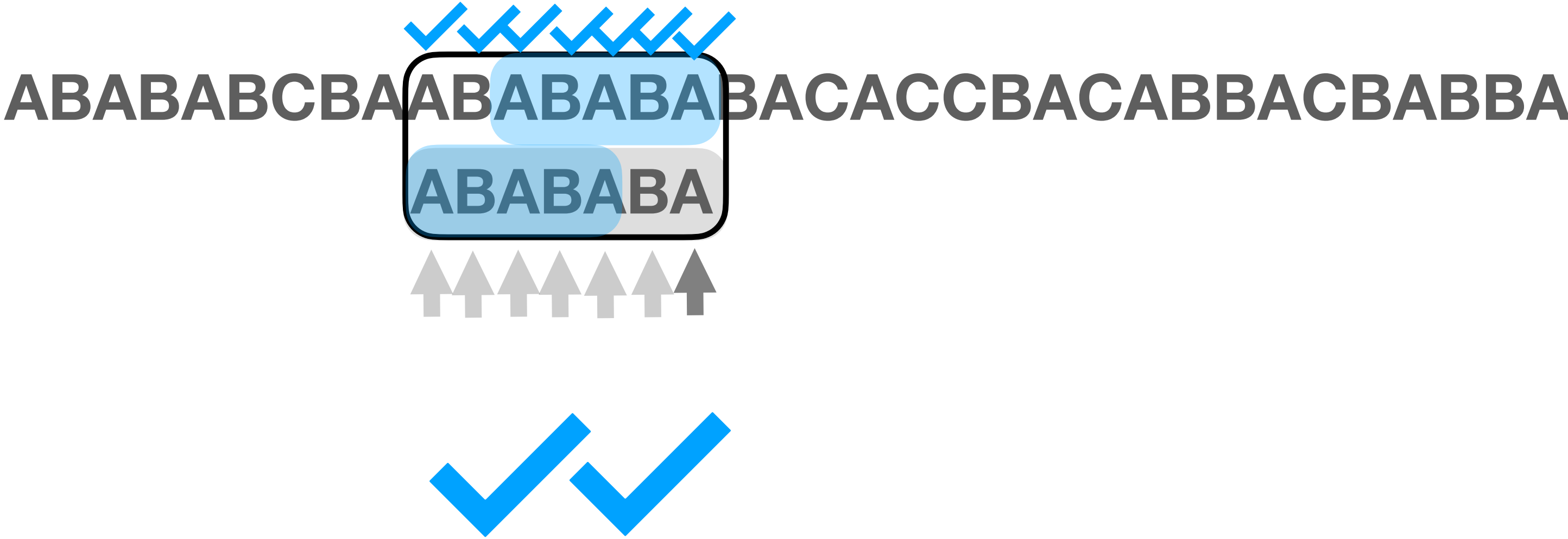
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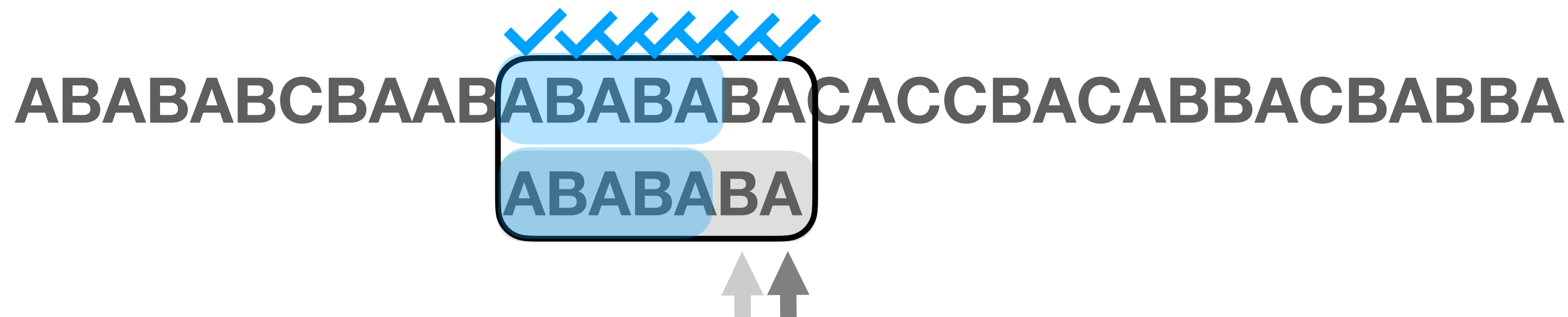
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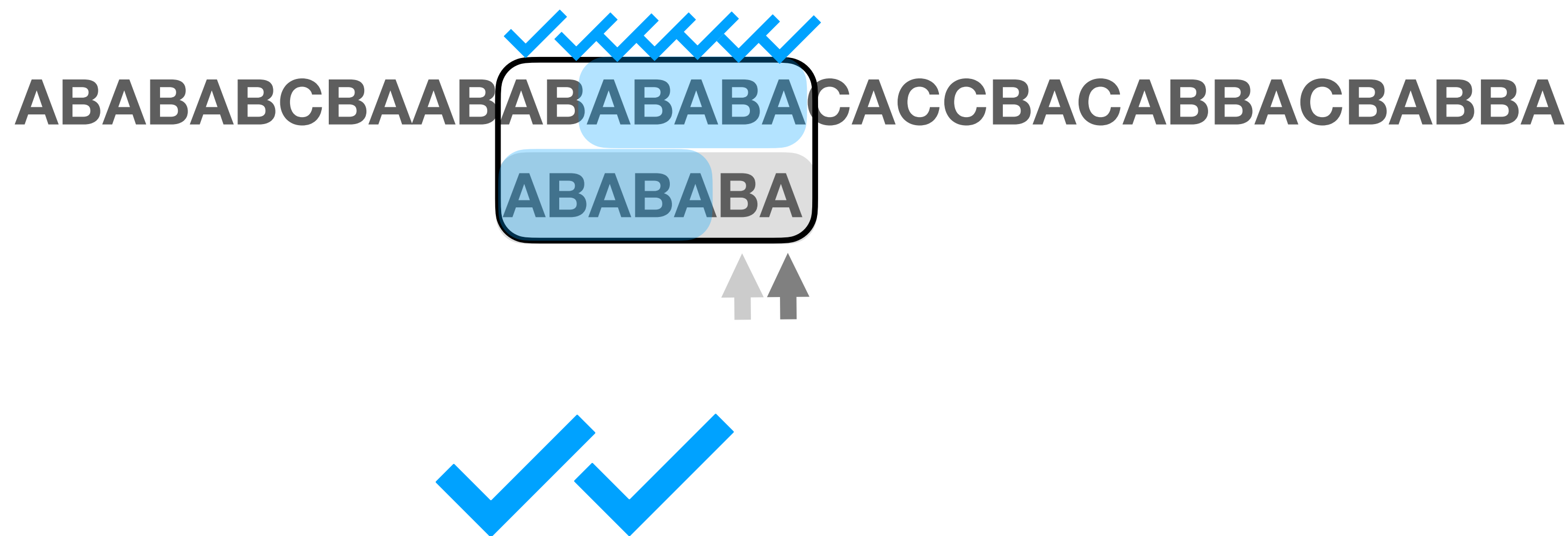
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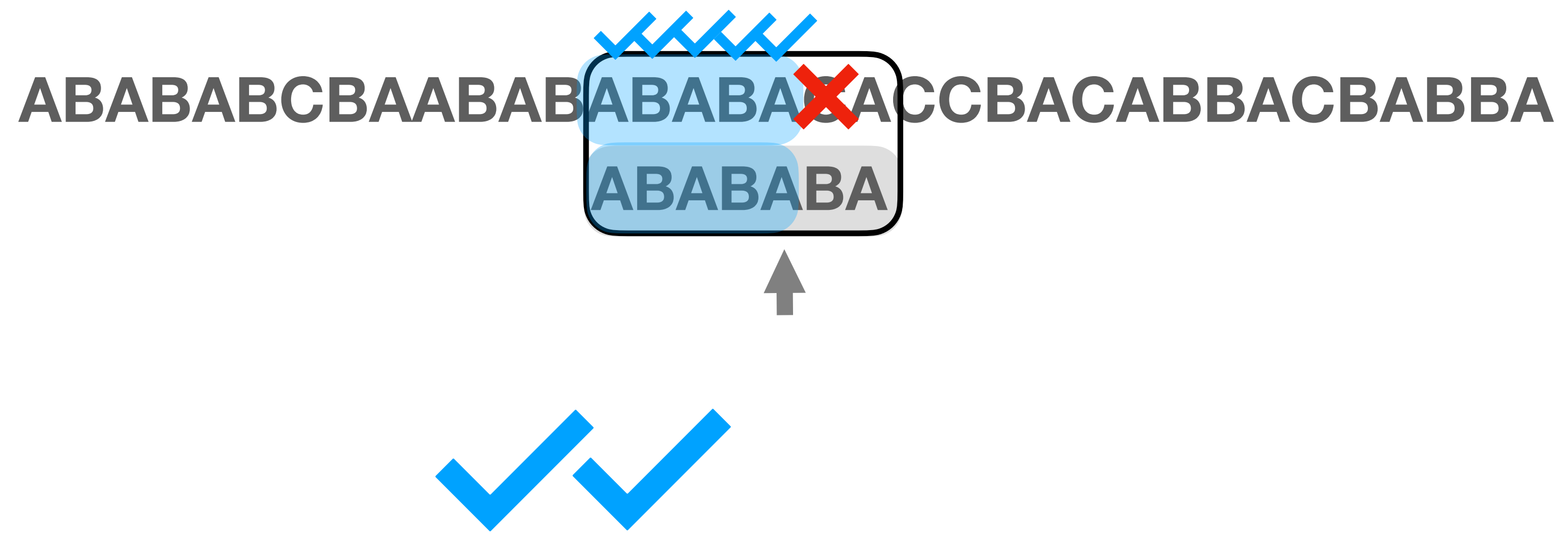
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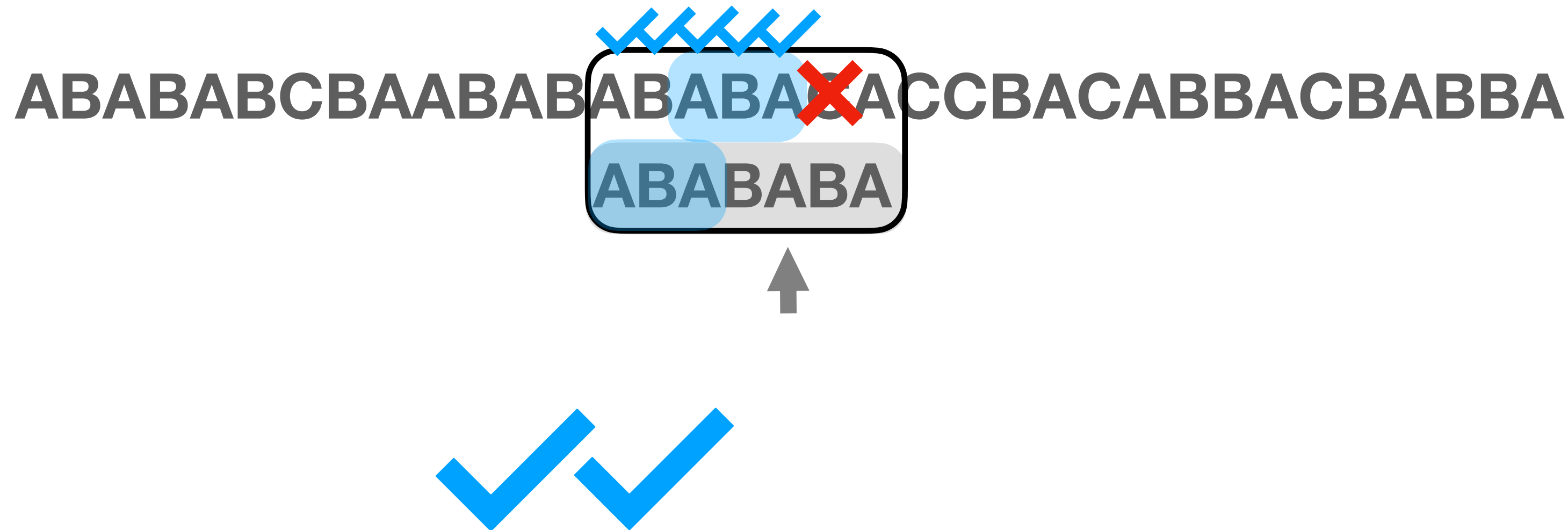
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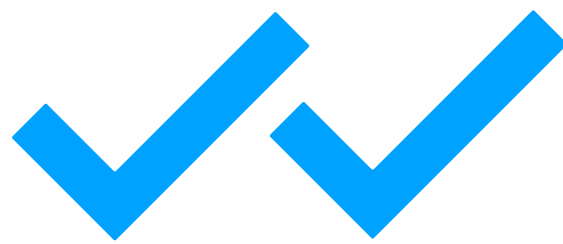
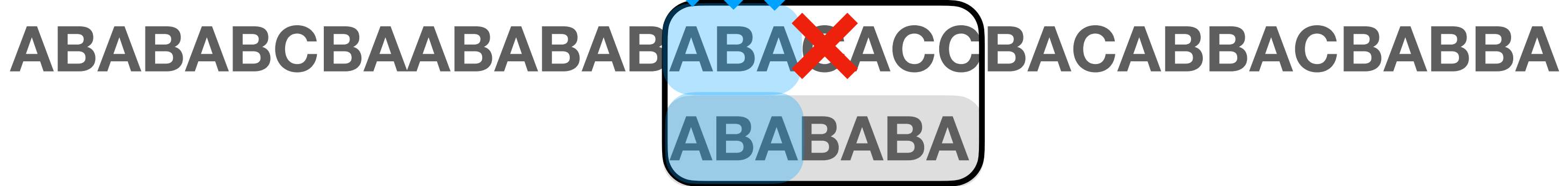
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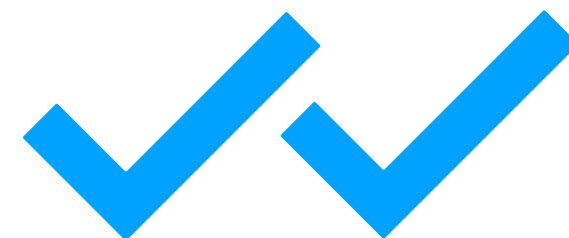
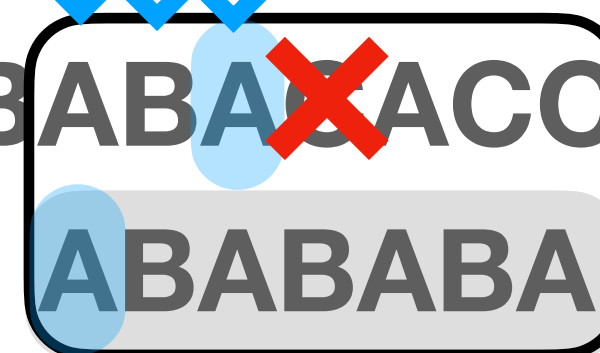
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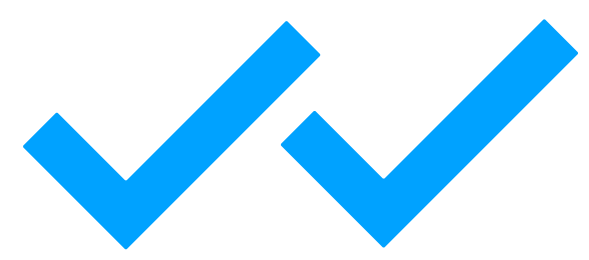
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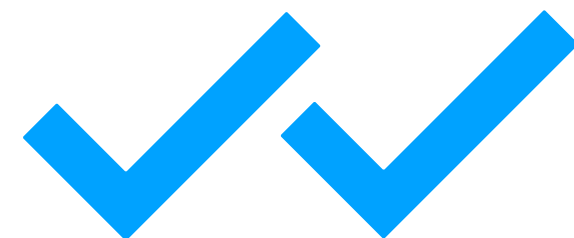
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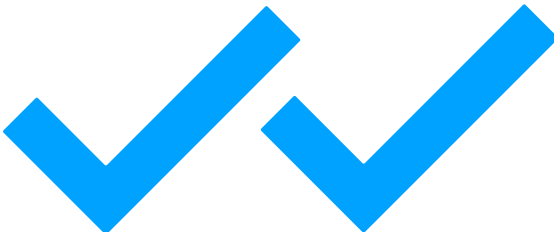
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"find a pattern of length m in an array of length n"

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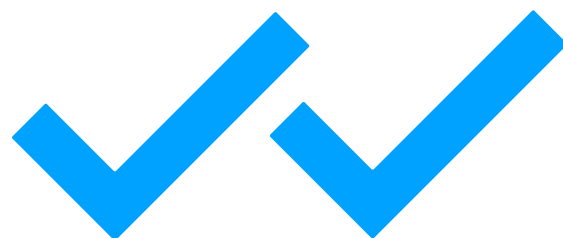
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A	AB	ABA	ABAB	ABABA	ABABAB	ABABABA
0	0	1	2	3	4	5



ABABABCBAABABABABACACCBACABBACBABBA

ABABABA

(The above text is highlighted in a grey box with a black border. A red 'X' is over the 'C' in the array above it, and a blue checkmark is above the 'A' in the pattern below it.)



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Computation Cost: $O(n + m)$

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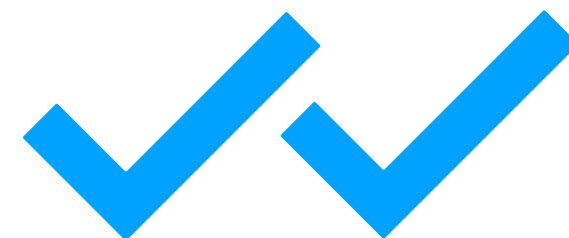
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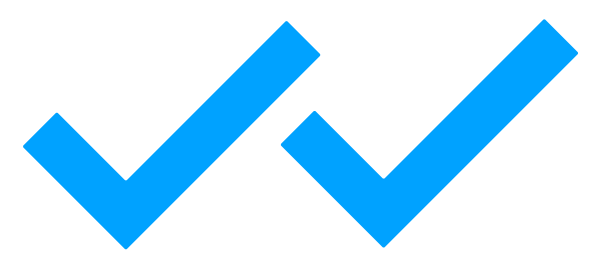
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ABABABCBAABABABABACACCC ~~**ACABBACBABBA**~~

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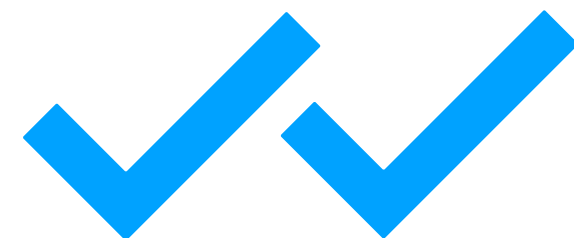
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✓

ABABABA

✗

↑



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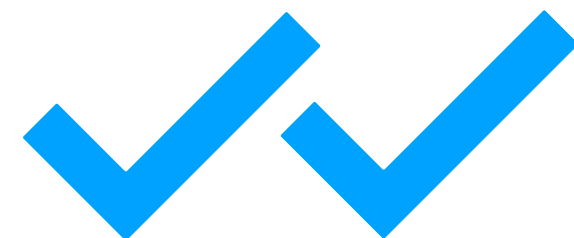
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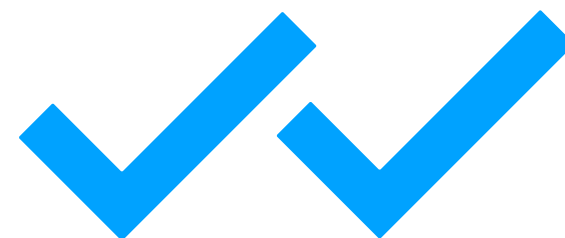
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Sorting Algorithms:

Considerations:

- **Computation time:**
 - **$O(1)$, $O(n)$, $O(n^2)$, $O(n \log(n))$, etc**
 - Worst case? Best case? Average case?
- **Stable sort:**
Is the order of elements with the same value maintained?
yes = **stable sort**
- **In place:**
Is the array sorted in place?
- **Auxiliary storage:**
How much extra storage is required?

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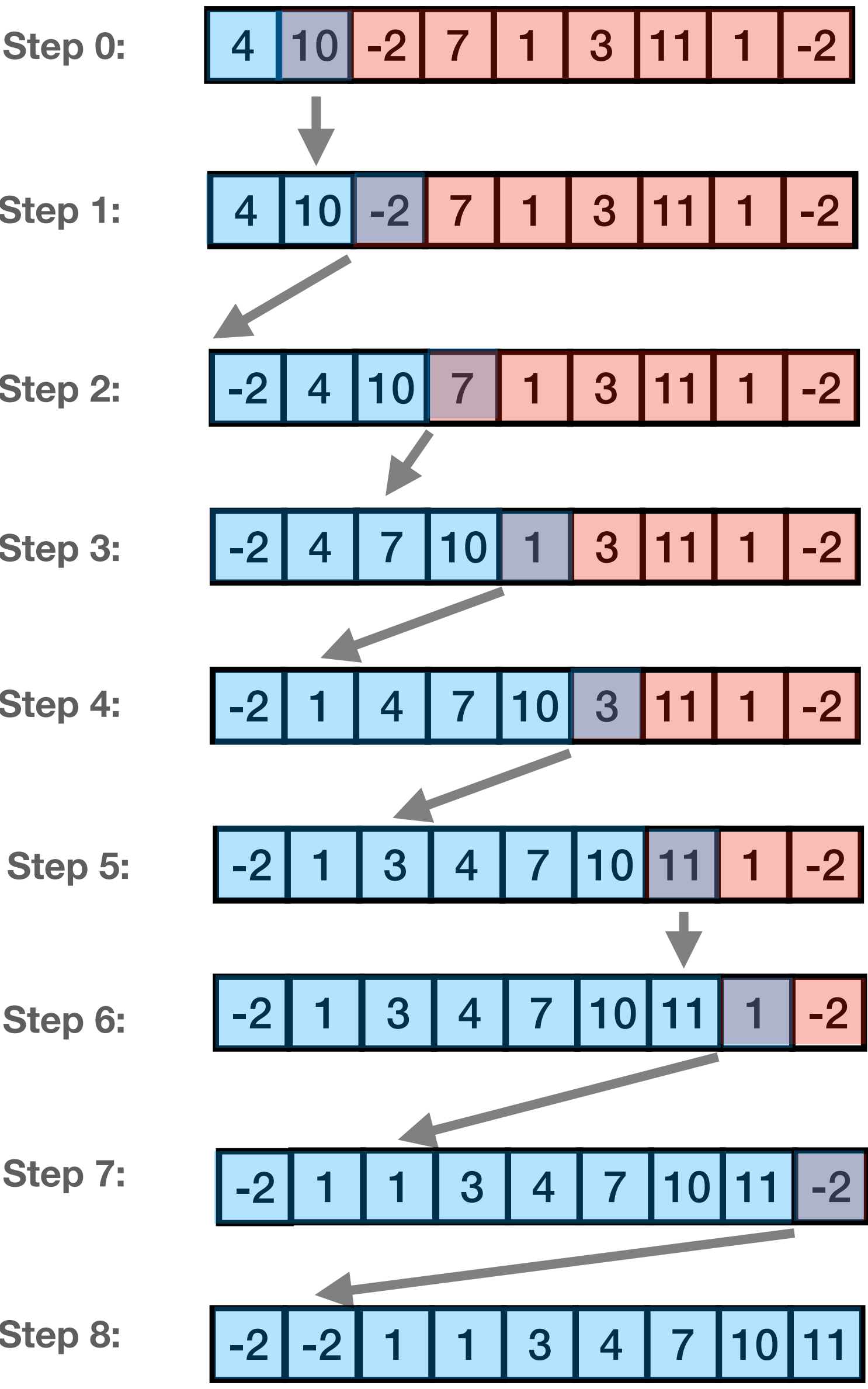
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Insertion Sort:



Computation Cost:
Worst case: $O(n^2)$
Stable sort: **True**

When to use:
Stable, Quick and dirty

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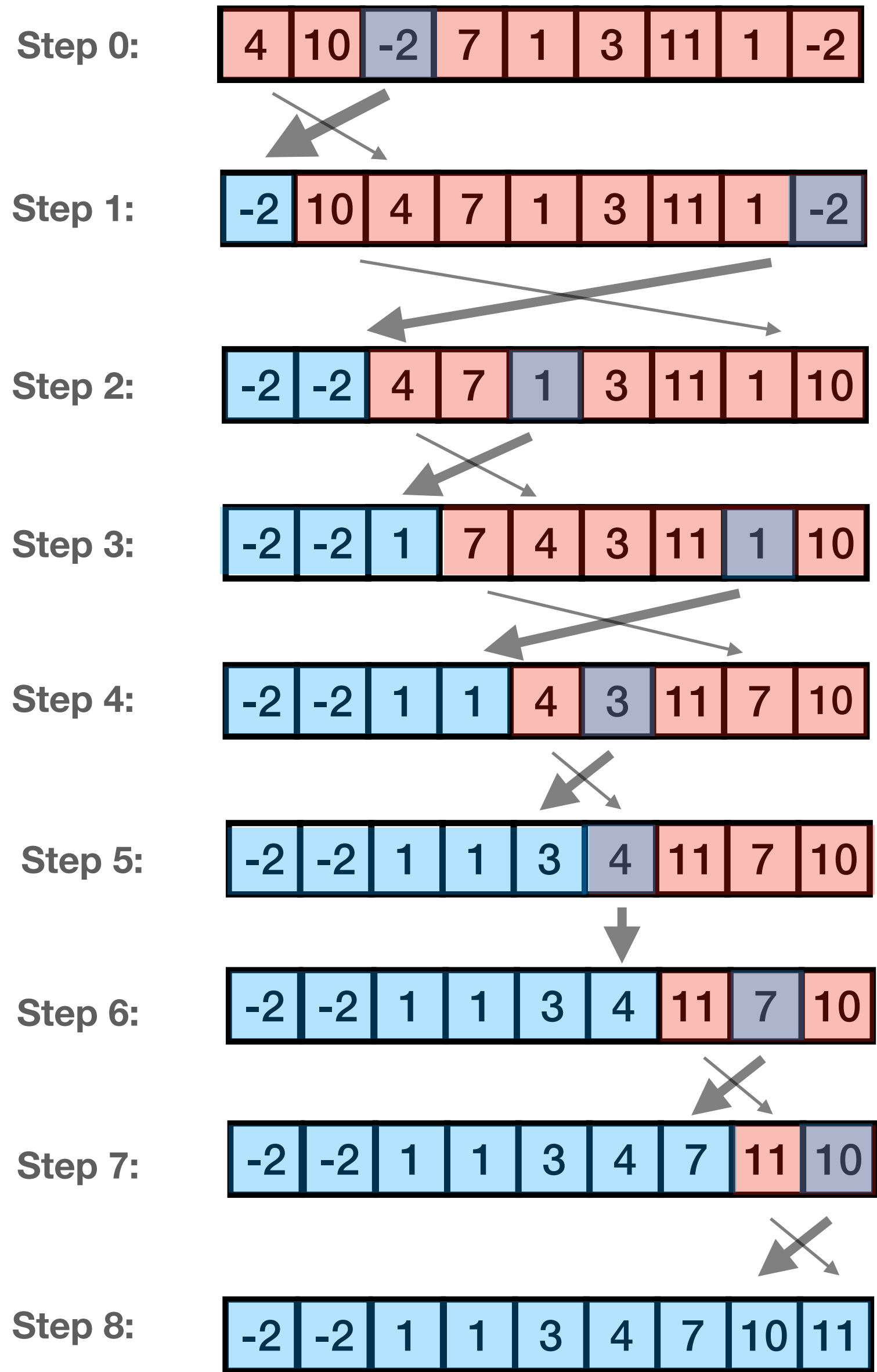
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Selection Sort:



Computation Cost:
Cost: $O(n^2)$
Stable sort: **False**
In place: **True**

When to use:
Quick and dirty

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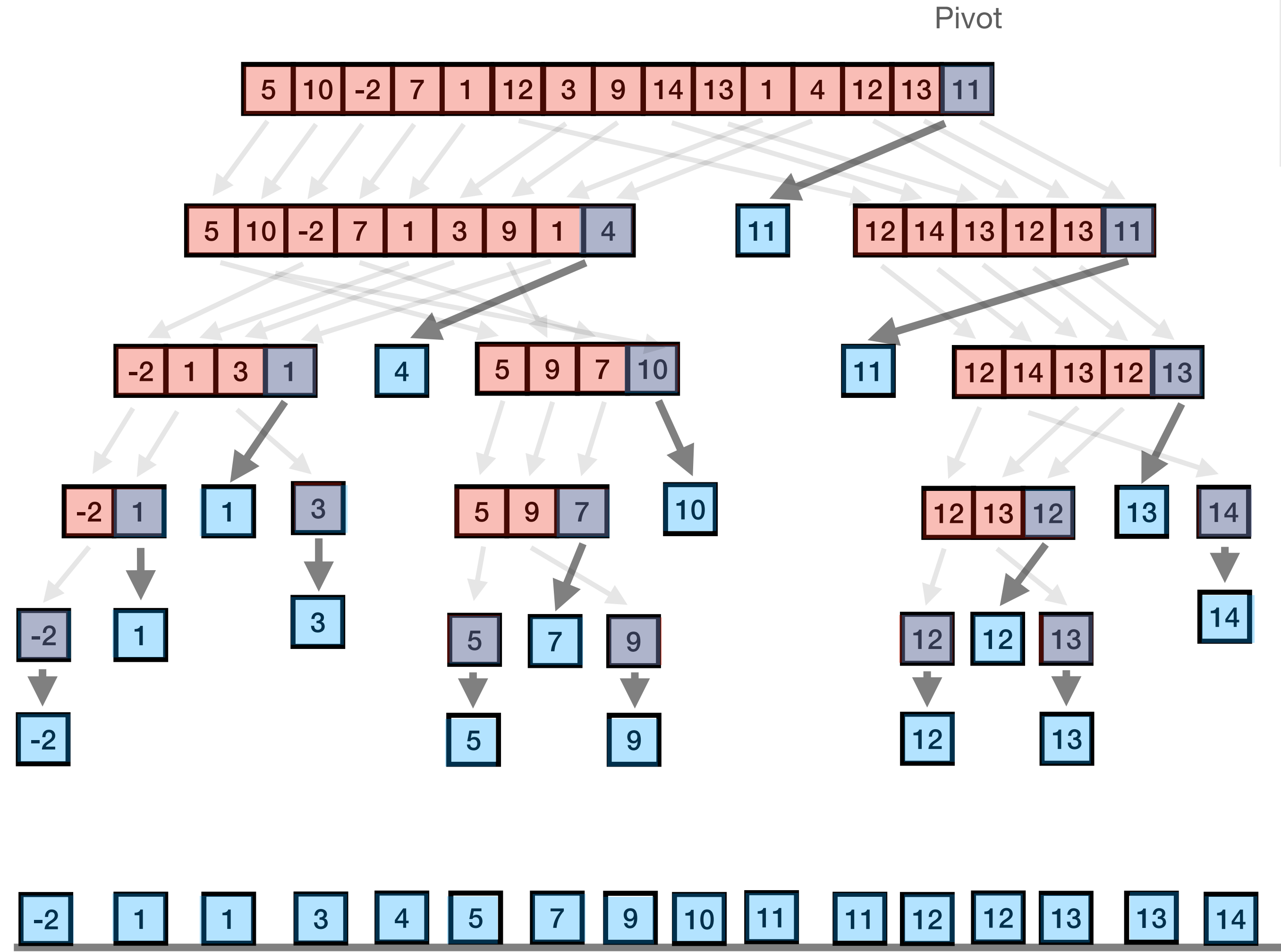
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Quick Sort:



Computation Cost:
 Average: $n \log(n)$
 Worst case: n^2 (rare)
 Extra storage: $\log(n)$
 Stable: **False**
 In place: **True**

When to use:
 when you don't need a stable sort and average case is more important than worst case. A good implementation uses $O(\log(n))$ auxiliary storage.

Note:
 Best performance happens when arrays split into similar sizes. Lopsided splits cause bad performance.

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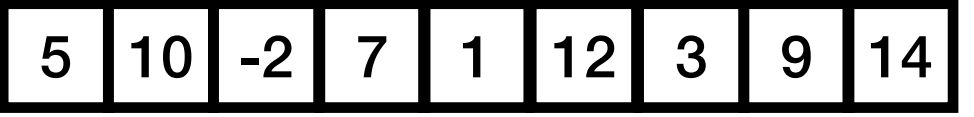
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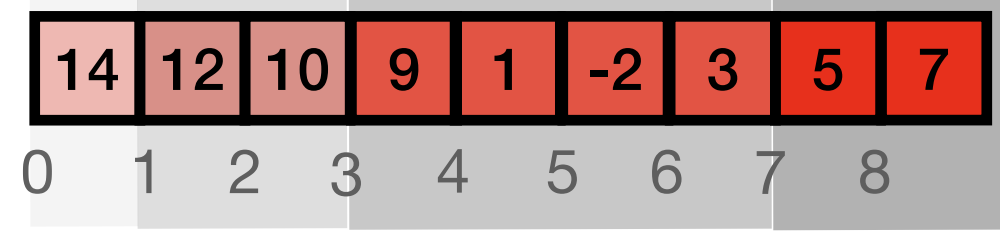
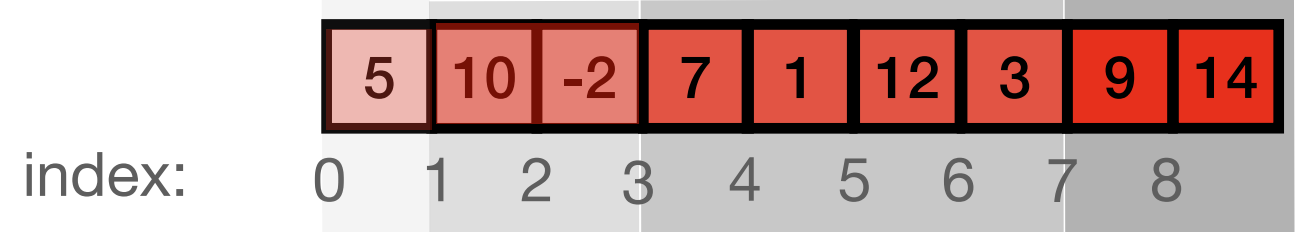
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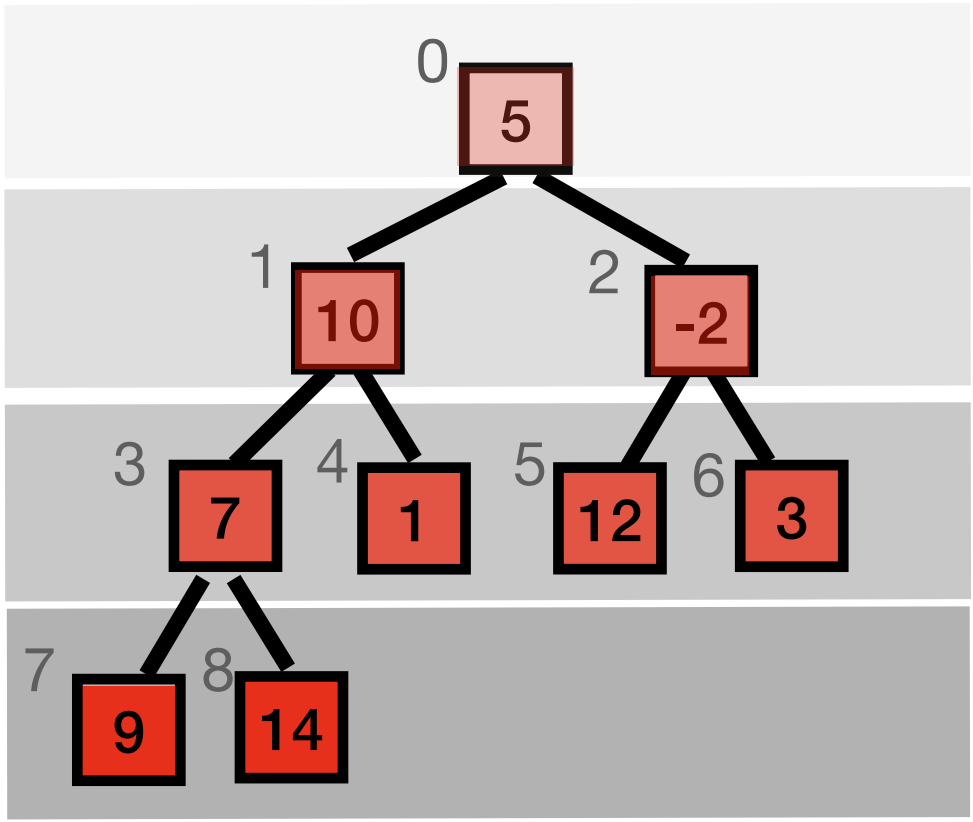
Heap Sort:



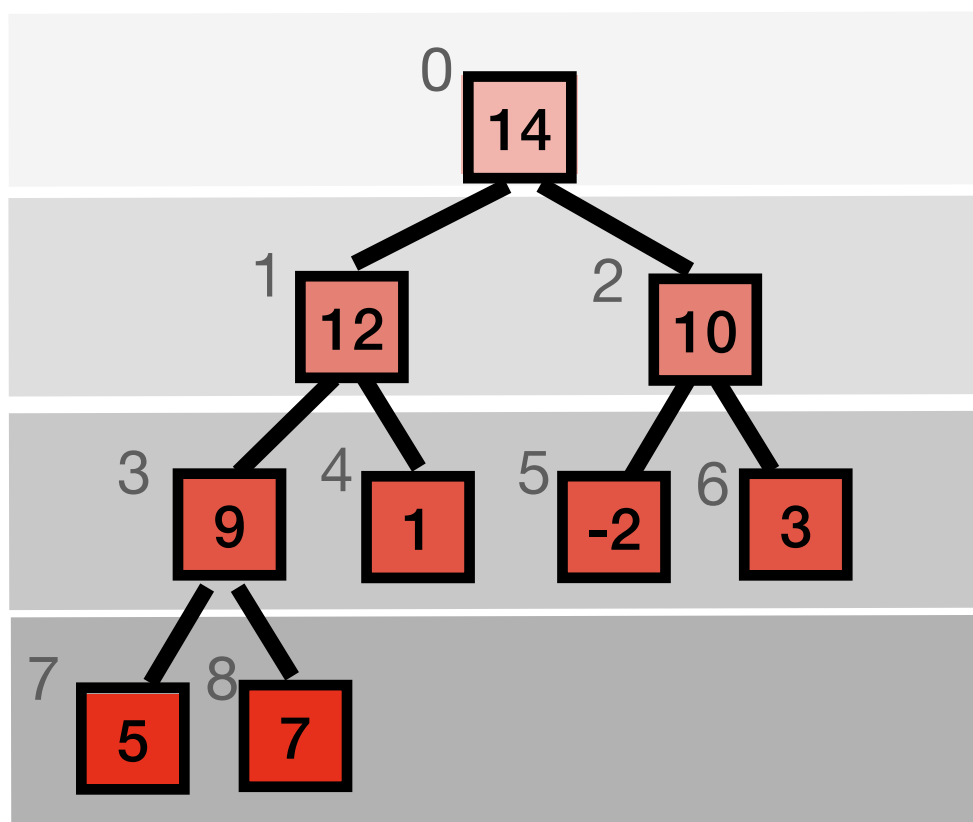
Uses a data structure called a heap...



Heap



Max Heap



Computation Cost:
 Worst case: $O(n \log(n))$
 Extra storage: $O(1)$
 In place: **possible**
 Stable: **False**

- usually slower than quicksort
- better worst case performance

When to use:
 when you don't need a stable sort and you care more about worst case performance than average. Also uses constant auxiliary storage space.

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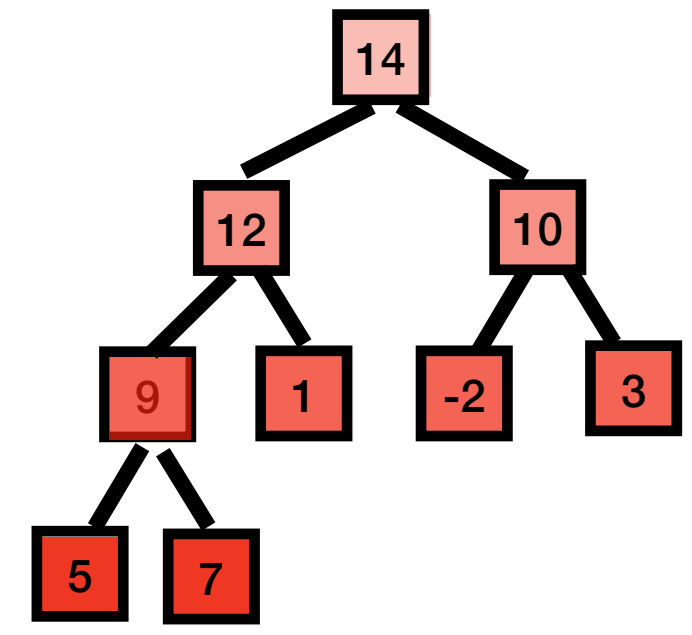
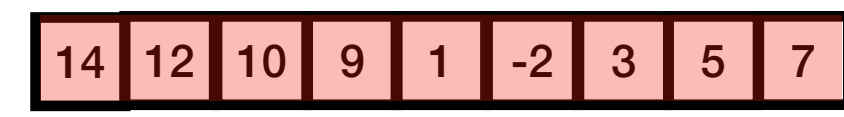
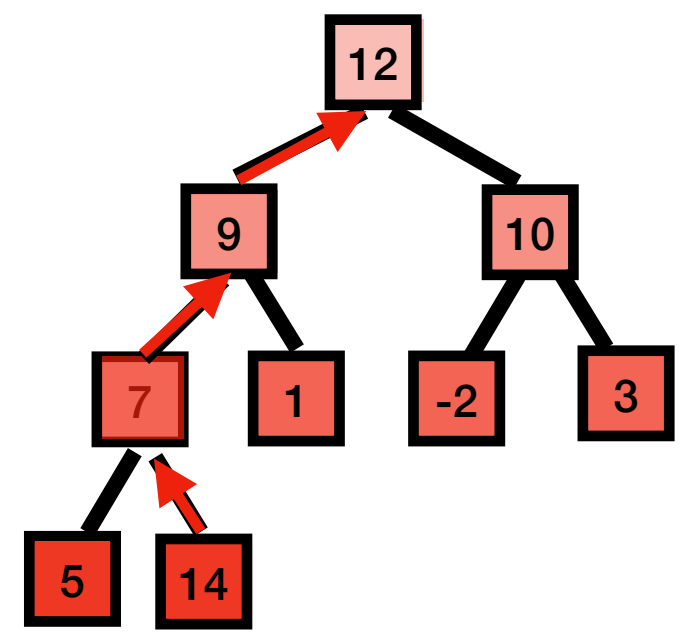
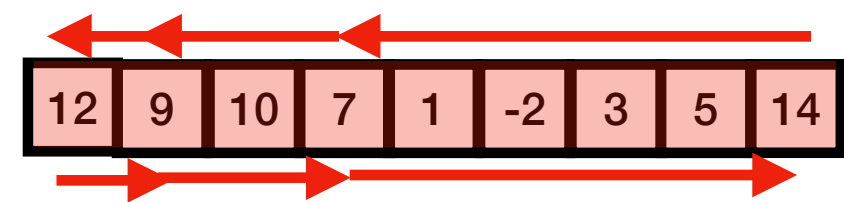
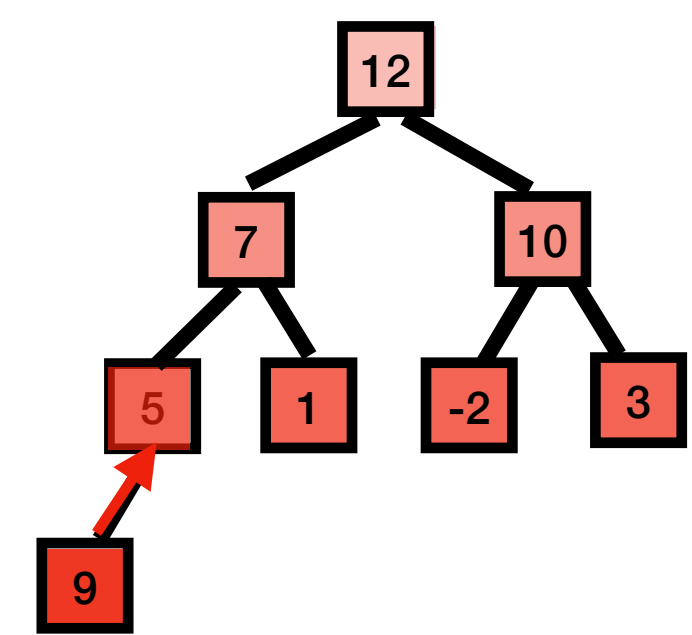
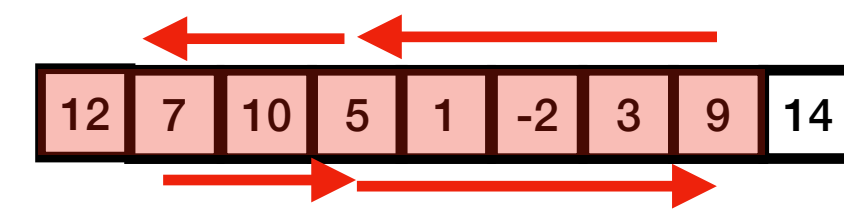
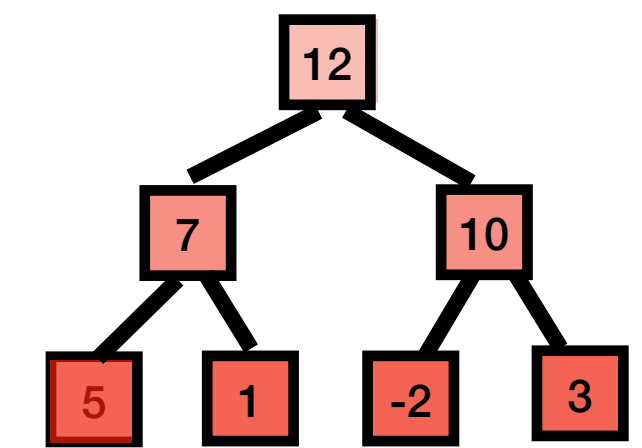
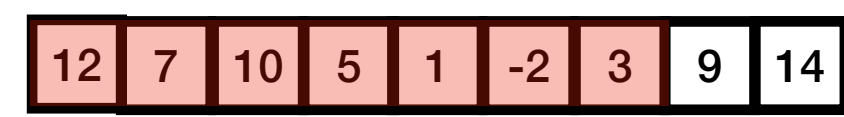
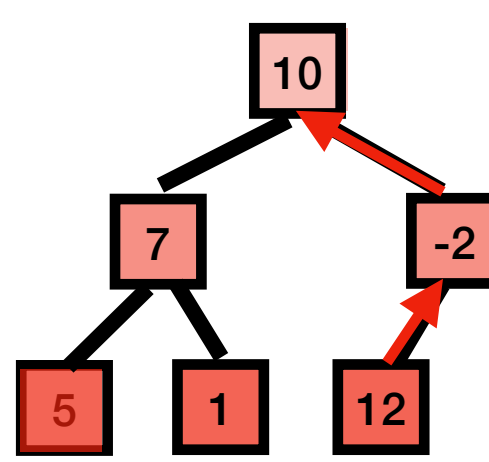
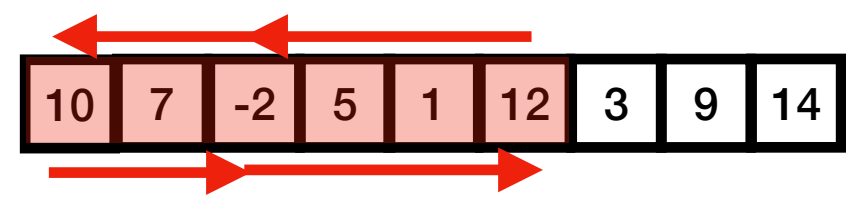
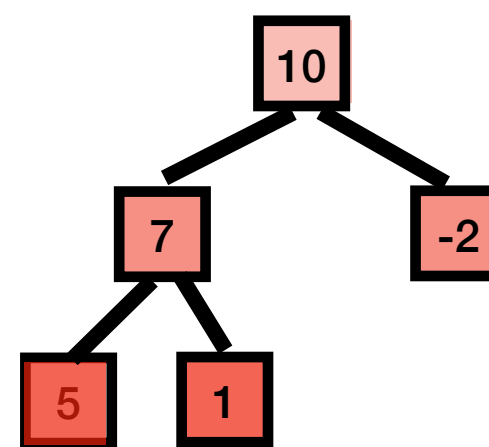
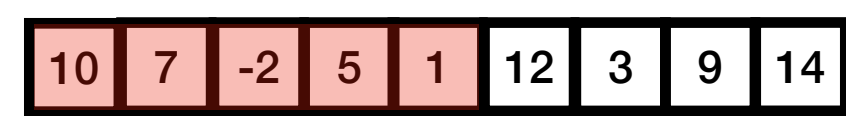
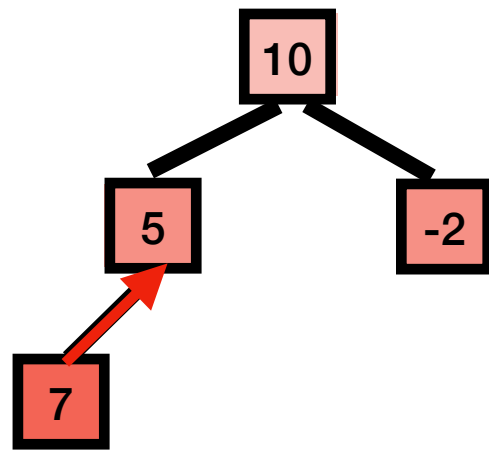
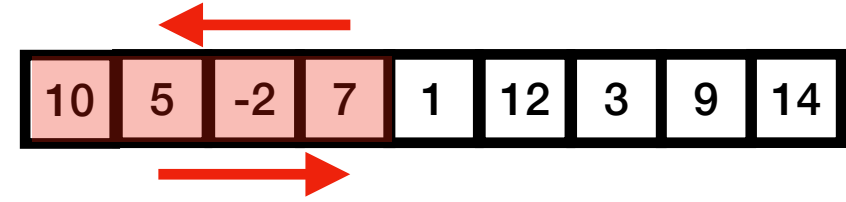
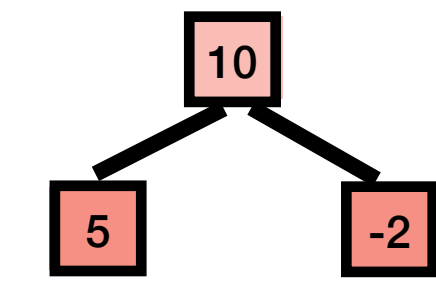
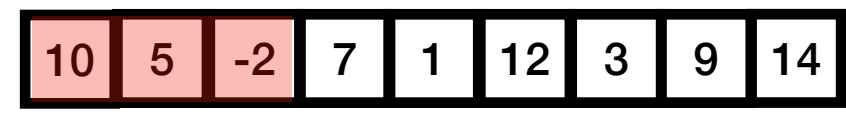
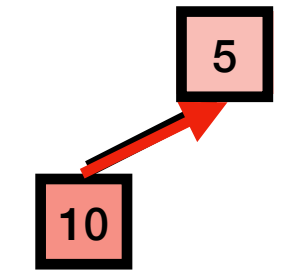
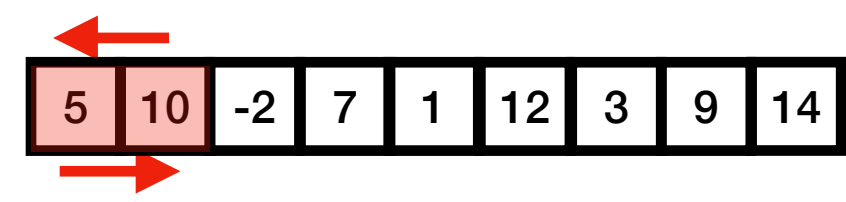
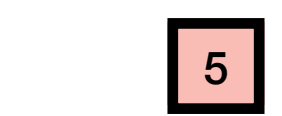
Fill Algorithm

- Flood Fill Algorithm

Heap Sort:



... build max heap



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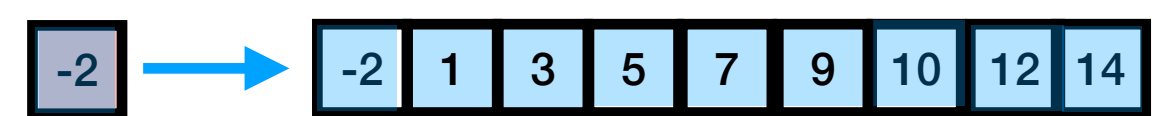
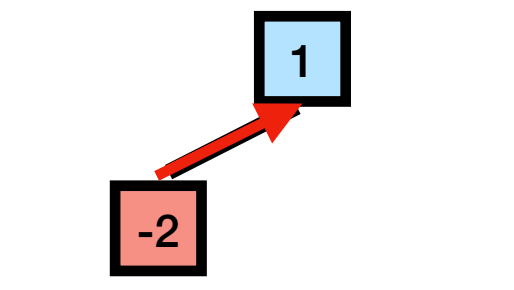
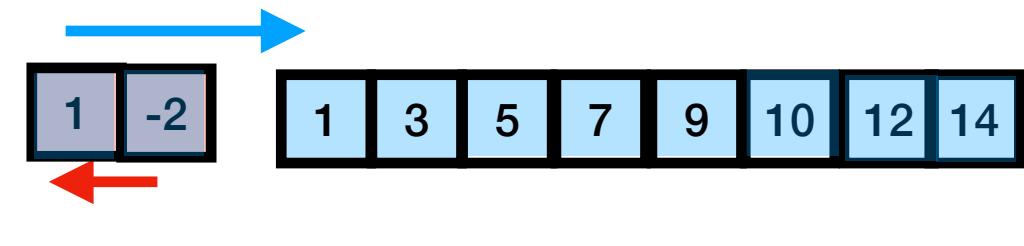
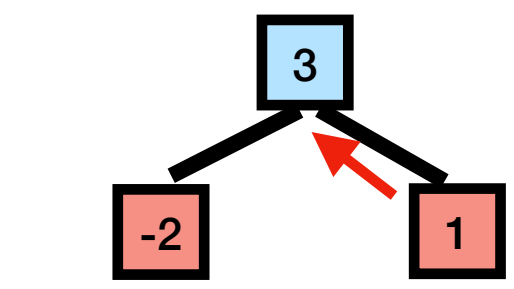
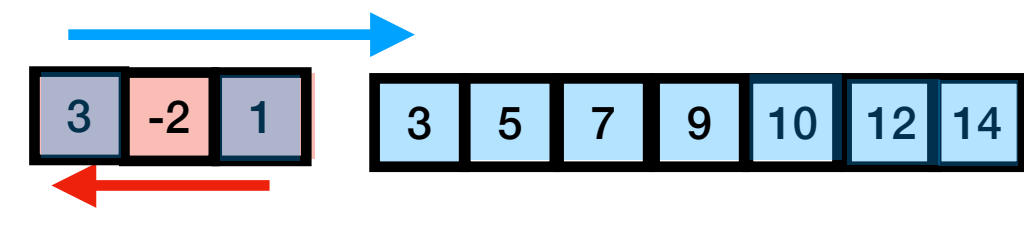
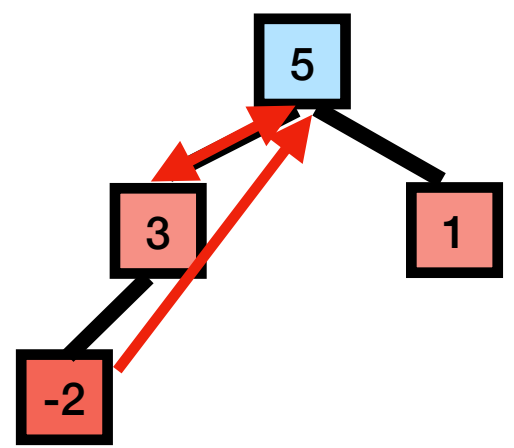
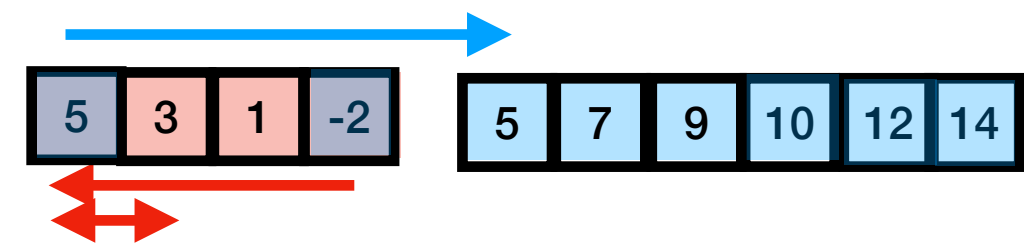
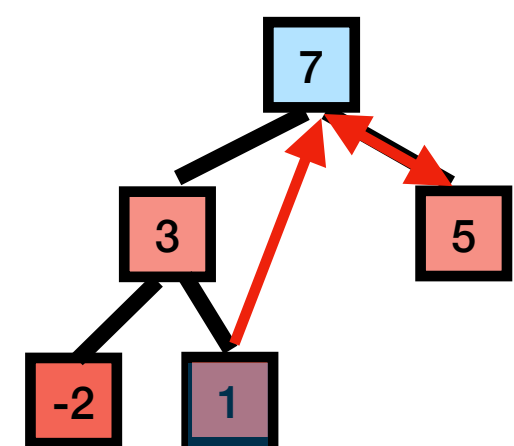
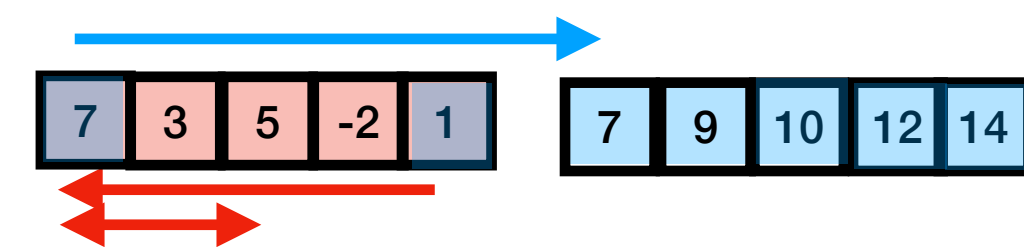
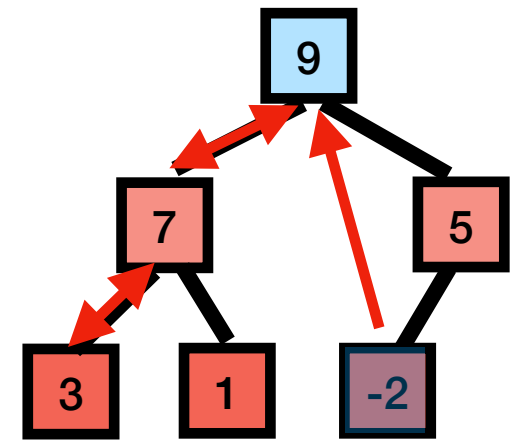
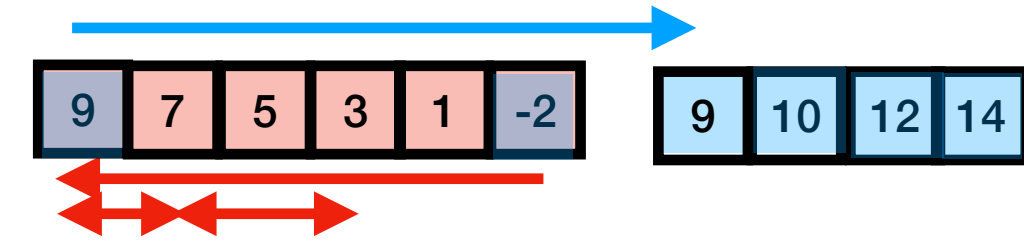
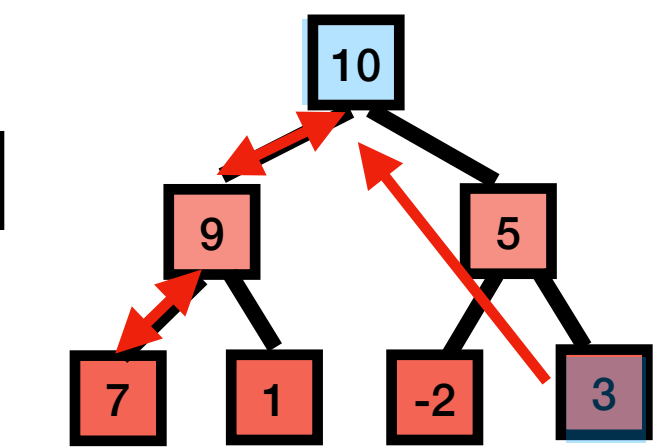
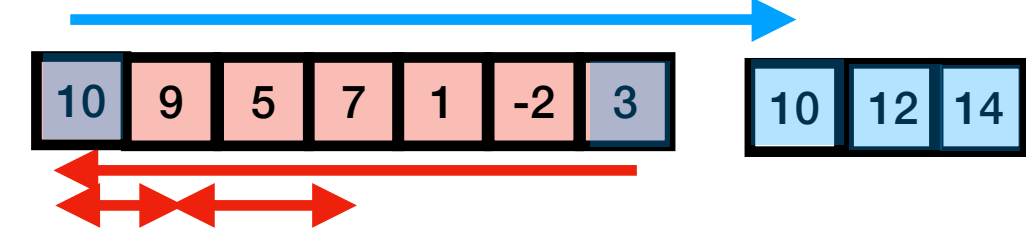
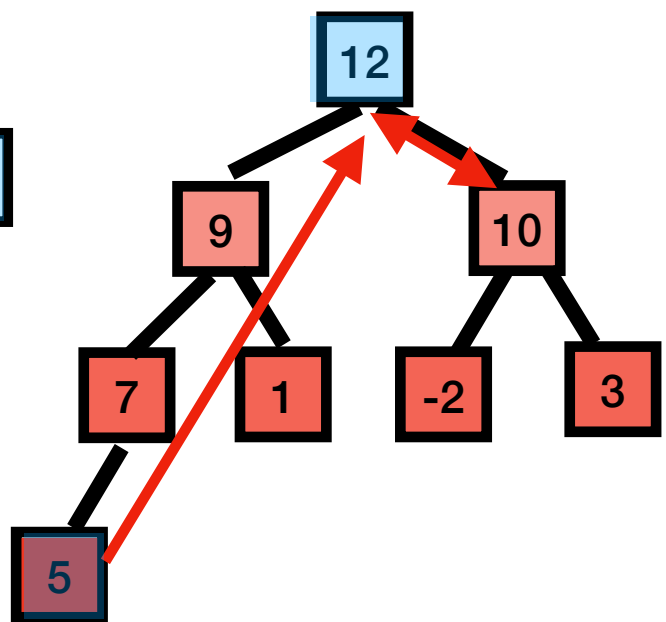
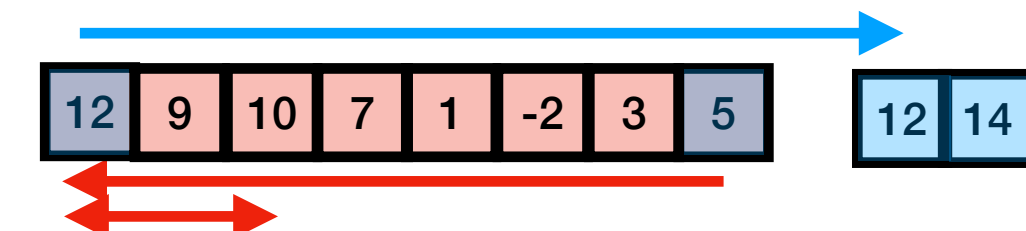
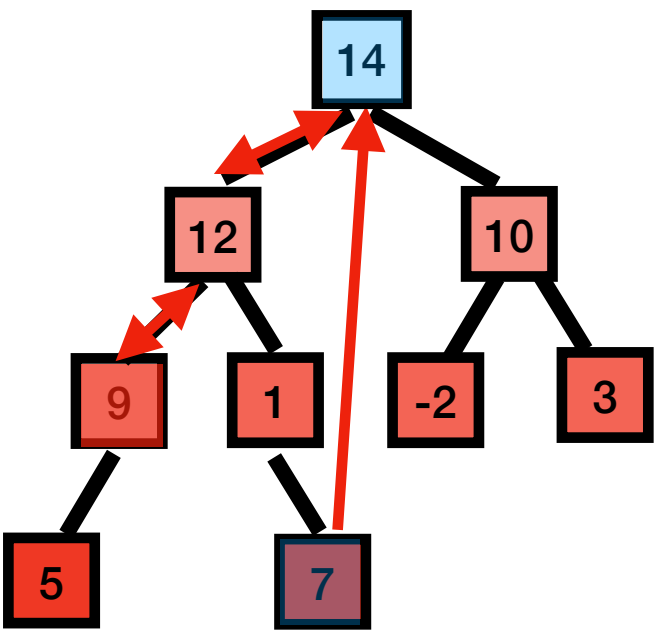
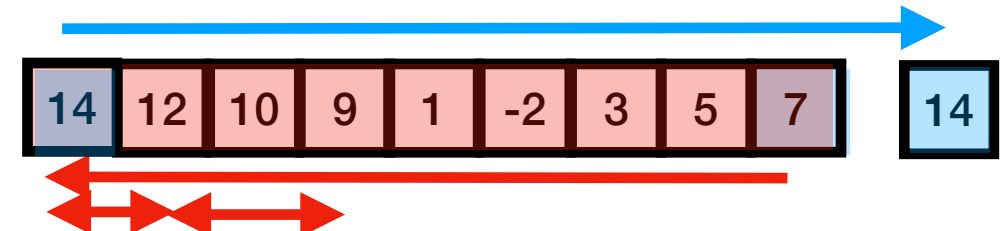
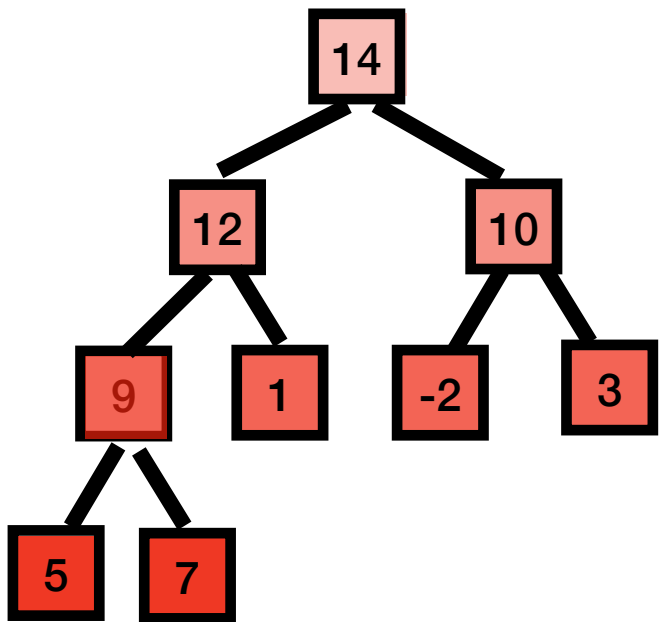
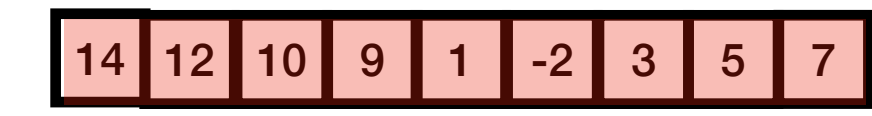
Fill Algorithm

- Flood Fill Algorithm

Heap Sort:

5 10 -2 7 1 12 3 9 14

... sort



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...sort in order consistent with graph

many consistent sorting orders...

Options:

	1, 2, 7	any order to start...
if 1 before 3	3, 2, 7	any order after 1...
	5	next to last
	4	last

Idea/Pseudo-code:

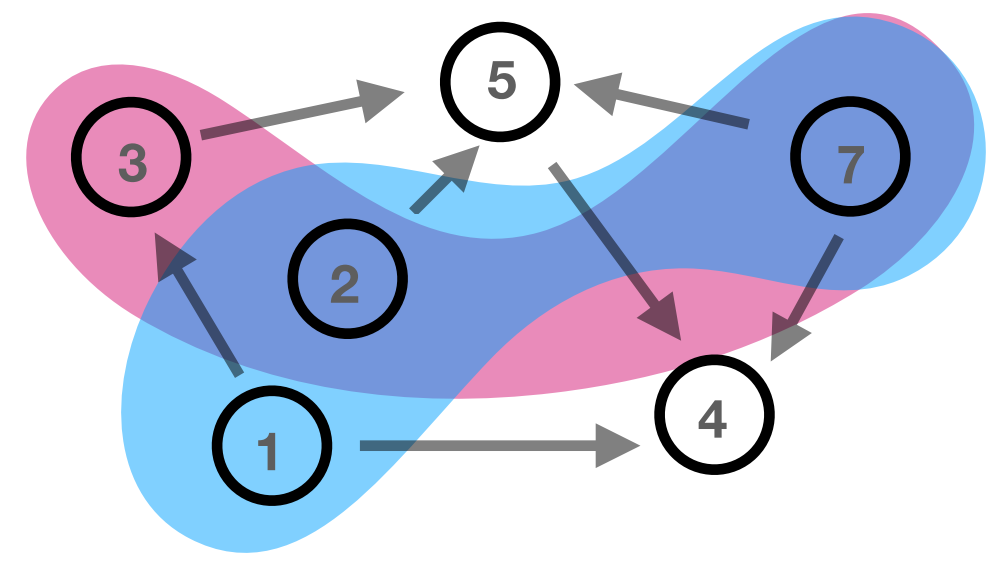
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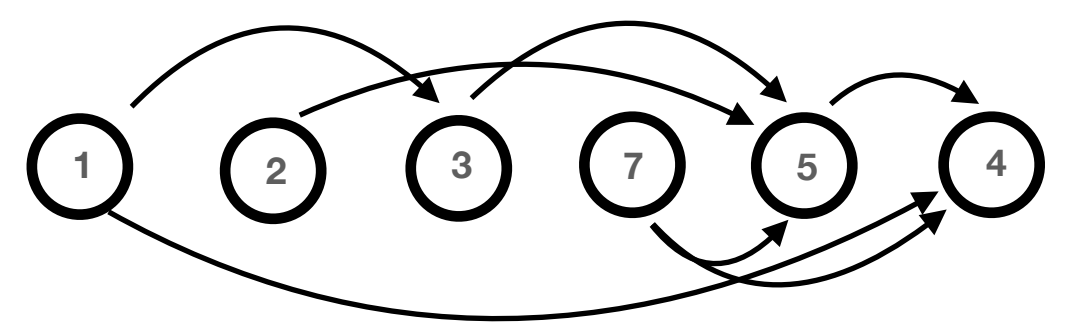
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 Add v into V

If any edges left in graph
 "Graph has at least one cycle"
 else:
 "S is a topological sort."



Computation Cost:
 Cost: $O(|V|+|E|)$



Acceptable sorts...

- 1, 2, 7, 3, 5, 4
- 1, 7, 2, 3, 5, 4
- 1, 3, 2, 7, 5, 4
- 2, 1, 3, 7, 5, 4
- 7, 2, 1, 3, 5, 4
- ⋮
- ⋮
- ⋮

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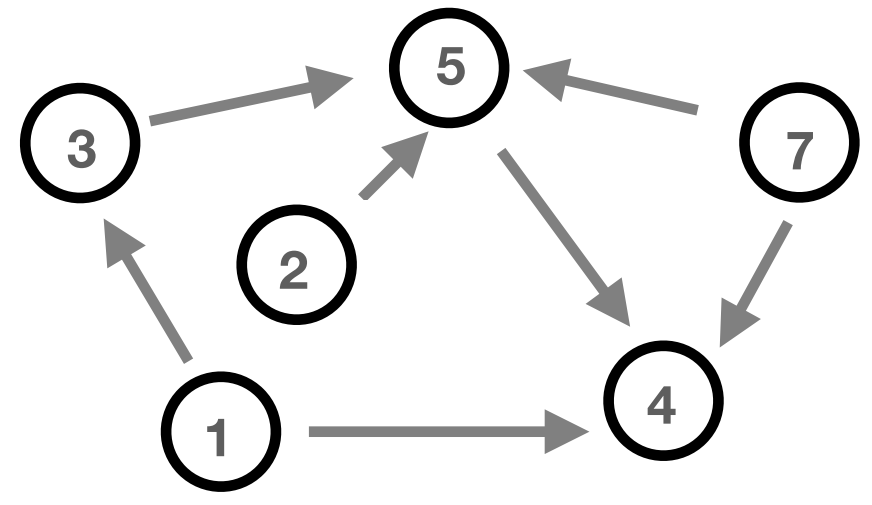
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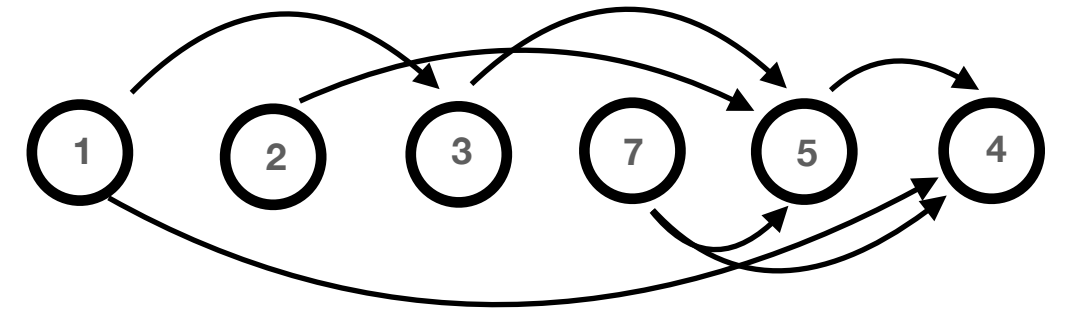
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Computation Cost:
 Cost: $O(|V|+|E|)$



Algorithm:

S = []; V = [2, 1, 7];

pick 2...

... move 2 to S and remove edge to 5
 ... (5 has other incoming edges)

S = [2]; V = [1, 7];

pick 1...

... move 1 to S and remove edges to 3 and 4
 ... add 3 to V, (4 has other incoming edges)

S = [2, 1]; V = [7, 3];

pick 7...

... move 7 to S and remove edges to 4 and 5
 ... (4 and 5 have other incoming edges)

S = [2, 1, 7]; V = [3];

pick 3...

... move 3 to S and remove edge to 5
 ... add 5 to V

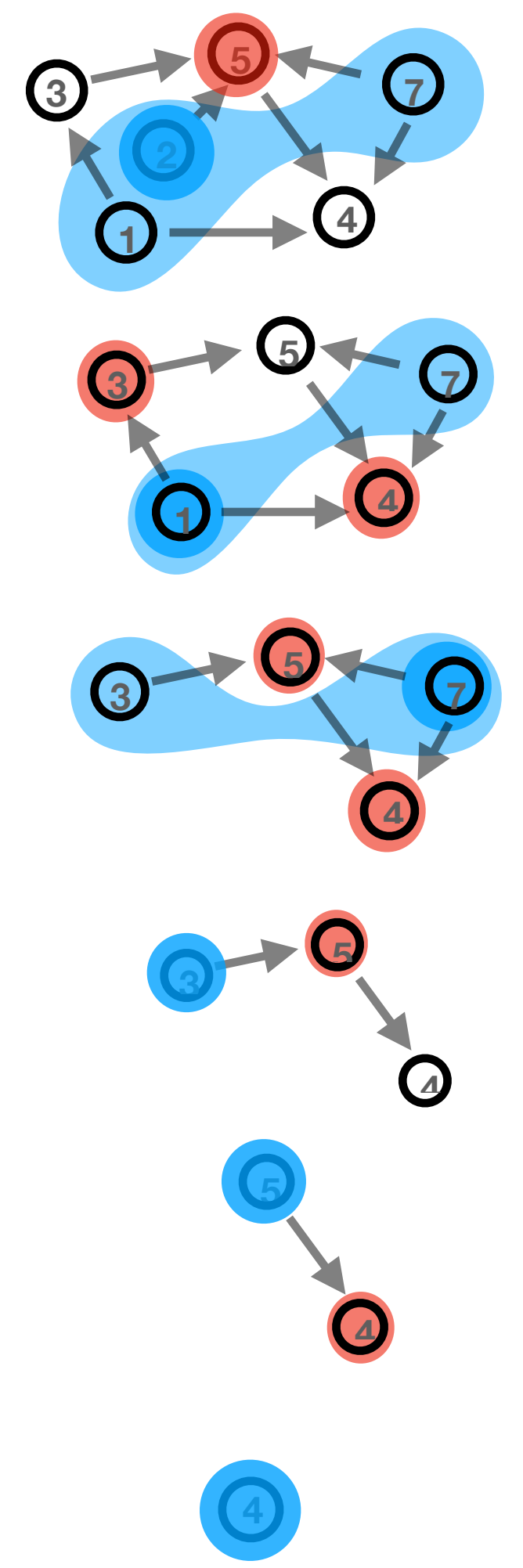
S = [2, 1, 7, 3]; V = [5];

pick 5...

... move 5 to S and remove edge to 4
 ... add 4 to V

S = [2, 1, 7, 3, 5]; V = [4];

pick 4...



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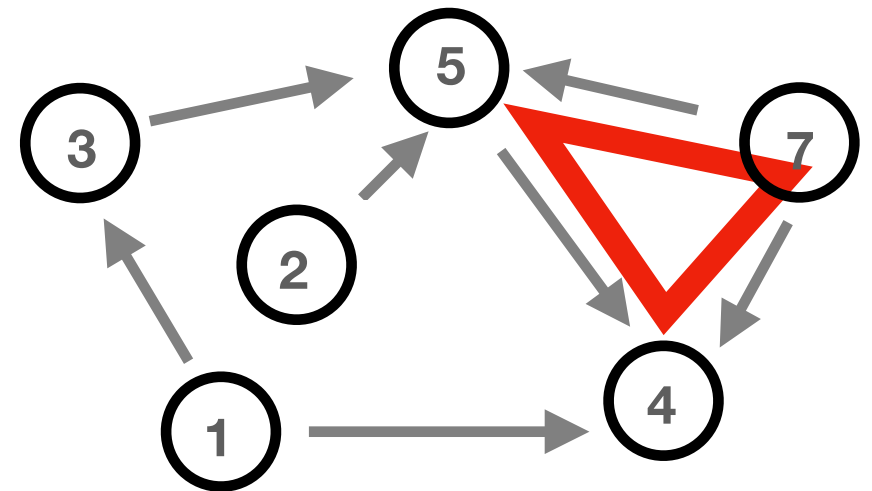
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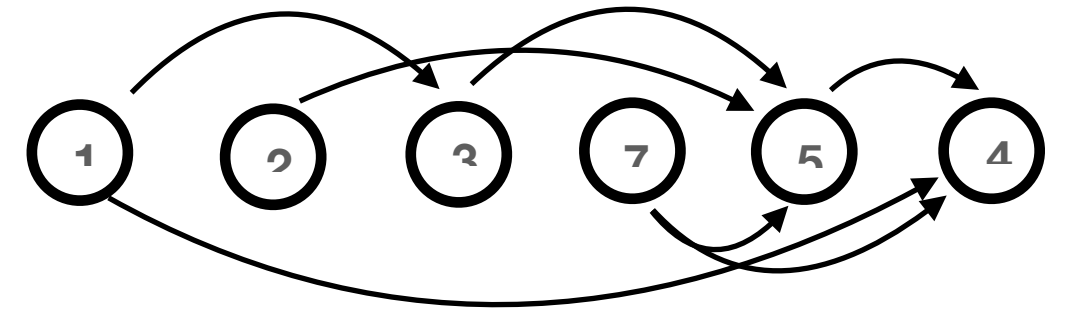
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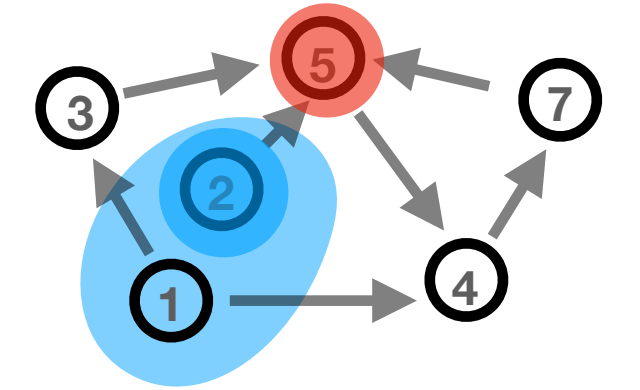


Algorithm:

S = []; V = [2, 1];

pick 2...

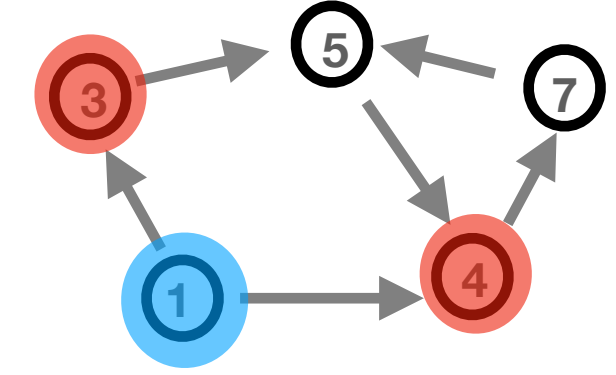
... move 2 to S and remove edge to 5
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S = [2]; V = [1];

pick 1...

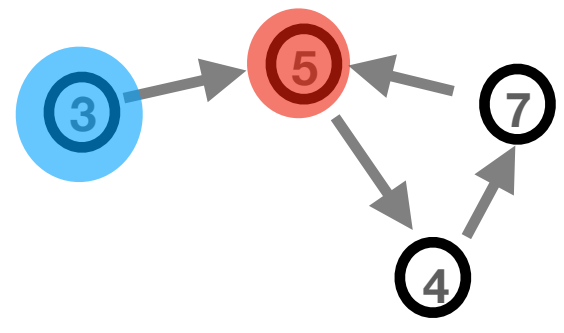
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S = [2, 1]; V = [3];

pick 3...

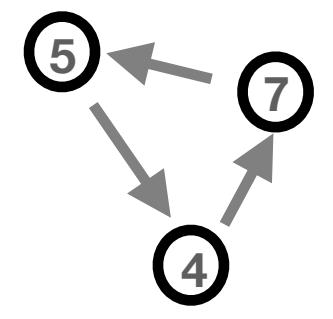
... move 3 to S and remove edge to 5
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S = [2, 1, 3]; V = [];

V empty but still edges left...

Graph contains a directed cycle.



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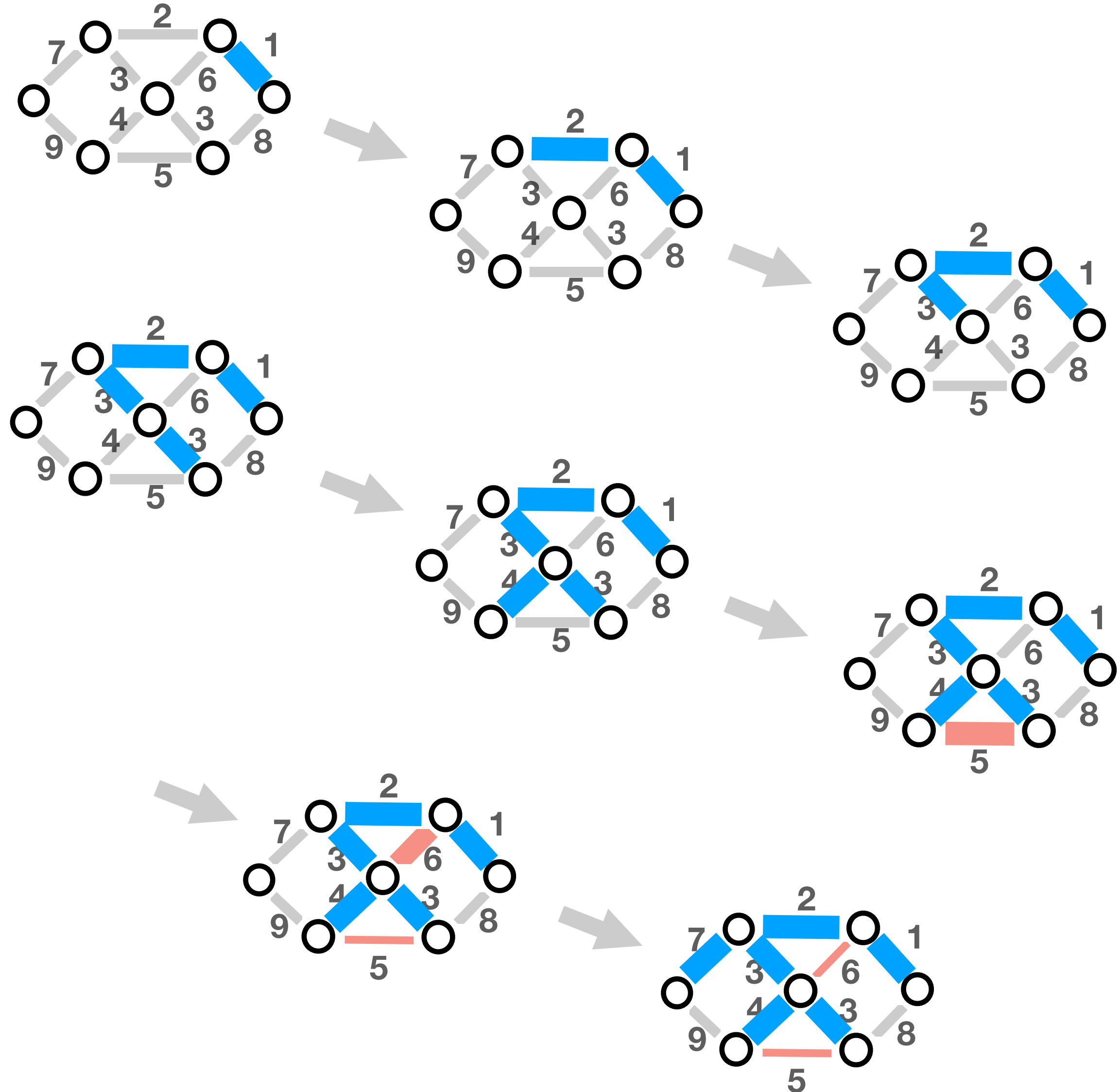
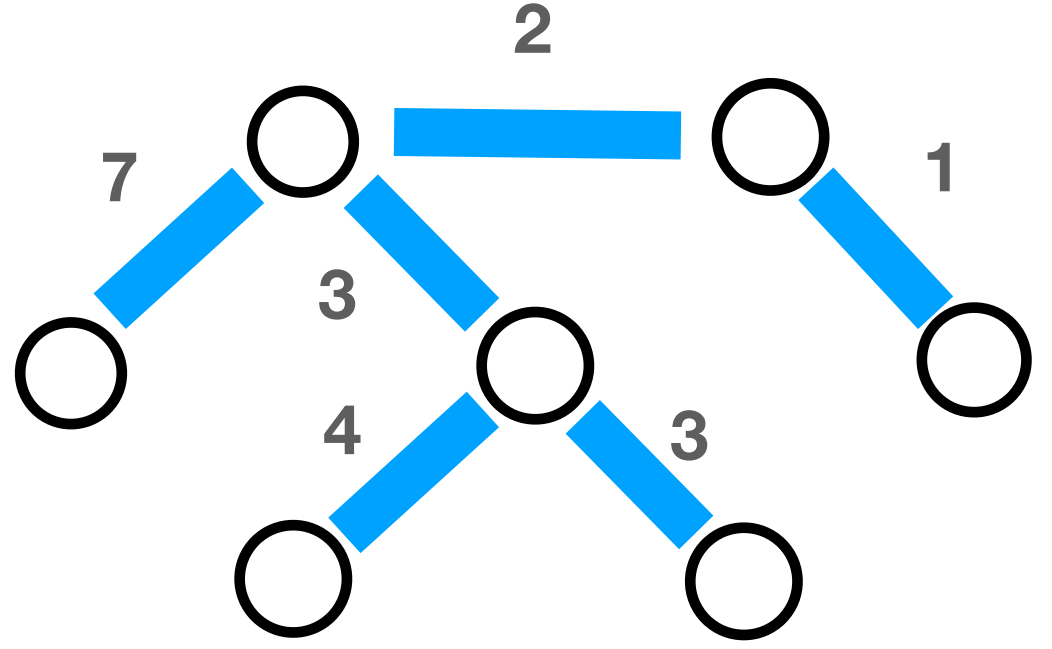
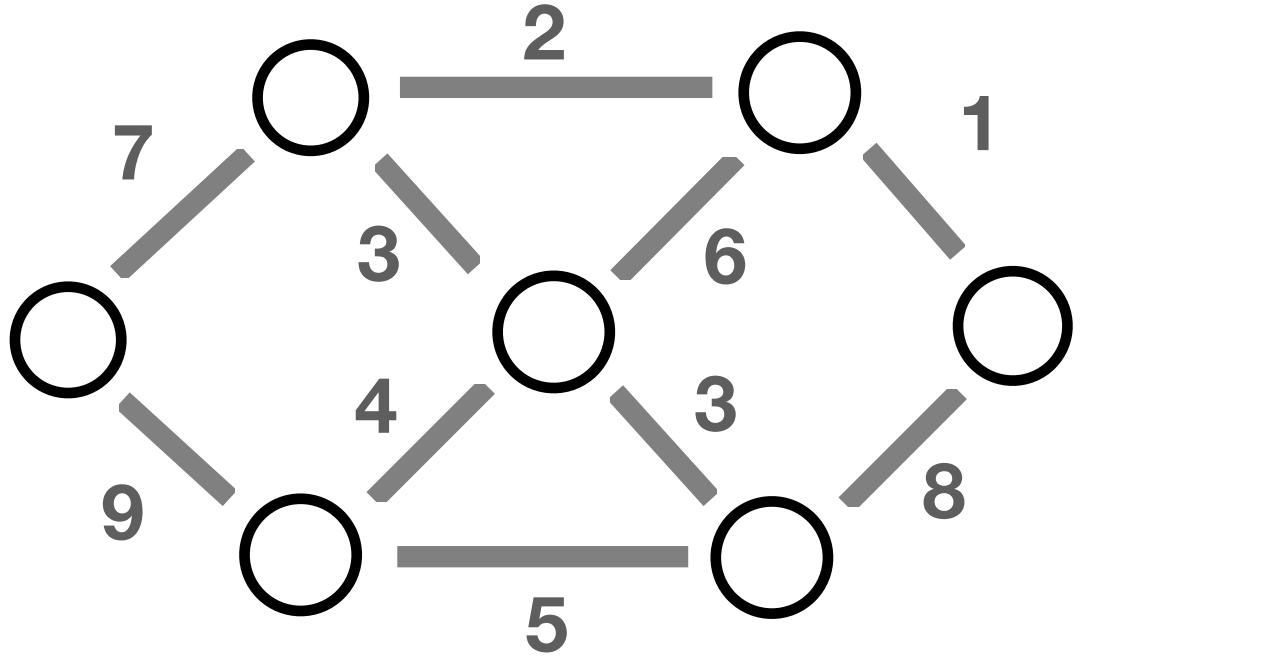
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Kruskal's Algorithm:

...for minimum spanning tree

- **N vertices**
- min spanning tree has **n-1 edges**



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Union-Find or Merge-Find

Data Structure: Disjoint Sets

Two Operations:

- **Find:** find set that contains any element (represented by *representative element*)
- **Union:** combine sets

each set often stored as a tree...

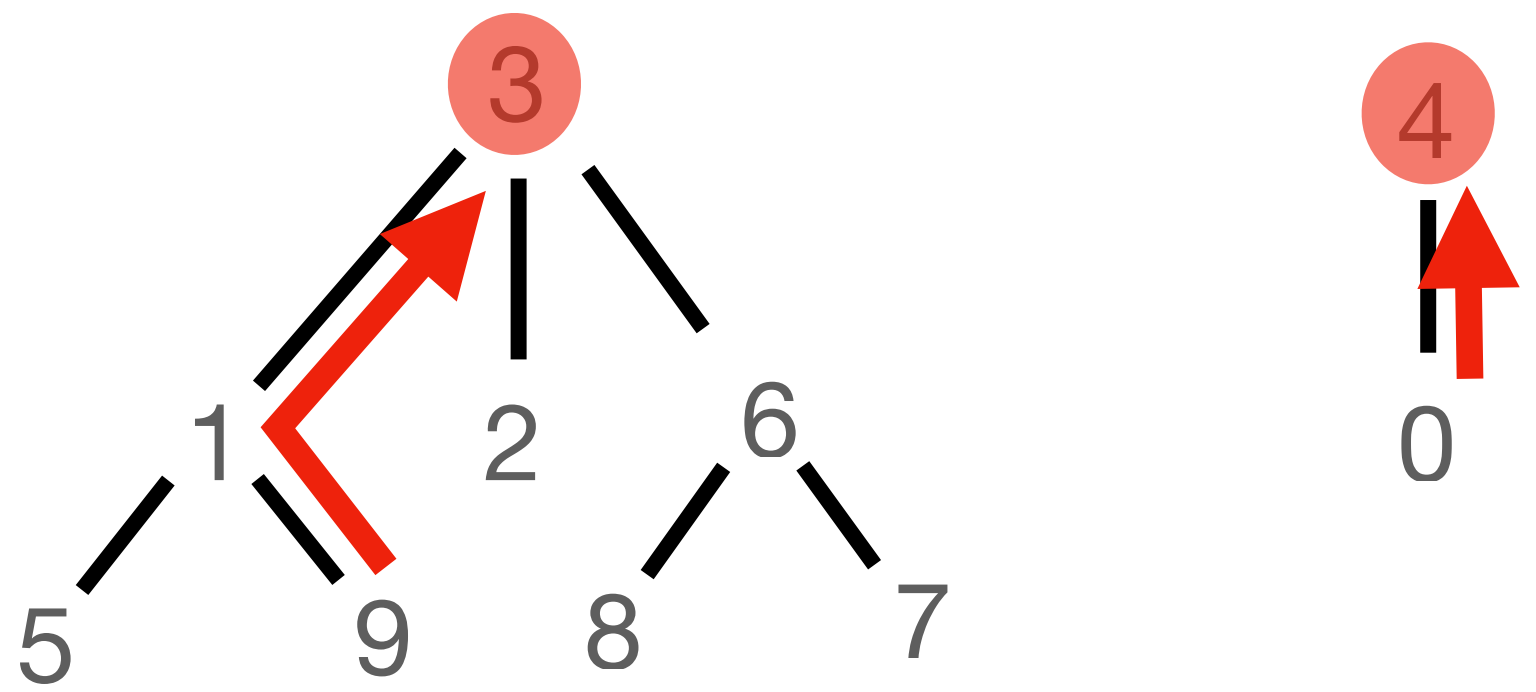
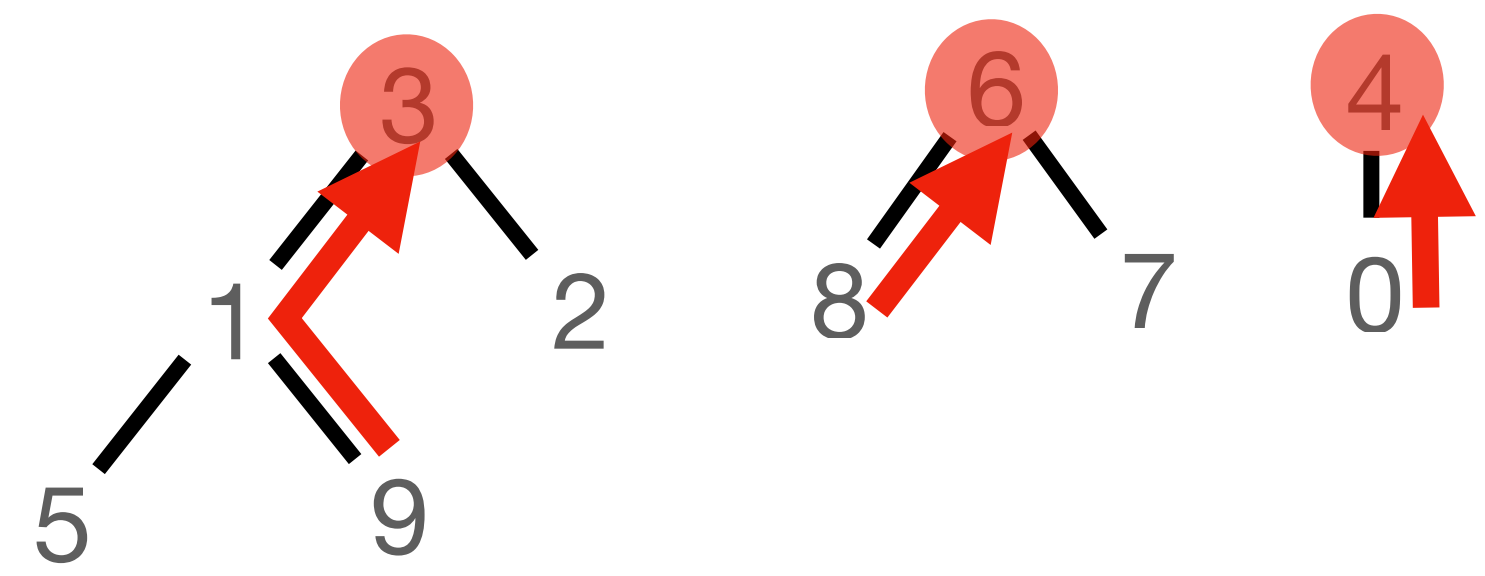
Disjoint-set Forest

- Set: represented as a tree
- Representative: root of tree
- Find:** follows parent nodes to the root
- Union:** attach root of one set to root of the other

Not better than other ways to implement sets (ie. linked lists, etc) unless

- 1. Union by rank**
Smaller (shallower) tree attached to root of larger (deeper) tree
- 2. Path compression**
Move nodes up to root whenever Find is called

Computation Cost:
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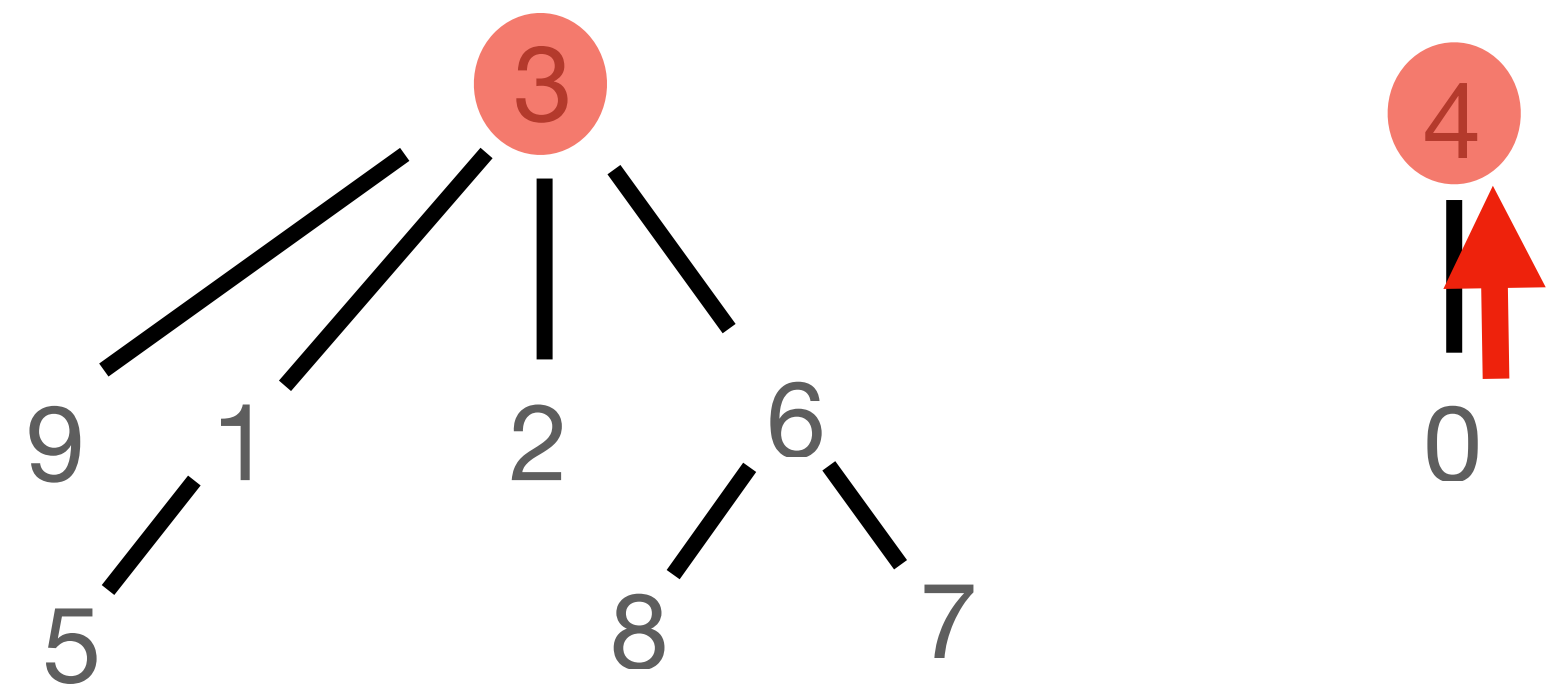
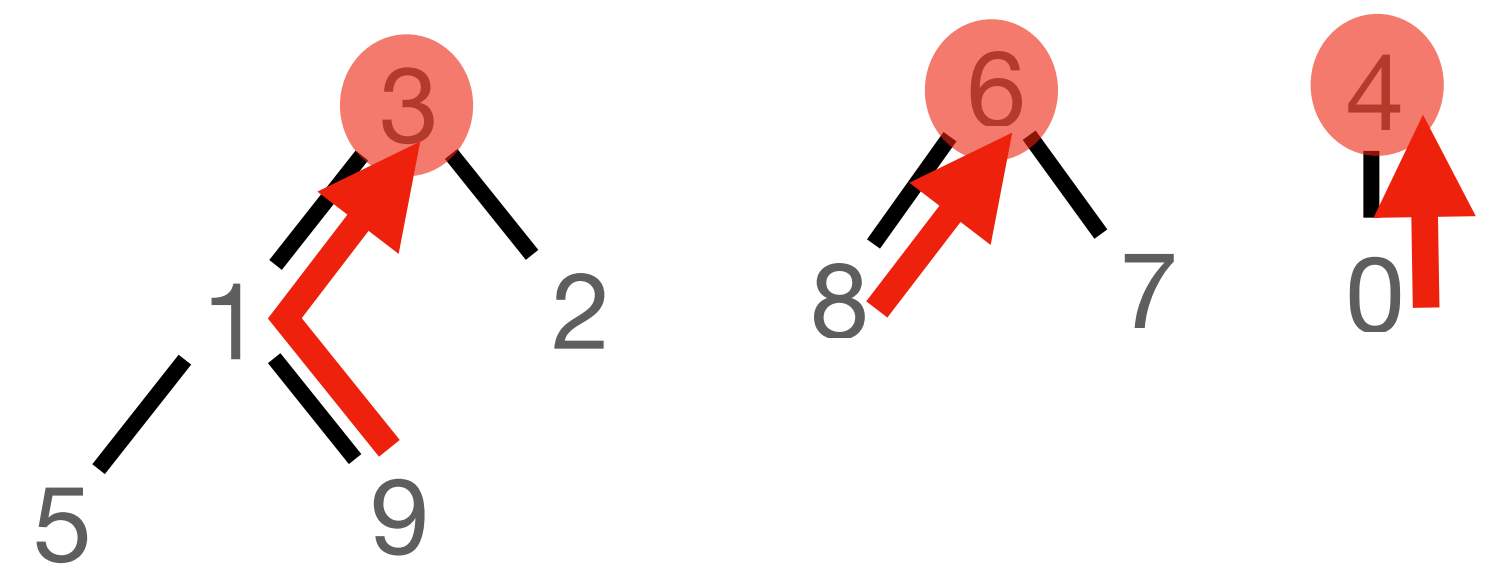
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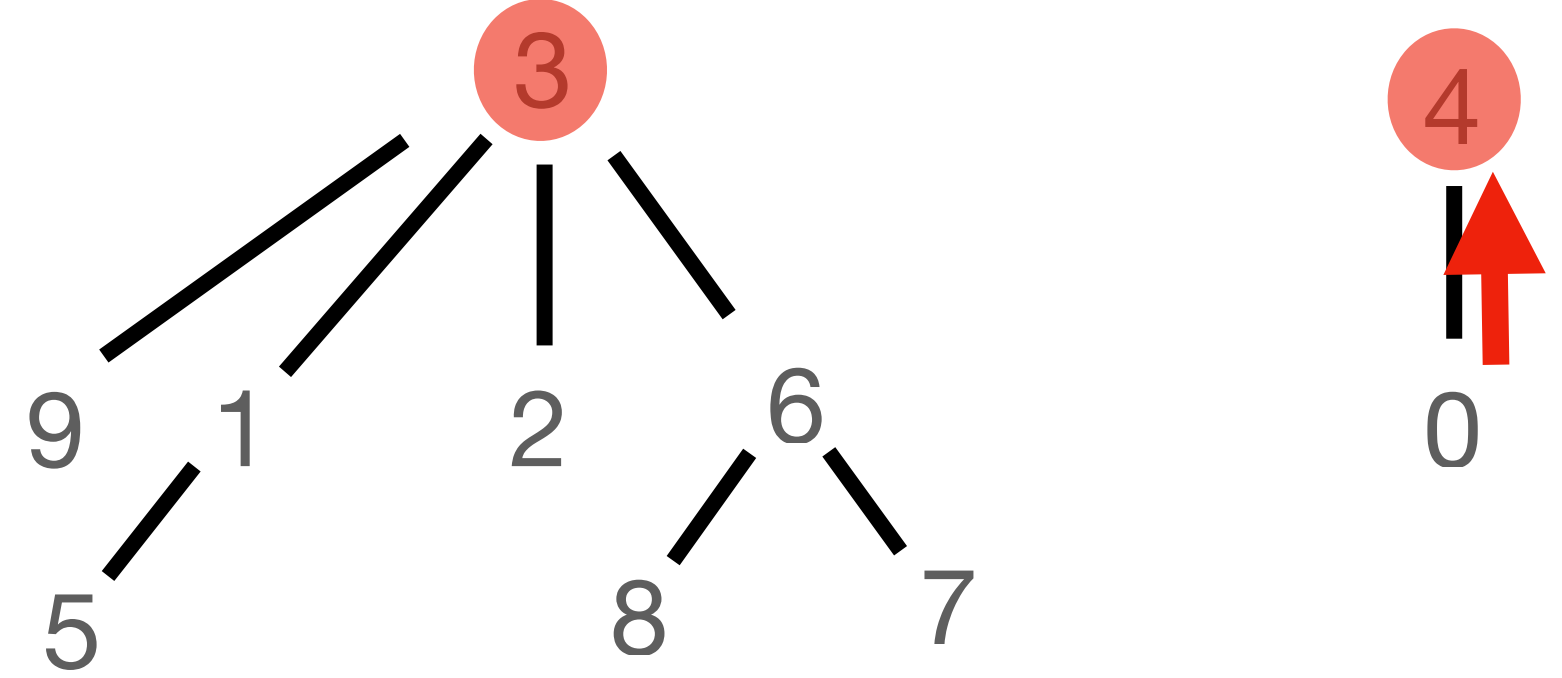
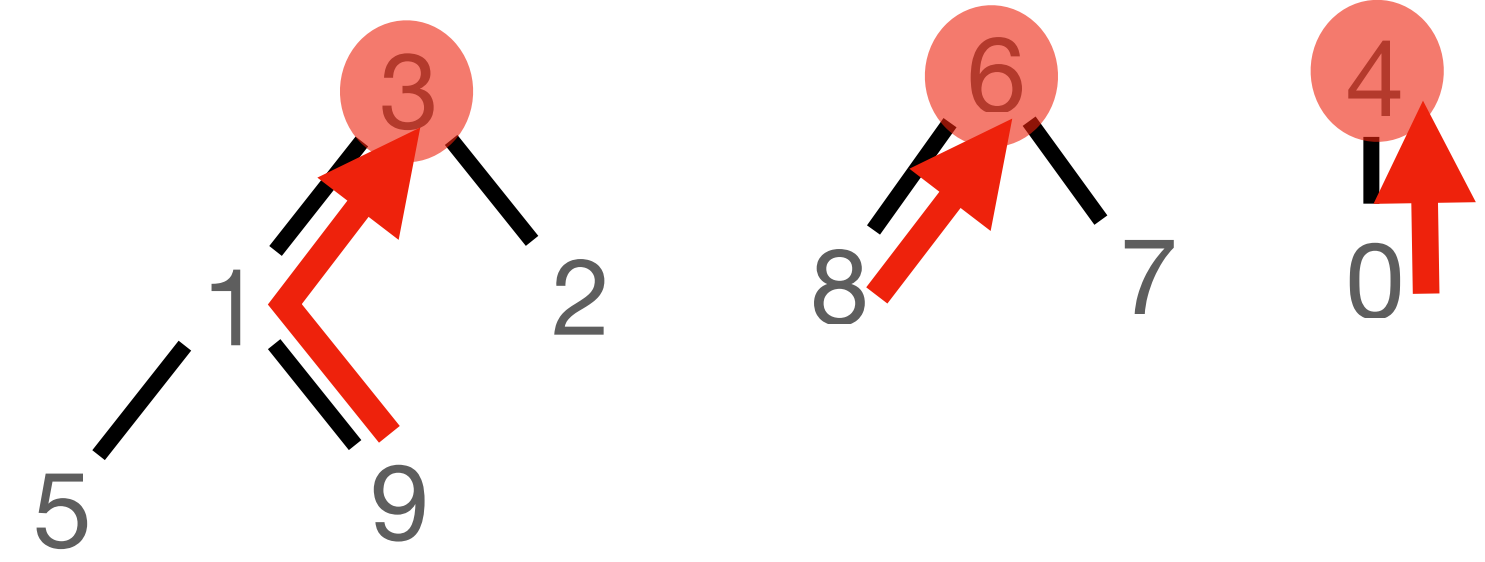
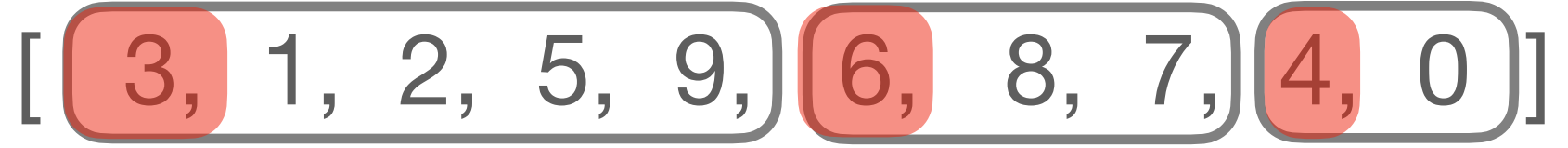
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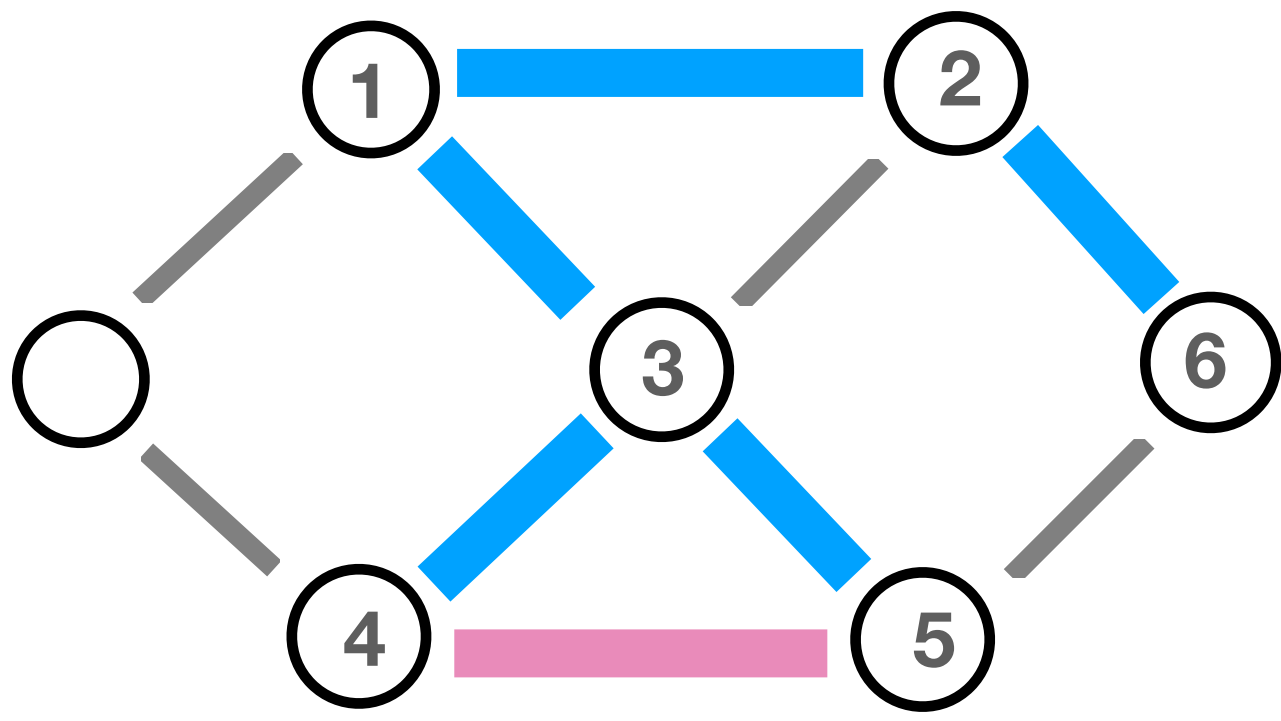
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Undirected Graph:

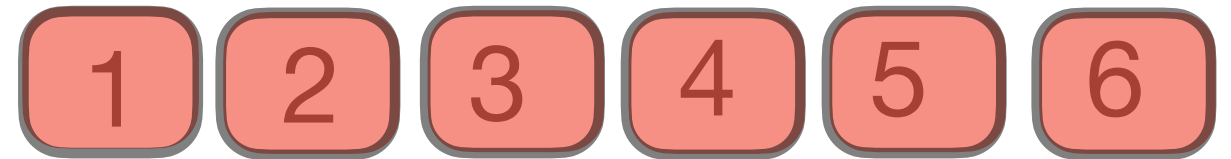
...minimum spanning tree step - cycle detection



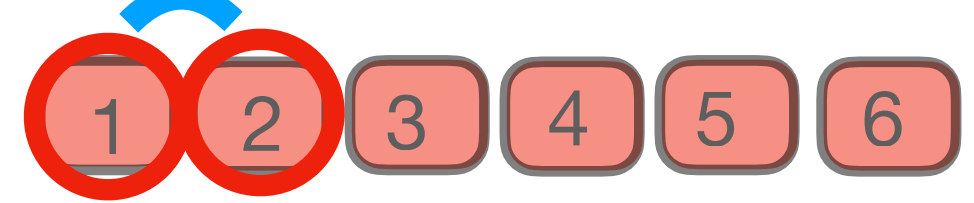
Cycle detected

Disjoint Set Forest

disjoint sets: one with each vertex



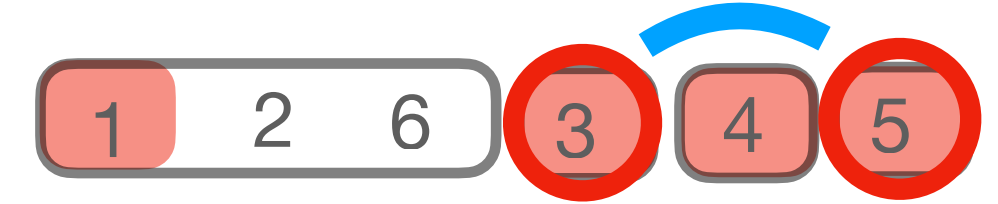
Loop over edges...



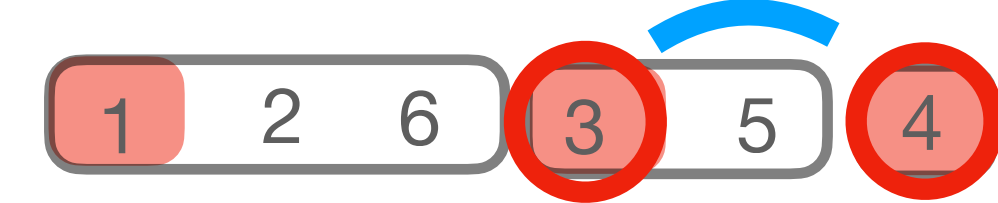
check if two vertices in the same set...
1 not = 2 ... so merge sets.



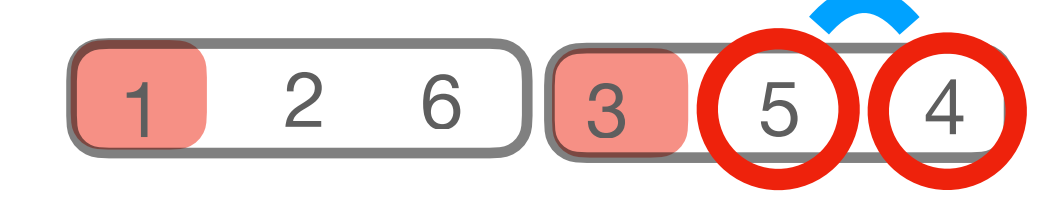
check if two vertices in the same set...
1 not = 6 ... so merge sets.



check if two vertices in the same set...
3 not = 5 ... so merge sets.

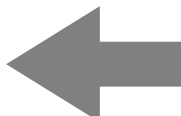
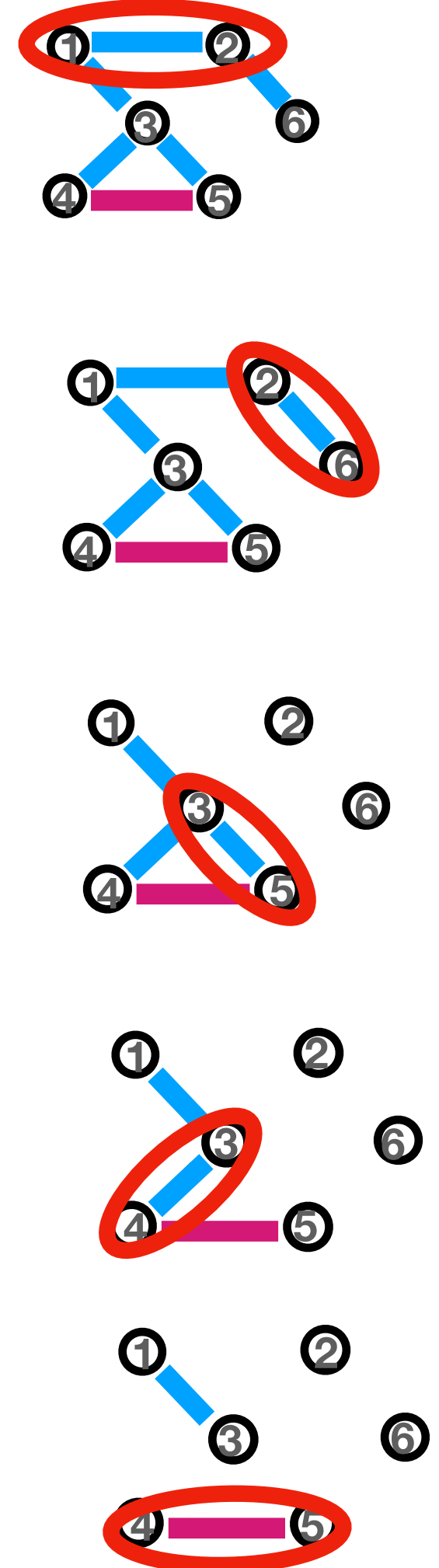


check if two vertices in the same set...
3 not = 5 ... so merge sets.



check if two vertices in the same set...
3 = 3 ... CYCLE DETECTED

Computation Cost:
Cost: $O(|V|^3)$



Algorithms

Search Algorithms

- Binary search
- Breadth First Search (BFS)
- Depth First Search (DFS)
- Kadane's algorithm
- KMP algorithm
- Quick select algorithm
- Boyer-Moore Majority Vote algorithm
- Euclid's algorithm

Sorting Algorithms

- Insertion Sort
- Selection Sort
- Quick Sort
- Merge Sort
- Heap Sort
- Introsort
- Bubble sort
- Non-comparison sorts
- Counting sorts
- Radix sort
- Bucket sort
- (Kahn's topological sort)

Graph Algorithms

- Sorting**
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 - Floyd's Cycle Detection Algorithm
 - Union-Find Cycle Detection
 - Kruskal's Algorithm
- Shortest path**
 - Dijkstra's Algorithm
 - Bellman Ford Algorithm
 - Floyd Warshall Algorithm
 - Lee Algorithm

Compression Algorithms

- Huffman Coding

Fill Algorithm

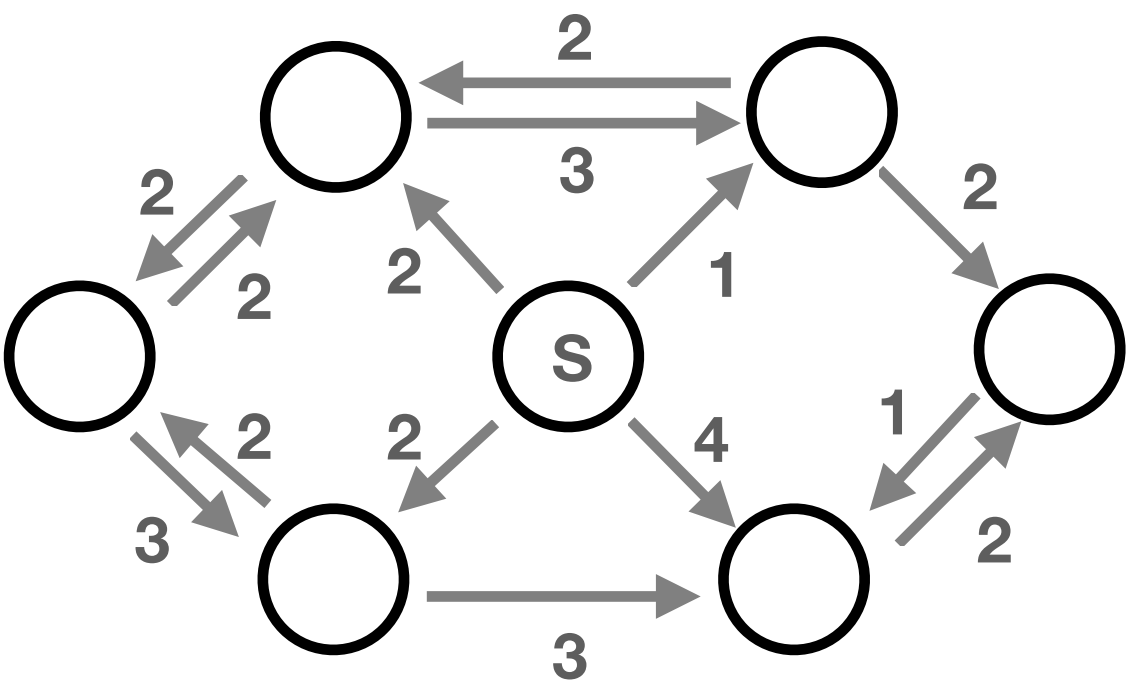
- Flood Fill Algorithm

Dijkstra's Algorithm:



Shortest path from source on a weighted graph.
Edges have only positive weights.

Graph: $G = (V,E)$



Idea/Pseudo-code:

Initialize:

Visited = {source}, cost = 0
Unvisited = { other nodes }, cost = INF

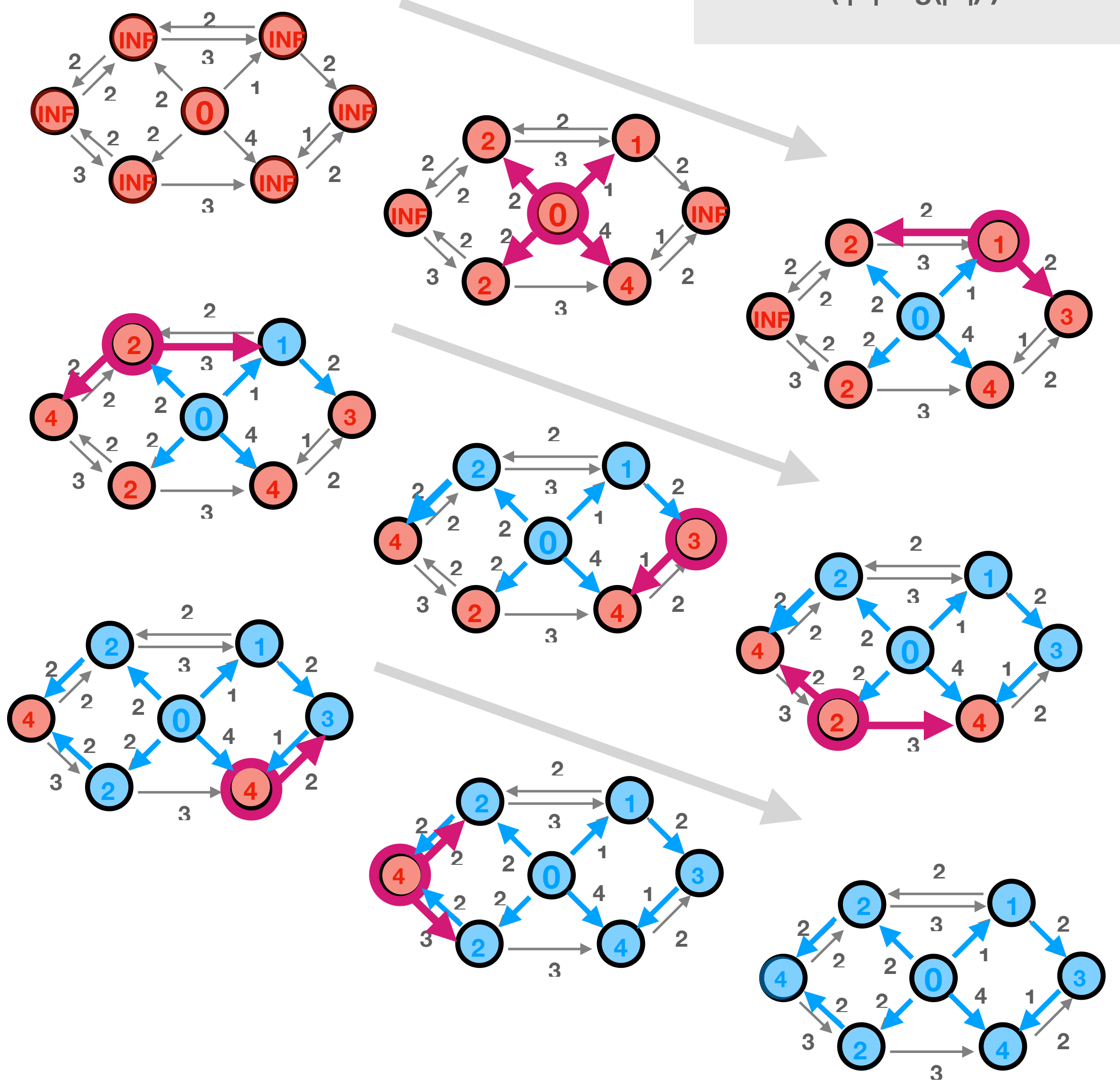
For each node:

Starting with lowest cost node...
1. update minimum cost to neighbors
2. Mark node as visited

Stop:

When each node has been visited or when desired node is reached.

Computation Cost:
Cost: $O(|E| \log(|V|))$



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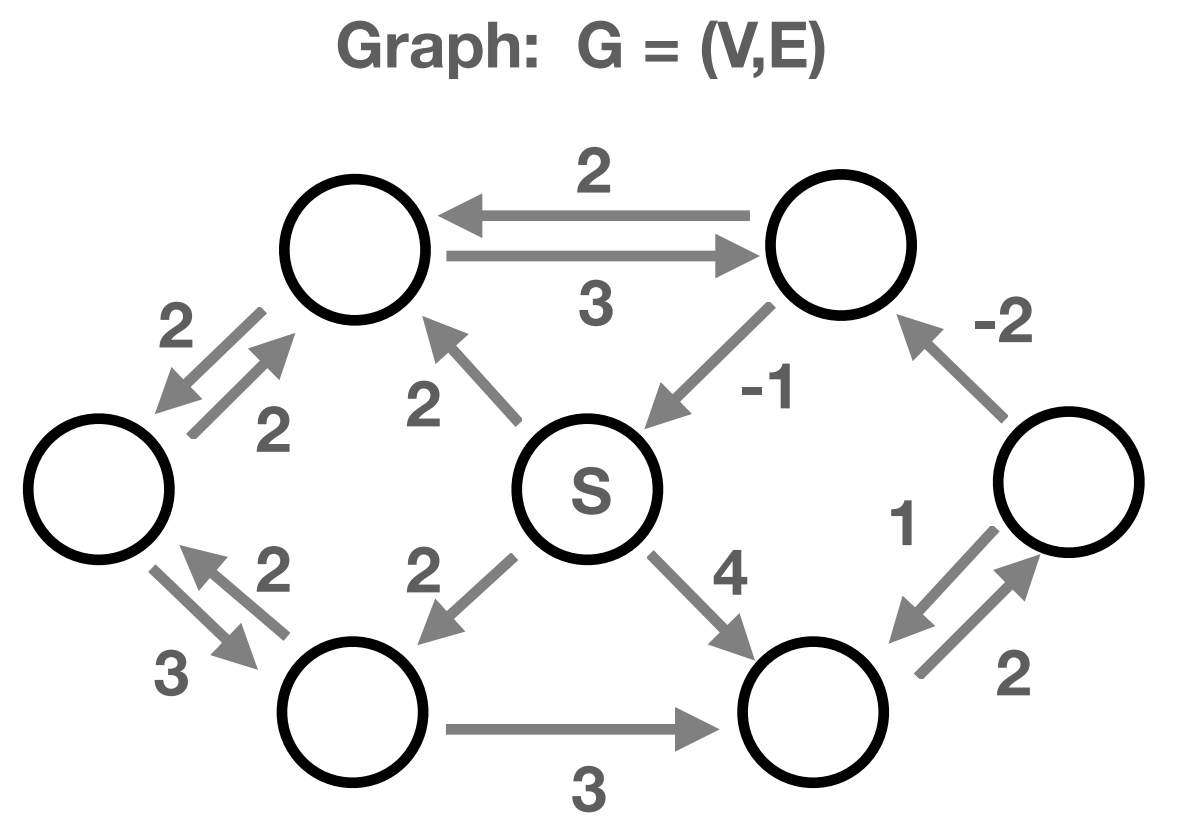
- Huffman Coding

Fill Algorithm

- Flood Fill Algorithm

Bellman Ford Algorithm:

Shortest path from source on a weighted graph.
Edges have positive and negative weights
Report negative weight cycles.



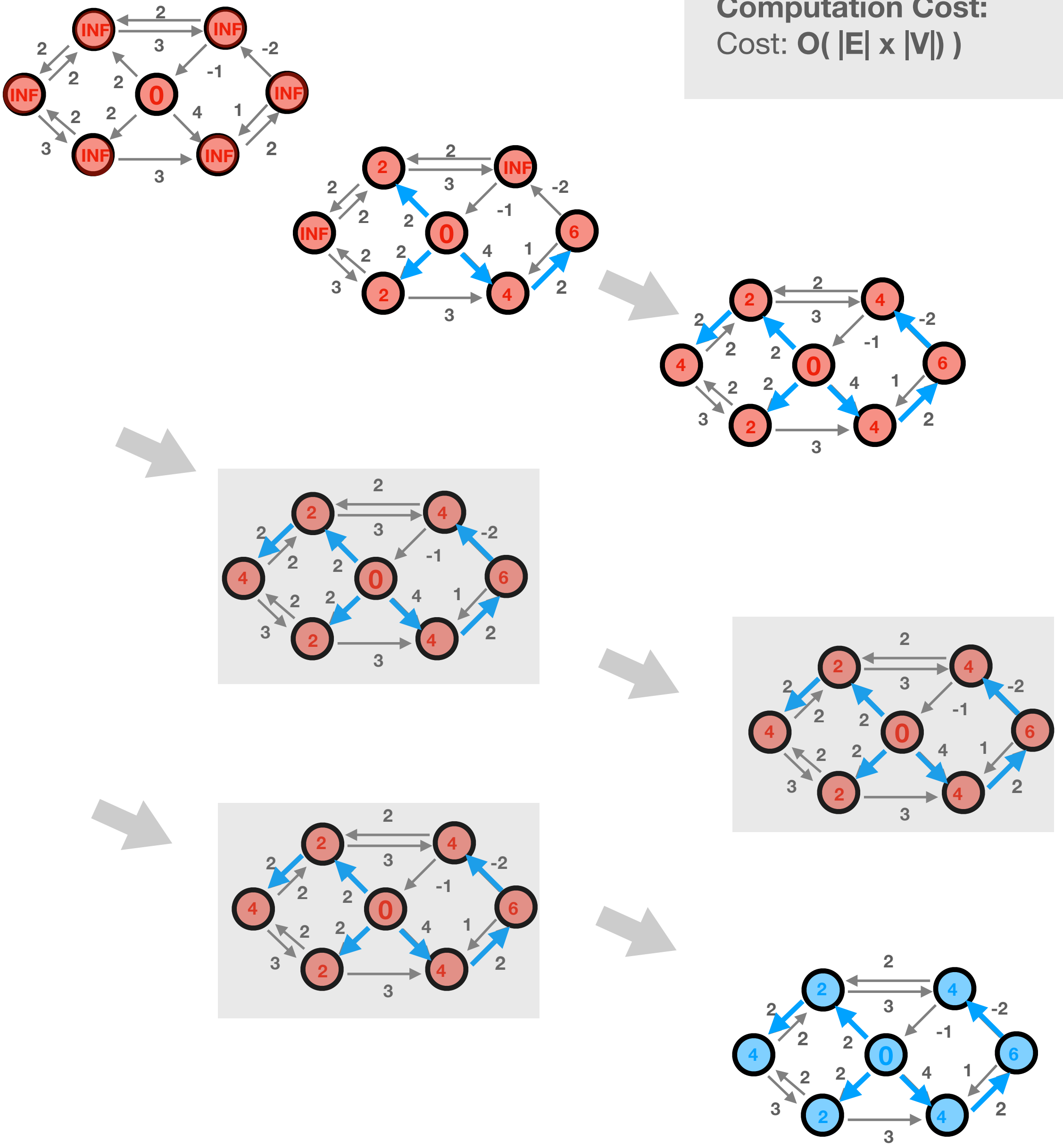
Idea/Pseudo-code:

Initialize:
Each node: cost = INF, parent = NULL

Source node: cost = 0

For $|V|-1$ iterations:
For each edge e :
Edge e : from u to v
If $cost(v) > cost(u) + cost(edge)$:
 $cost(v) = cost(u) + cost(edge)$
 $parent(v) = u$

Else if $cost(v) < cost(u) + cost(edge)$:
Report negative cycle



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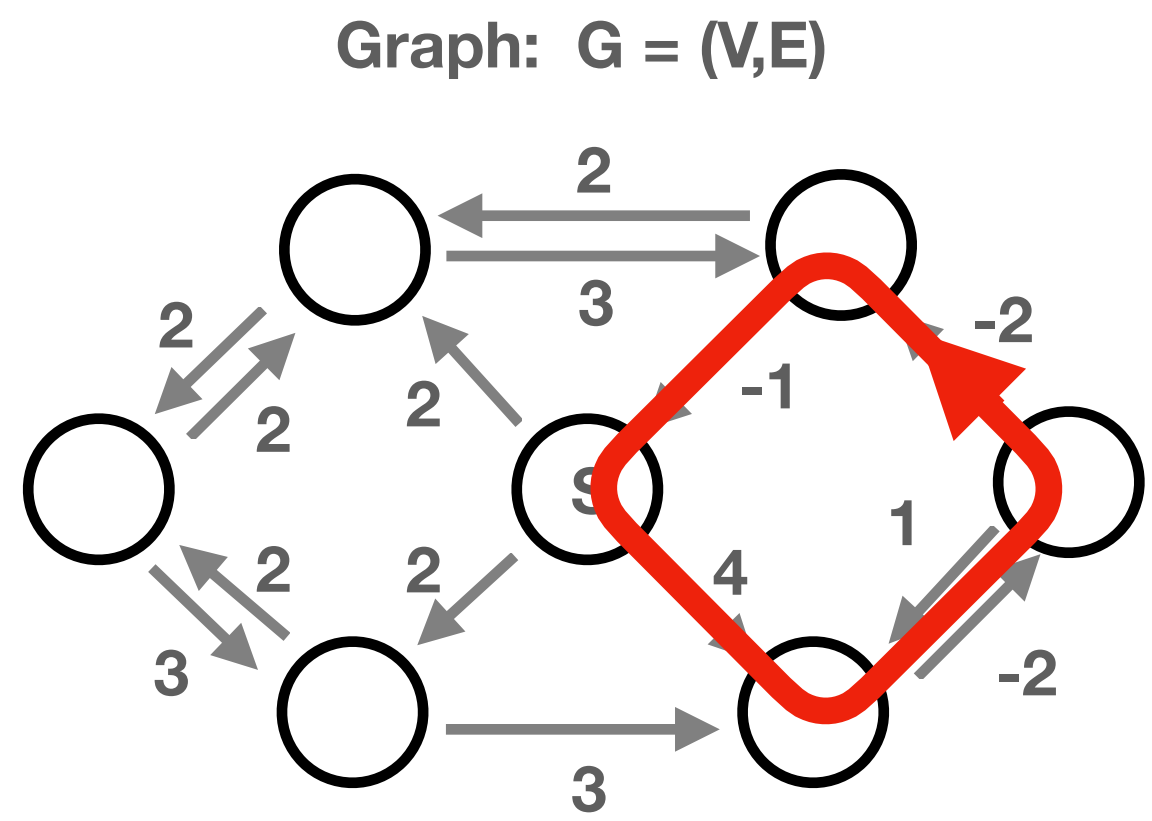
- Huffman Coding

Fill Algorithm

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Bellman Ford Algorithm:

...with negative weight cycle...



... total weight is -1

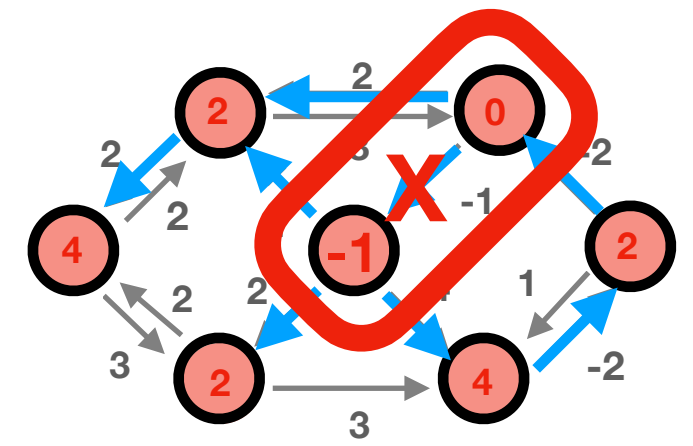
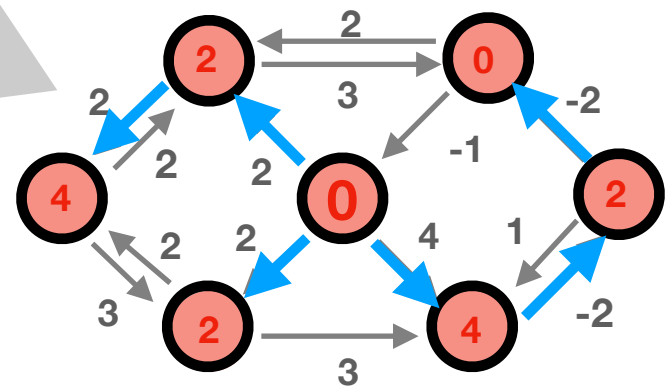
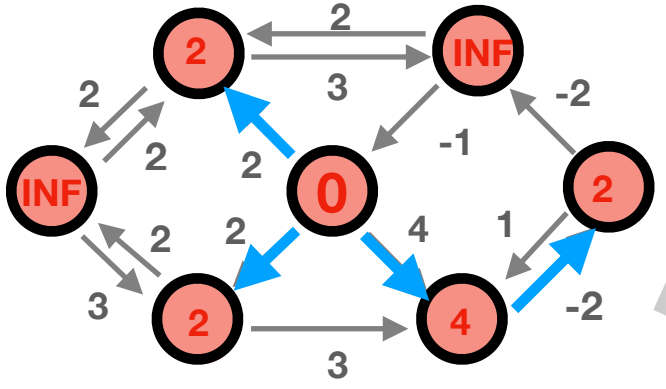
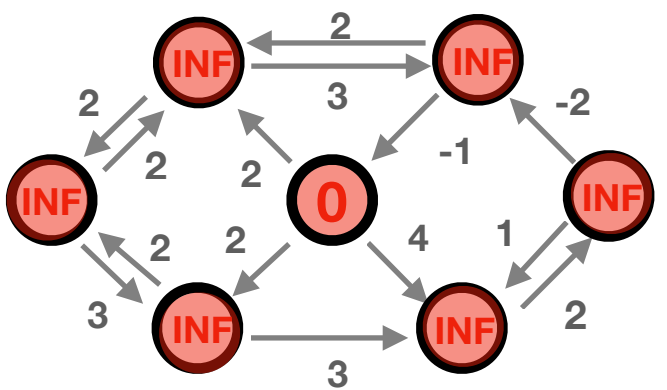
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 $cost(v) = cost(u) + cost(edge)$
 $parent(v) = u$

Else if $cost(v) < cost(u) + cost(edge)$:
Report negative cycle



Report cycle w/
negative weights

STOP

Computation Cost:
Cost: $O(|E| \times |V|)$

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- Lee Algorithm

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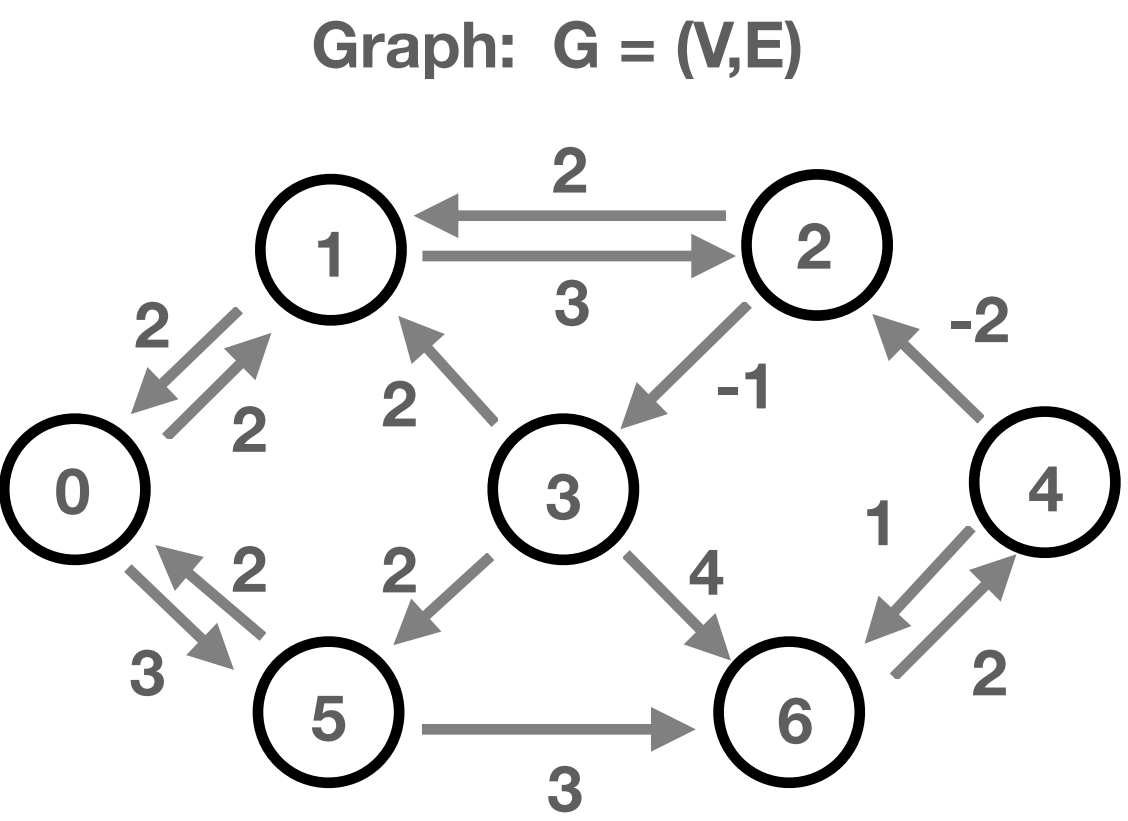
- Huffman Coding

Fill Algorithm

- Flood Fill Algorithm

Floyd - Warshall algorithm:

Shortest path from every node...
...to every other node



Idea/Pseudo-code:

Initialize:
 dist matrix: $|V| \times |V|$
 Set diagonal elements = 0
 if edge from i to j in graph:
 dist[i][j] = weight;

For k = 0 to $|V|-1$:
 For j = 0 to $|V|-1$:
 For i = 0 to $|V|-1$:
 If dist[i][k] + dist[k][j] < dist[i][j]:
 dist[i][j] = dist[i][k] + dist[k][j]

After:
 if dist[i][i] is negative for any i:
 Report negative cycle

Computation Cost:
 Cost: $O(|V|^3)$

dist =

	0	1	2	3	4	5	6
0	0	INF	INF	INF	INF	INF	INF
1	INF	0	INF	INF	INF	INF	INF
2	INF	INF	0	INF	INF	INF	INF
3	INF	INF	INF	0	INF	INF	INF
4	INF	INF	INF	INF	0	INF	INF
5	INF	INF	INF	INF	INF	0	INF
6	INF	INF	INF	INF	INF	INF	0

dist =

	0	1	2	3	4	5	6
0	0	2	INF	INF	INF	3	INF
1	2	0	3	INF	INF	INF	INF
2	INF	2	0	-1	INF	INF	INF
3	INF	2	INF	0	INF	2	4
4	INF	INF	-2	INF	0	INF	1
5	2	INF	INF	INF	INF	0	3
6	INF	INF	INF	INF	2	INF	0

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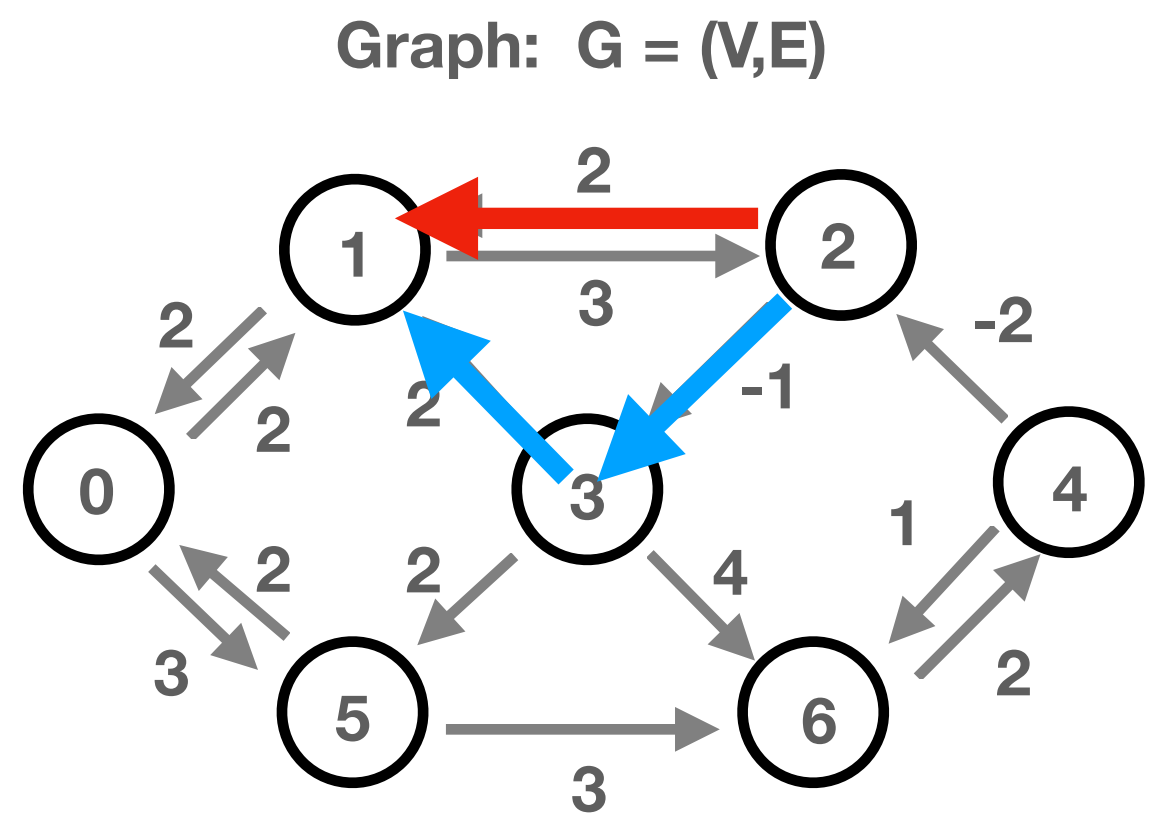
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Fill Algorithm

- Flood Fill Algorithm

Floyd - Warshall algorithm:

Shortest path from every node...
...to every other node



Idea/Pseudo-code:

Initialize:

dist matrix: $|V| \times |V|$
Set diagonal elements = 0
if edge from i to j in graph:
 $dist[i][j] = weight;$

For k = 0 to $|V|-1$:
 For j = 0 to $|V|-1$:
 For i = 0 to $|V|-1$:
 If $dist[i][k] + dist[k][j] < dist[i][j]$:
 $dist[i][j] = dist[i][k] + dist[k][j]$

After:
if $dist[i][i]$ is negative for any i:
 Report negative cycle

Computation Cost:
Cost: $O(|V|^3)$

dist =

	0	1	2	3	4	5	6
0	0	INF	INF	INF	INF	INF	INF
1	INF	0	INF	INF	INF	INF	INF
2	INF	INF	0	INF	INF	INF	INF
3	INF	INF	INF	0	INF	INF	INF
4	INF	INF	INF	INF	0	INF	INF
5	INF	INF	INF	INF	INF	0	INF
6	INF	INF	INF	INF	INF	INF	0

dist =

	0	1	2	3	4	5	6
0	0	2	INF	INF	INF	3	INF
1	2	0	3	INF	INF	INF	INF
2	INF	2	0	-1	INF	INF	INF
3	INF	2	INF	0	INF	2	4
4	INF	INF	-2	INF	0	INF	1
5	2	INF	INF	INF	INF	0	3
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dist =

	0	1	2	3	4	5	6
0	0	2	INF	INF	INF	3	INF
1	2	0	3	INF	INF	INF	INF
2	INF	2	0	-1	INF	INF	INF
3	INF	2	INF	0	INF	2	4
4	INF	INF	-2	INF	0	INF	1
5	2	INF	INF	INF	INF	0	3
6	INF	INF	INF	INF	2	INF	0

If $2 > -1 + 2$:

$dist[2,1] = -1 + 2 = 1$

dist =

	0	1	2	3	4	5	6
0	0	2	INF	INF	INF	3	INF
1	2	0	3	INF	INF	INF	INF
2	INF	1	0	-1	INF	INF	INF
3	INF	2	INF	0	INF	2	4
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Huffman Coding - Data Compression

Computation Cost:
Cost: $O(n \log(n))$

Fixed length encoding: "every character takes 8 bits."

Variable length encoding: "Use less memory for more frequency characters."

Character	A	B	C	D	E
Frequency	11	3	7	5	20
Binary Encoding	1	100	10	11	0

Problem:

"Encoding is ambiguous"

Ex.

010010111



EAEAEAAA

EBCDA

EAEECAD

...etc

Solution:

"No code (for a character) can be a prefix of another code"

Character	E	A	C	D	B
Frequency	20	11	7	5	3
Binary Encoding	0	10	110	1110	1111

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Huffman Coding - Data Compression

Huffman coding:

Create binary tree

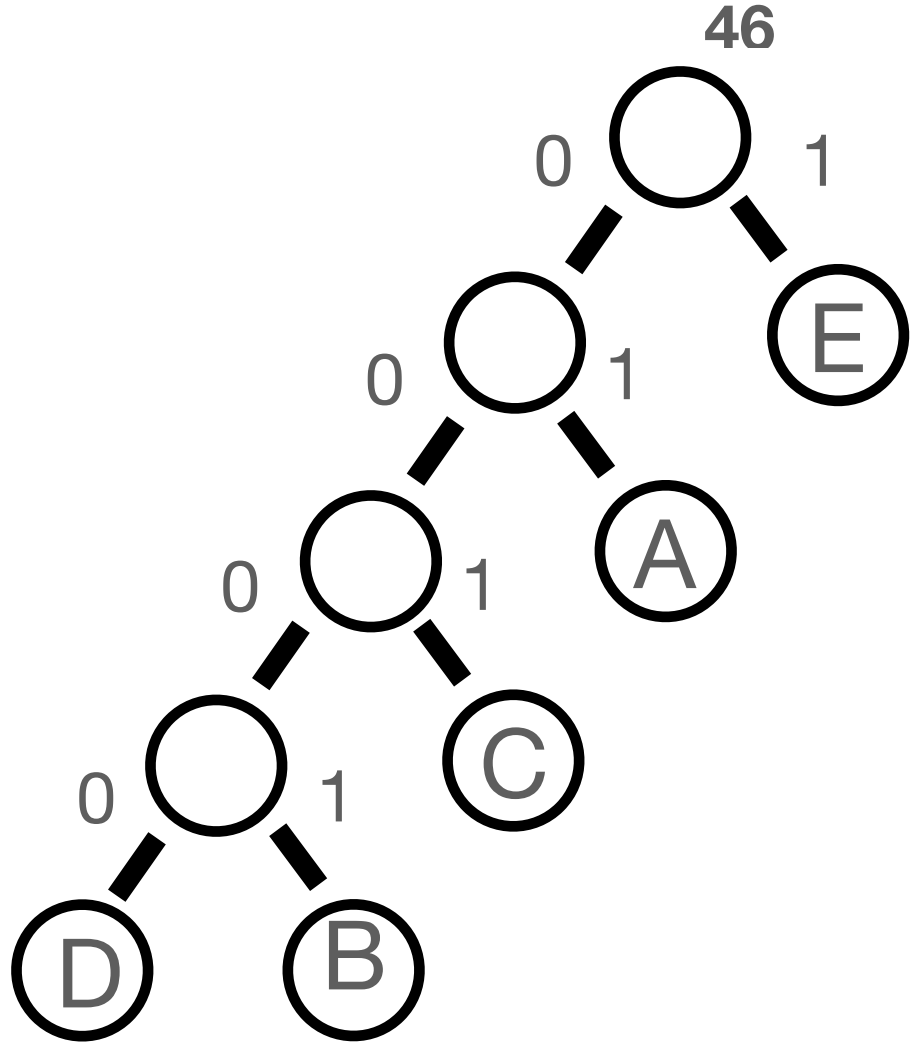
Required data structure:

Priority queue

Character	E	A	C	D	B
Frequency	20	11	7	5	3
Binary Encoding	1	01	001	0001	0000

Binary Tree:

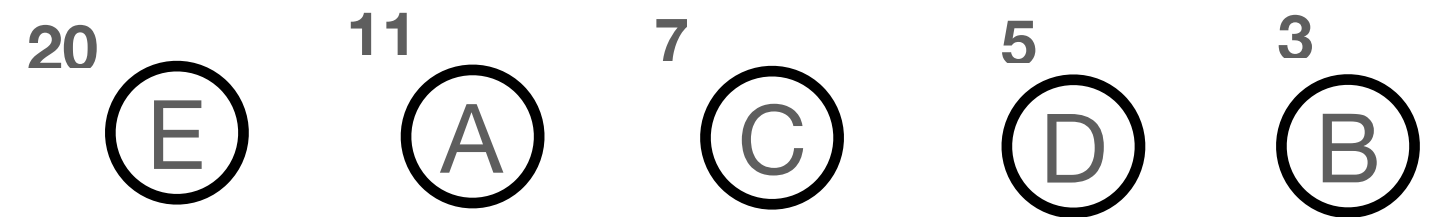
weights along branches give encoding



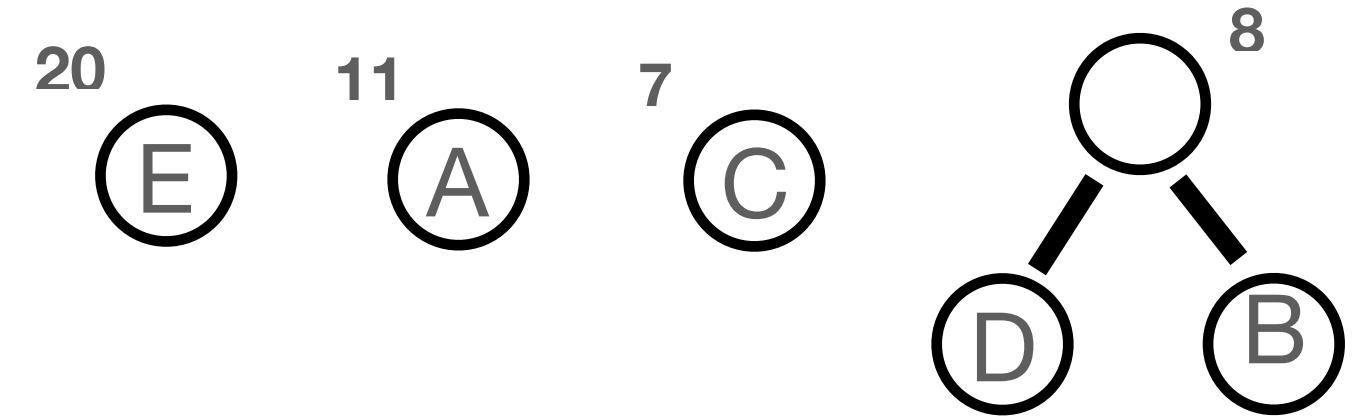
Note the ordering:

...binary tree
... more frequent characters (or groups of characters) assigned 0

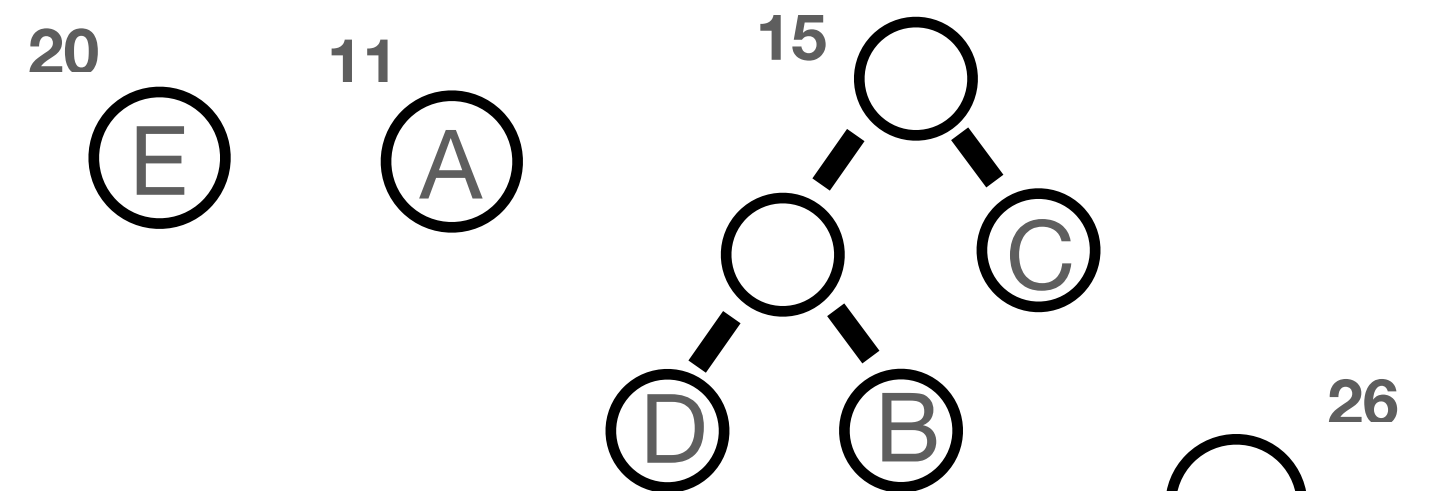
Computation Cost:
Cost: $O(n \log(n))$



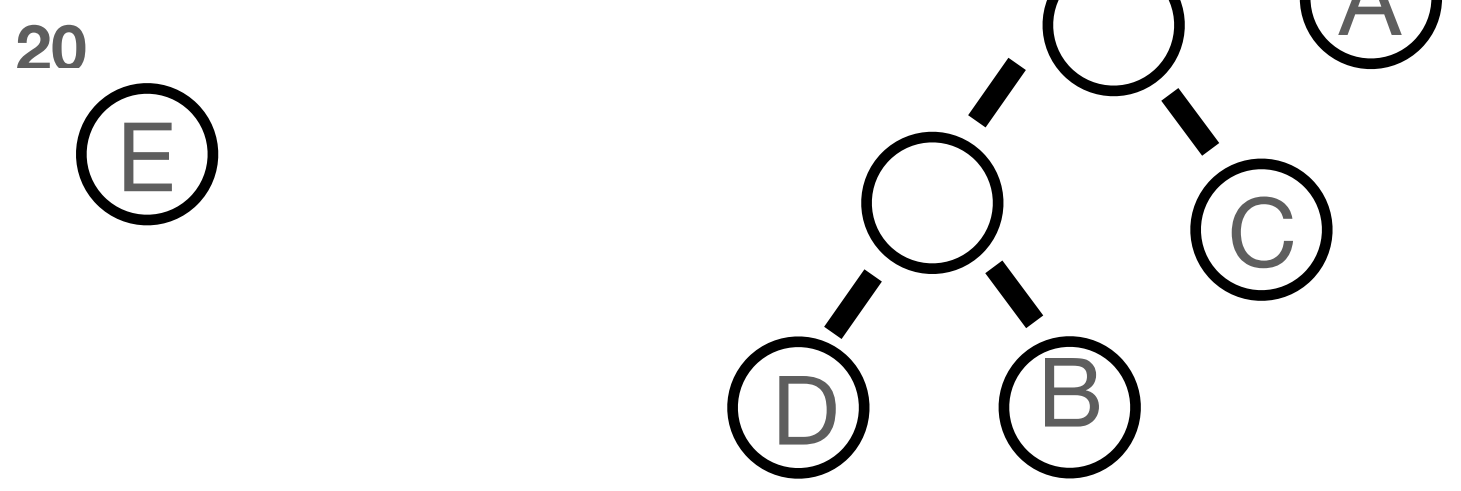
Pick two elements with lowest frequency...



Pick two elements with lowest frequency...



Pick two elements with lowest frequency...



and so on...