Plotting & Drawing - 2D/3D

Python & Matlab

Winter 2022 - Dan Calderone

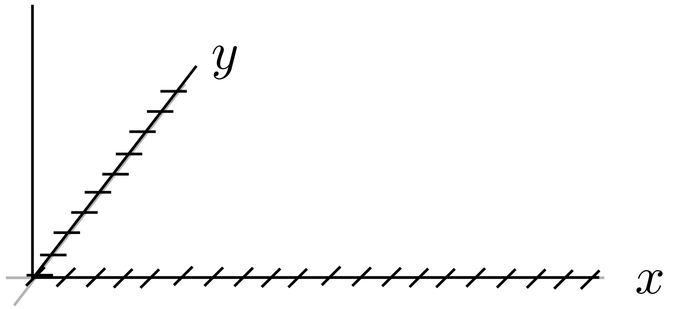
Meshgrid - Surface

Meshgrid …plot f(x,y)

x = [0,1,2,3,4,5,6,7,8,9]

y = [0,1,2,3,4,5,6,7,8,9]

$$z = f(x, y)$$



X,Y = meshgrid(x,y)

```
def height(x,y):
      return // height of surface
```

Z = height(X,Y) apply function to each array element wise

values in Z give surface heights.

surf(X,Y,Z) plot surface heights

$j \rightarrow$

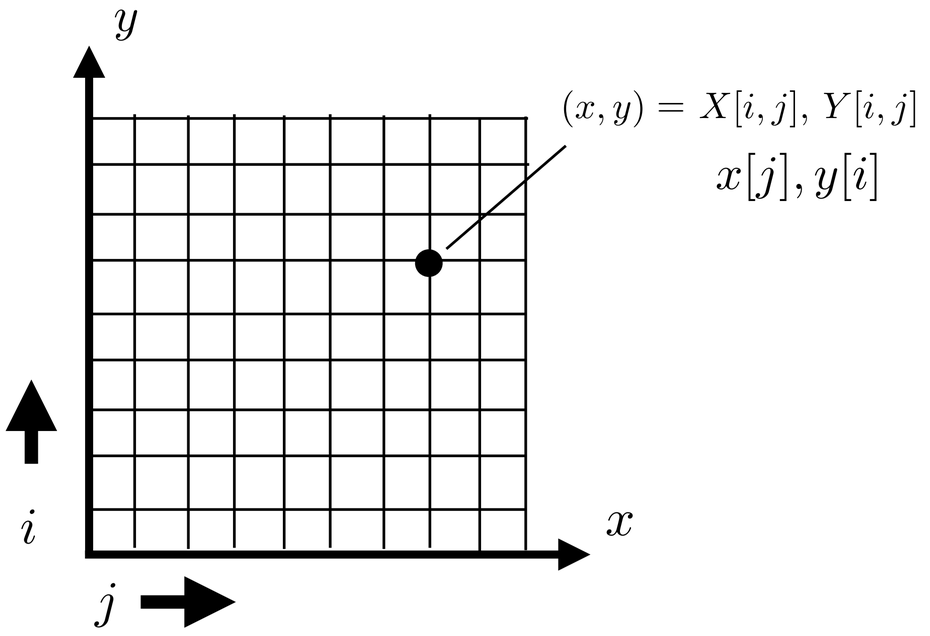
```
X = [ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9 ]
```

varies along 2nd index

$i \downarrow$

```
Y = [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
      2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
      3, 3, 3, 3, 3, 3, 3, 3, 3, 3,
      4, 4, 4, 4, 4, 4, 4, 4, 4, 4,
      5, 5, 5, 5, 5, 5, 5, 5, 5, 5,
      6, 6, 6, 6, 6, 6, 6, 6, 6, 6,
      7, 7, 7, 7, 7, 7, 7, 7, 7, 7,
      8, 8, 8, 8, 8, 8, 8, 8, 8, 8,
      9, 9, 9, 9, 9, 9, 9, 9, 9, 9]
```

varies along 1st index



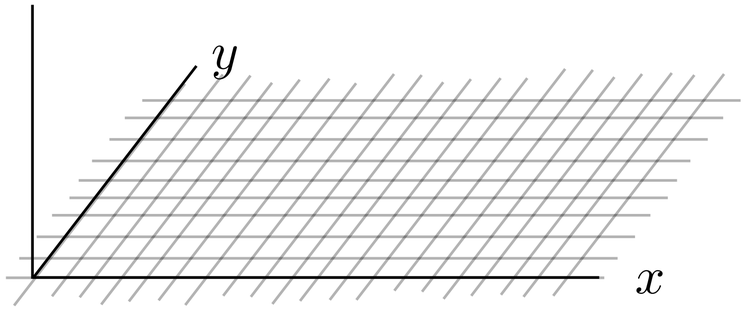
Meshgrid - Surface

Meshgrid …plot f(x,y)

x = [0,1,2,3,4,5,6,7,8,9]

y = [0,1,2,3,4,5,6,7,8,9]

$$z = f(x, y)$$



X,Y = meshgrid(x,y)

```
def height(x,y):
      return // height of surface
```

Z = height(X,Y) apply function to each array element wise

values in Z give surface heights.

surf(X,Y,Z) plot surface heights

j →

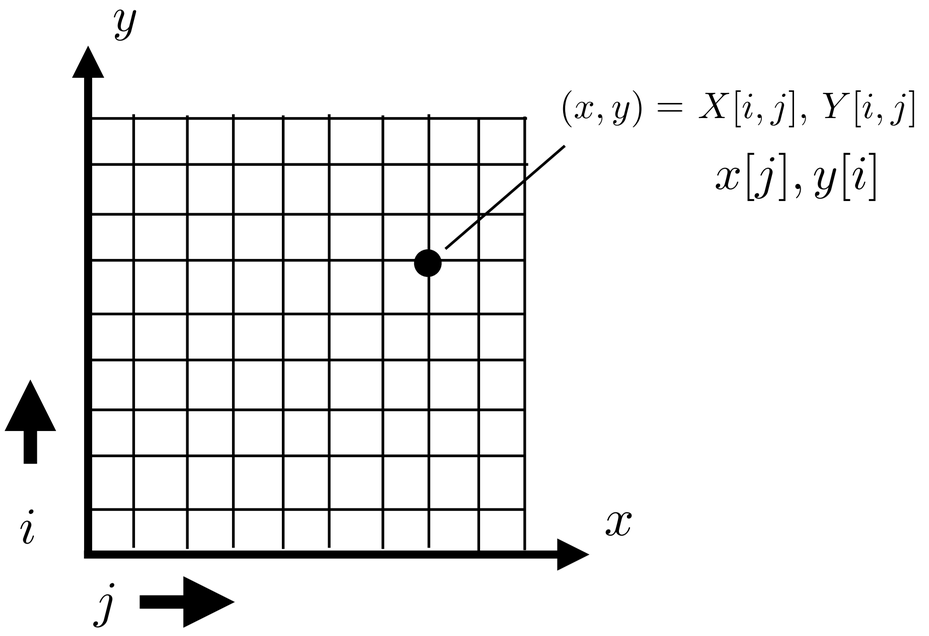
```
X = [ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9 ]
```

varies along 2nd index

i ↓

```
Y = [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
      2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
      3, 3, 3, 3, 3, 3, 3, 3, 3, 3,
      4, 4, 4, 4, 4, 4, 4, 4, 4, 4,
      5, 5, 5, 5, 5, 5, 5, 5, 5, 5,
      6, 6, 6, 6, 6, 6, 6, 6, 6, 6,
      7, 7, 7, 7, 7, 7, 7, 7, 7, 7,
      8, 8, 8, 8, 8, 8, 8, 8, 8, 8,
      9, 9, 9, 9, 9, 9, 9, 9, 9, 9]
```

varies along 1st index

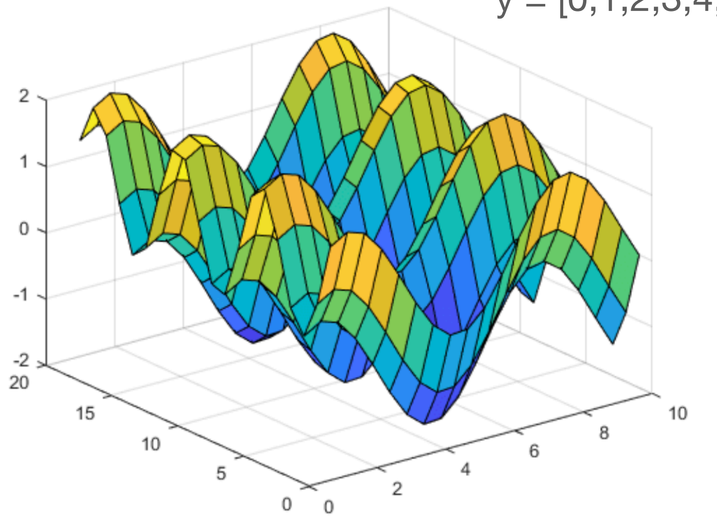


Meshgrid - Surface

Meshgrid …plot f(x,y)

x = [0,1,2,3,4,5,6,7,8,9]

y = [0,1,2,3,4,5,6,7,8,9]



X,Y = meshgrid(x,y)

```
def height(x,y):
      return // height of surface
```

Z = height(X,Y) apply function to each array element wise

values in Z give surface heights.

surf(X,Y,Z) plot surface heights

j →

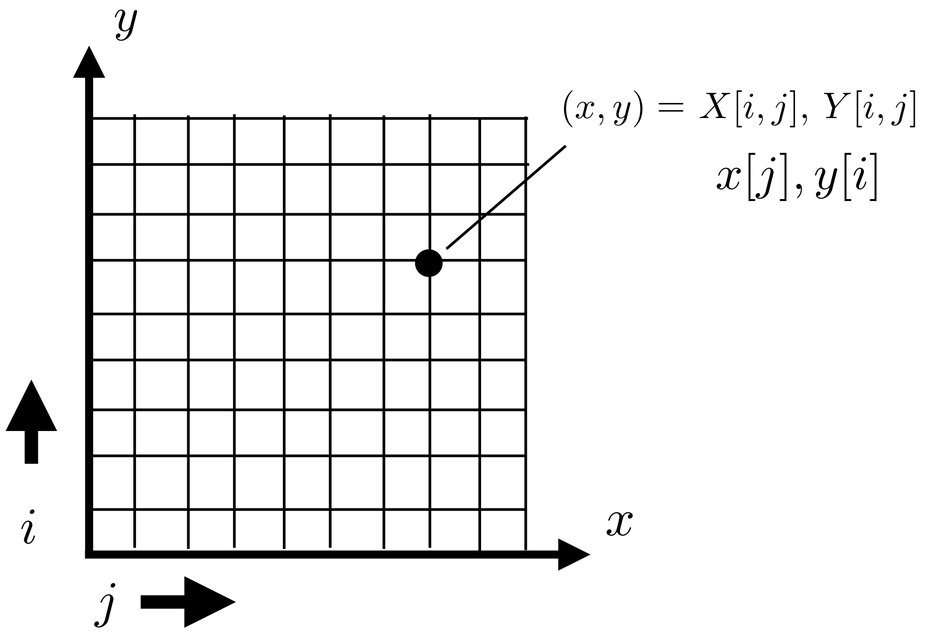
```
X = [ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9 ]
```

varies along 2nd index

i ↓

```
Y = [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
      2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
      3, 3, 3, 3, 3, 3, 3, 3, 3, 3,
      4, 4, 4, 4, 4, 4, 4, 4, 4, 4,
      5, 5, 5, 5, 5, 5, 5, 5, 5, 5,
      6, 6, 6, 6, 6, 6, 6, 6, 6, 6,
      7, 7, 7, 7, 7, 7, 7, 7, 7, 7,
      8, 8, 8, 8, 8, 8, 8, 8, 8, 8,
      9, 9, 9, 9, 9, 9, 9, 9, 9, 9]
```

varies along 1st index

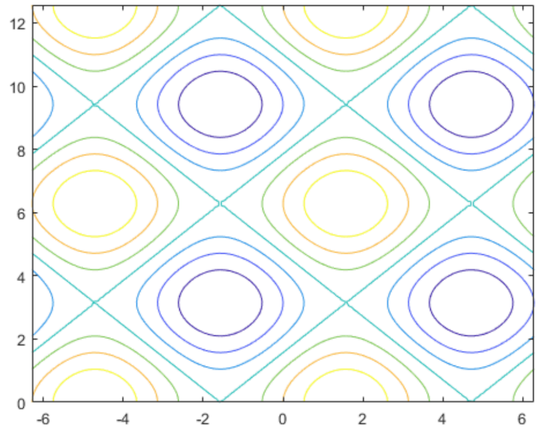


Meshgrid - Contours

Meshgrid …plot f(x,y)

x = [0,1,2,3,4,5,6,7,8,9]

y = [0,1,2,3,4,5,6,7,8,9]



X,Y = meshgrid(x,y)

```
def height(x,y):
      return // height of surface
```

Z = height(X,Y) apply function to each array element wise

values in Z give surface heights.

contour(X,Y,Z) plot contours of surface

j →

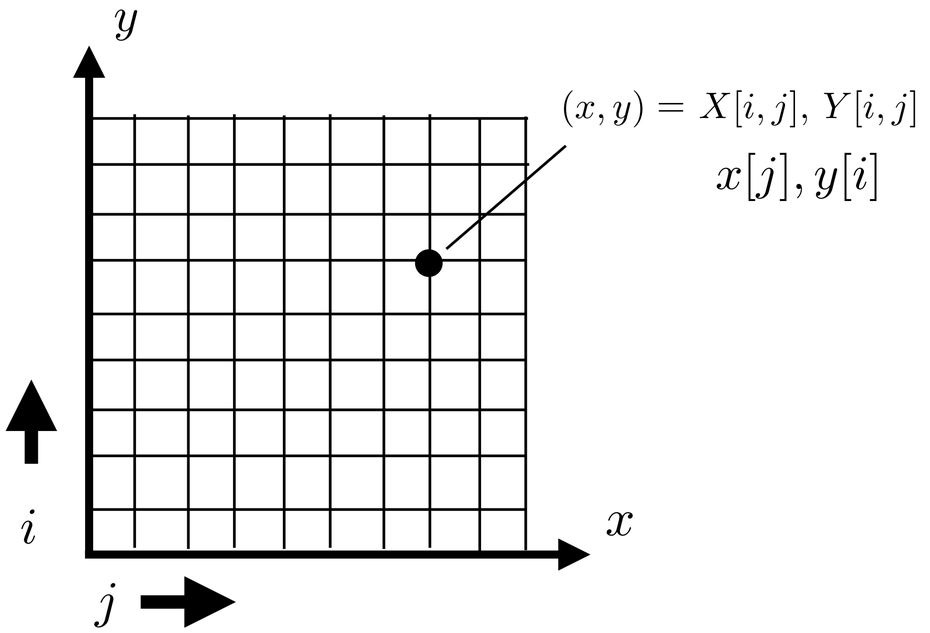
```
X = [ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
      0, 1, 2, 3, 4, 5, 6, 7, 8, 9 ]
```

varies along 2nd index

i ↓

```
Y = [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
      2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
      3, 3, 3, 3, 3, 3, 3, 3, 3, 3,
      4, 4, 4, 4, 4, 4, 4, 4, 4, 4,
      5, 5, 5, 5, 5, 5, 5, 5, 5, 5,
      6, 6, 6, 6, 6, 6, 6, 6, 6, 6,
      7, 7, 7, 7, 7, 7, 7, 7, 7, 7,
      8, 8, 8, 8, 8, 8, 8, 8, 8, 8,
      9, 9, 9, 9, 9, 9, 9, 9, 9, 9]
```

varies along 1st index

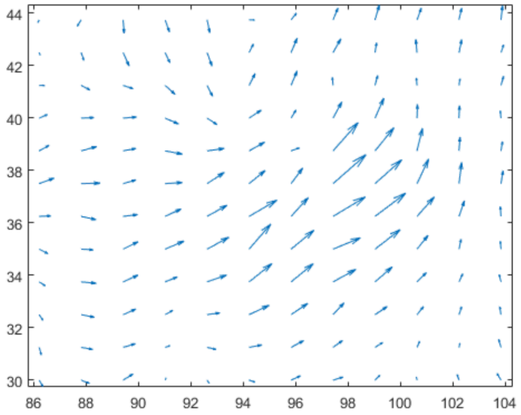


Meshgrid - Vector field

Meshgrid …plot f(x,y)

x = [0,1,2,3,4,5,6,7,8,9]

y = [0,1,2,3,4,5,6,7,8,9]



X,Y = meshgrid(x,y)

j →

X = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
0, 1, 2, 3, 4, 5, 6, 7, 8, 9;
0, 1, 2, 3, 4, 5, 6, 7, 8, 9]

varies along 2nd index

Y = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
3, 3, 3, 3, 3, 3, 3, 3, 3, 3,
4, 4, 4, 4, 4, 4, 4, 4, 4, 4,
5, 5, 5, 5, 5, 5, 5, 5, 5, 5,
6, 6, 6, 6, 6, 6, 6, 6, 6, 6,
7, 7, 7, 7, 7, 7, 7, 7, 7, 7,
8, 8, 8, 8, 8, 8, 8, 8, 8, 8,
9, 9, 9, 9, 9, 9, 9, 9, 9, 9]

i ↓

varies along 1st index

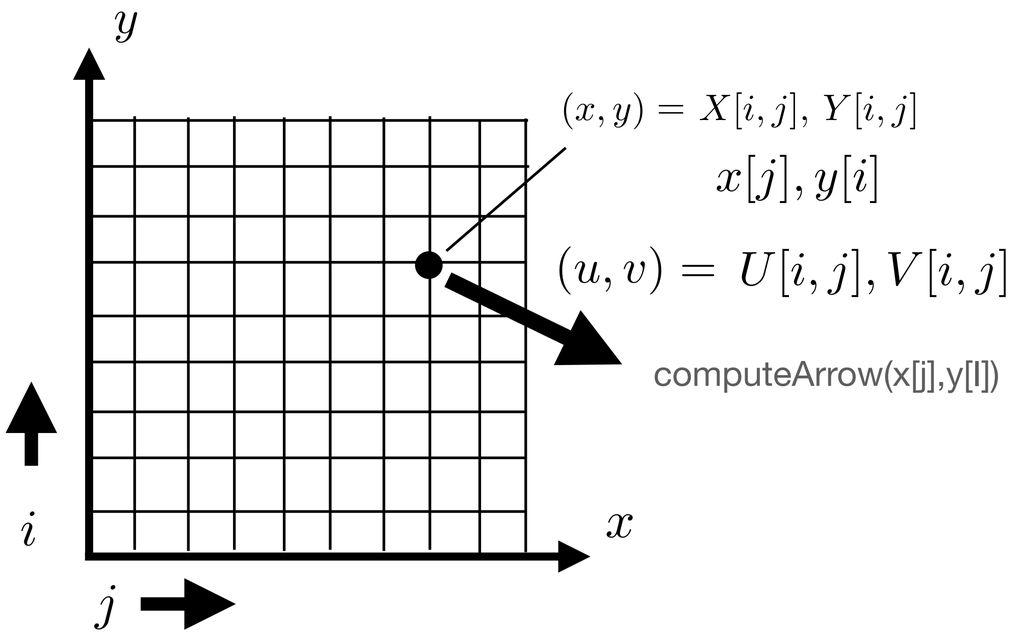
quiver(X,Y,U,V)

vectors located at points

```
def computeArrow(x,y):
      return // returns arrow vector

U = np.zeros(np.shape(X))
V = np.zeros(np.shape(X))

for i in range(nx):
    for j in range(ny):
        arrow = computeArrow(X[i,j],Y[i,j]);
        U[i,j] = arrow[0]
        V[i,j] = arrow[1]
```



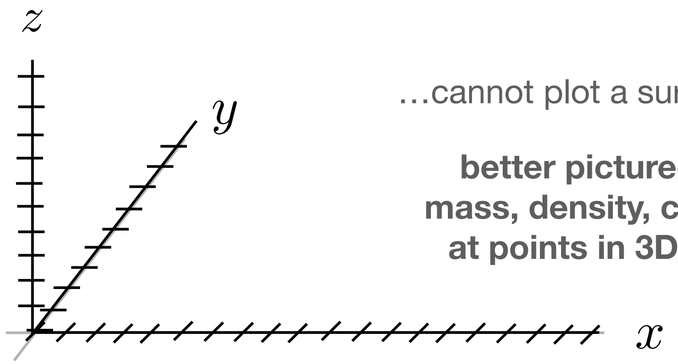
Meshgrid 3D

Meshgrid …plot f(x,y,z)

x = [0,1,2,3,4,5,6,7,8,9]

y = [0,1,2,3,4,5,6,7,8,9]

z = [0,1,2,3,4,5,6,7,8,9]



…cannot plot a surface in 4D

better pictured as a mass, density, color, etc at points in 3D space

X,Y,Z = meshgrid(x,y,z)

```
def density(x,y,z):
    return // density at a point
```

W = density(X,Y,Z) apply function to each array element wise

values in W give density at x,y,z point

```
X = [[ [ 0, 0, 0, 0 ],
       [ 1, 1, 1, 1 ],
       [ 2, 2, 2, 2 ],
       [ 3, 3, 3, 3 ] ],

     [[ 0, 0, 0, 0 ],
      [ 1, 1, 1, 1 ],
      [ 2, 2, 2, 2 ],
      [ 3, 3, 3, 3 ]],

     [[ 0, 0, 0, 0 ],
      [ 1, 1, 1, 1 ],
      [ 2, 2, 2, 2 ],
      [ 3, 3, 3, 3 ]]]
```

varies along 2nd index

```
Y = [[ [ 0, 0, 0, 0 ],
       [ 0, 0, 0, 0 ],
       [ 0, 0, 0, 0 ],
       [ 0, 0, 0, 0 ] ],

     [ [ 1, 1, 1, 1 ],
       [ 1, 1, 1, 1 ],
       [ 1, 1, 1, 1 ],
       [ 1, 1, 1, 1 ] ],

     [ [ 2, 2, 2, 2 ],
       [ 2, 2, 2, 2 ],
       [ 2, 2, 2, 2 ],
       [ 2, 2, 2, 2 ] ],

     [ [ 3, 3, 3, 3 ],
       [ 3, 3, 3, 3 ],
       [ 3, 3, 3, 3 ],
       [ 3, 3, 3, 3 ] ] ]
```

varies along 1st index

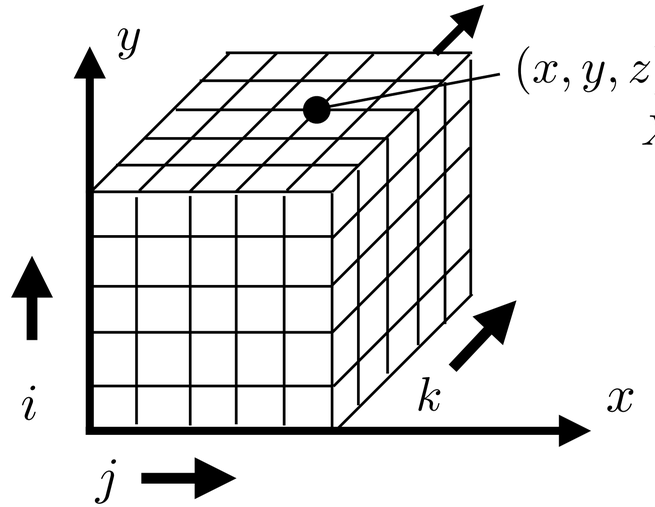
```
Z = [[ [ 0, 1, 2, 3 ],
       [ 0, 1, 2, 3 ],
       [ 0, 1, 2, 3 ],
       [ 0, 1, 2, 3 ] ],

     [ [ 0, 1, 2, 3 ],
       [ 0, 1, 2, 3 ],
       [ 0, 1, 2, 3 ],
       [ 0, 1, 2, 3 ] ],

     [ [ 0, 1, 2, 3 ],
       [ 0, 1, 2, 3 ],
       [ 0, 1, 2, 3 ],
       [ 0, 1, 2, 3 ] ],

     [ [ 0, 1, 2, 3 ],
       [ 0, 1, 2, 3 ],
       [ 0, 1, 2, 3 ],
       [ 0, 1, 2, 3 ] ] ]
```

varies along 3rd index



$$(x, y, z) = X[i,j,k], Y[i,j,k], Z[i,j,k]$$

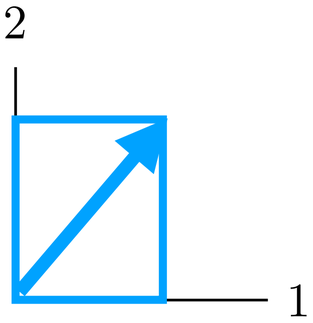
$$x[j], y[i], z[k]$$

Axes & Coordinates - 2D

x = [0.5, 0.75]

AXES = [[1 , 0],
[0 , 1]]

x @ AXES



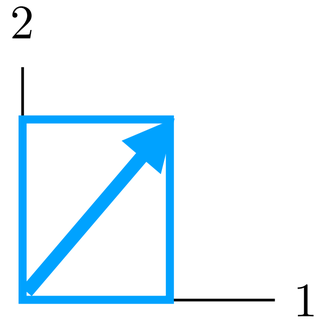
Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix}$$

Axes & Coordinates - 2D

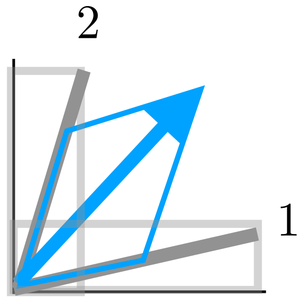
x = [0.5, 0.75] AXES = [[1 , 0],
[0 , 1]]

x @ AXES



x = [0.5, 0.75] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

x @ CRDS @ AXES



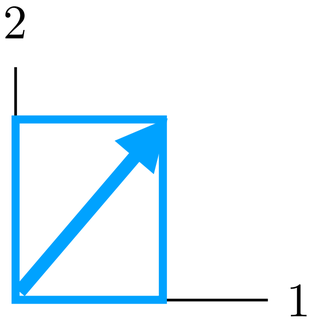
Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix}$$

Axes & Coordinates - 2D

x = [0.5, 0.75] AXES = [[1 , 0],
[0 , 1]]

x @ AXES

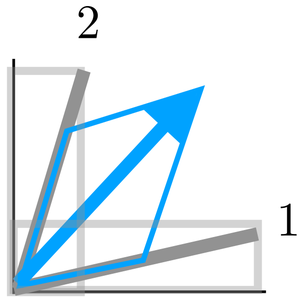


Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \end{bmatrix}}_{A} = \begin{bmatrix} x_{11}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{12}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{21}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{22}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \end{bmatrix}$$

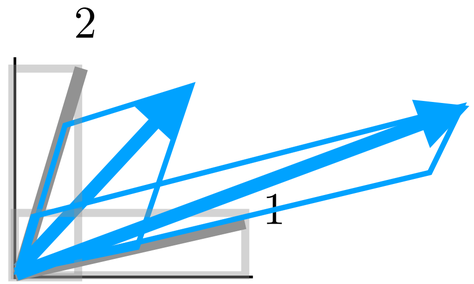
x = [0.5, 0.75] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

x @ CRDS @ AXES



x = [[0.5, 0.7],
[2.0, 0.3]] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

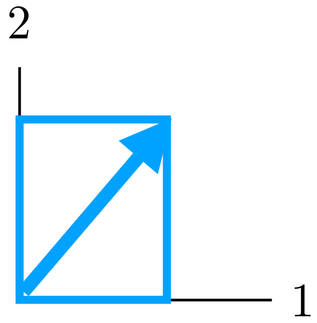
x @ CRDS @ AXES



Axes & Coordinates - 2D

x = [0.5, 0.75] AXES = [[1 , 0],
[0 , 1]]

x @ AXES

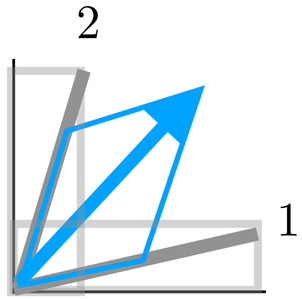


Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{32} \\ x_{31} & x_{32} \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \end{bmatrix}}_{A} = \begin{bmatrix} x_{11}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{12}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{21}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{22}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{31}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{32}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \end{bmatrix}$$

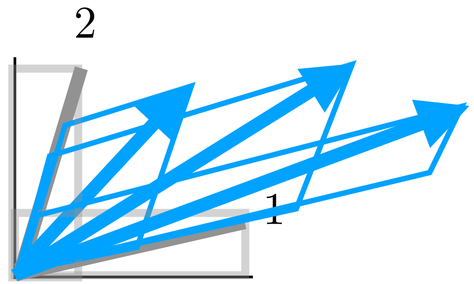
x = [0.5, 0.75] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

x @ CRDS @ AXES



x = [[0.5, 0.7],
[2.0, 0.3],
[1.2, 0.5]] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

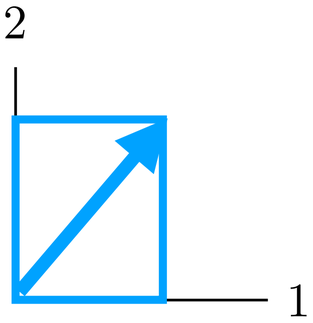
x @ CRDS @ AXES



Axes & Coordinates - 2D Shapes

x = [0.5, 0.75] AXES = [[1 , 0],
[0 , 1]]

x @ AXES

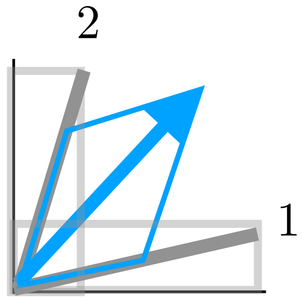


Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{32} \\ x_{31} & x_{32} \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \end{bmatrix}}_{A} = \begin{bmatrix} x_{11}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{12}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{21}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{22}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{31}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{32}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \end{bmatrix}$$

x = [0.5, 0.75] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

x @ CRDS @ AXES

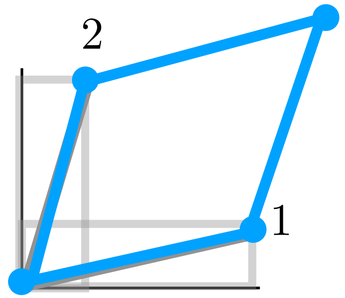


Unit cube

SHAPE = [[0 , 0],
[1 , 0],
[1 , 1],
[0 , 1]]

CRDS = [[1.0, 0.2],
[0.2, 1.0]]

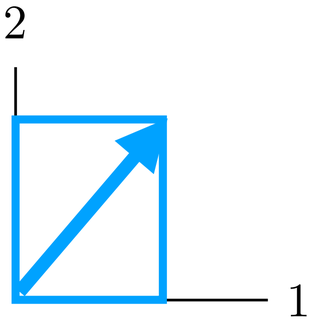
SHAPE @ CRDS @ AXES



Axes & Coordinates - 2D Shapes

x = [0.5, 0.75] AXES = [[1 , 0],
[0 , 1]]

x @ AXES

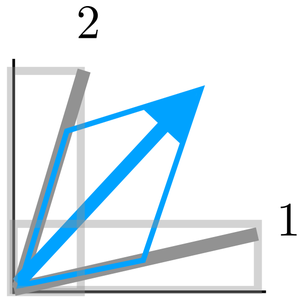


Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{32} \\ x_{31} & x_{32} \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \end{bmatrix}}_{A} = \begin{bmatrix} x_{11}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{12}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{21}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{22}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{31}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{32}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \end{bmatrix}$$

x = [0.5, 0.75] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

x @ CRDS @ AXES

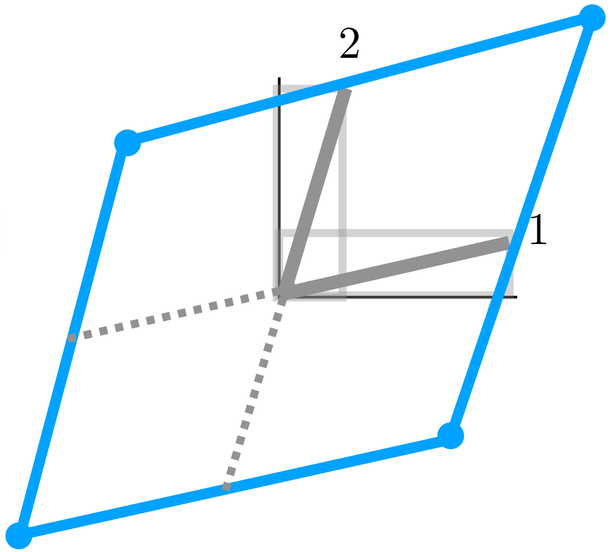


Inf-norm ball

SHAPE = [[-1,-1],
[1,-1],
[1, 1],
[-1, 1]]

CRDS = [[1.0, 0.2],
[0.2, 1.0]]

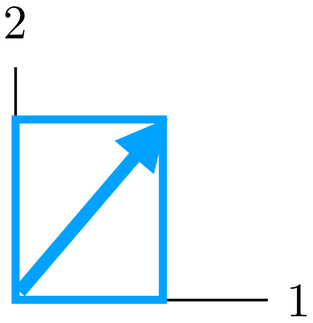
SHAPE @ CRDS @ AXES



Axes & Coordinates - 2D Shapes

x = [0.5, 0.75] AXES = [[1 , 0],
[0 , 1]]

x @ AXES

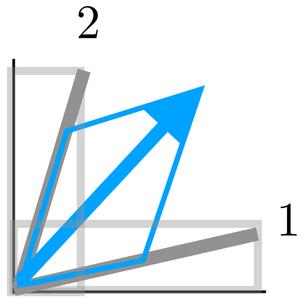


Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{32} \\ x_{31} & x_{32} \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \end{bmatrix}}_{A} = \begin{bmatrix} x_{11}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{12}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{21}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{22}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{31}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{32}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \end{bmatrix}$$

x = [0.5, 0.75] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

x @ CRDS @ AXES

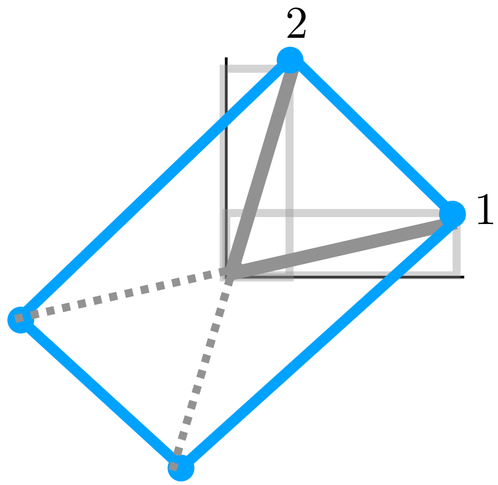


Diamond

SHAPE = [[1, 0],
[0, 1],
[-1, 0],
[0,-1]]

CRDS = [[1.0, 0.2],
[0.2, 1.0]]

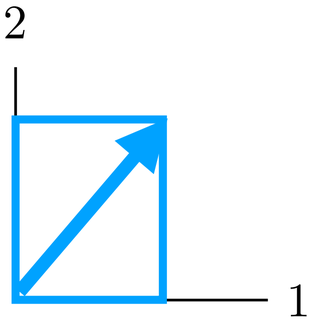
SHAPE @ CRDS @ AXES



Axes & Coordinates - 2D Shapes

x = [0.5, 0.75] AXES = [[1 , 0],
[0 , 1]]

x @ AXES

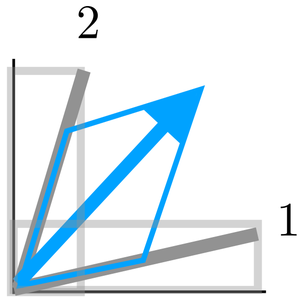


Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{32} \\ x_{31} & x_{32} \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \end{bmatrix}}_{A} = \begin{bmatrix} x_{11}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{12}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{21}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{22}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{31}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{32}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \end{bmatrix}$$

x = [0.5, 0.75] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

x @ CRDS @ AXES



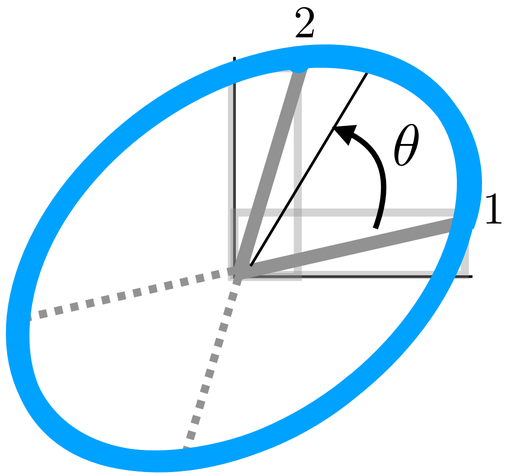
Unit circle

SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

θ

[cos(6.2), sin(6.2)]]

CRDS = [[1.0, 0.2],
[0.2, 1.0]]

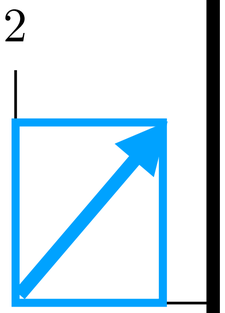


SHAPE @ CRDS @ AXES

Axes & Coordinates - 2D Sh

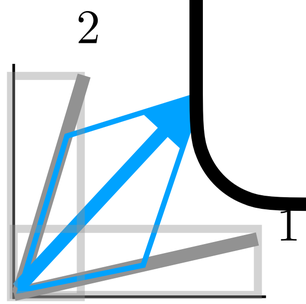
x = [0.5, 0.75] AXES = [[1 , 0],
[0 , 1]]

x @ AXES



x = [0.5, 0.75] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

x @ CRDS @ AXES



Ellipse: Axis-Length Representation

2 x 2 matrix	2 x 2 rotation		2 x 2 rotation	
CRDS @ AXES	$=$	$\begin{bmatrix} & U & \end{bmatrix}$	$\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$	$\begin{bmatrix} & V^T & \end{bmatrix}$

Singular Value Decomposition

$$= \begin{bmatrix} | & | \\ U_1 & U_2 \\ | & | \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} - & V_1^T & - \\ - & V_2^T & - \end{bmatrix}$$

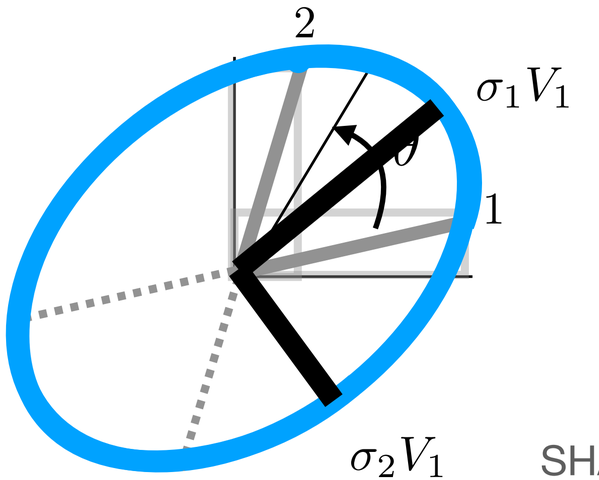
Unit circle

SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

θ

[cos(6.2), sin(6.2)]]

CRDS = [[1.0, 0.2],
[0.2, 1.0]]



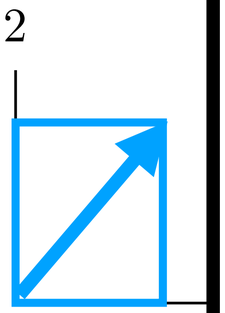
SHAPE @ CRDS @ AXES

$$\begin{bmatrix} | & & | \\ U_1 & \cdots & U_n \\ | & & | \end{bmatrix}$$

Axes & Coordinates - 2D Sh

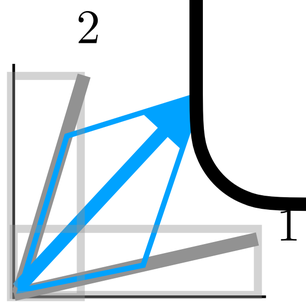
x = [0.5, 0.75] AXES = [[1 , 0],
[0 , 1]]

x @ AXES



x = [0.5, 0.75] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

x @ CRDS @ AXES



Ellipse: Axis-Length Representation

2 x 2 matrix | **2 x 2 rotation** | **2 x 2 rotation**

$$\text{CRDS @ AXES} = \begin{bmatrix} & U & \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} & V^T & \end{bmatrix}$$

Singular Value Decomposition

$$= \begin{bmatrix} | & | \\ U_1 & U_2 \\ | & | \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} - & V_1^T & - \\ - & V_2^T & - \end{bmatrix}$$

← Axis 1
← Axis 2

Length 1 (↑ σ_1) Length 2 (↑ σ_2)

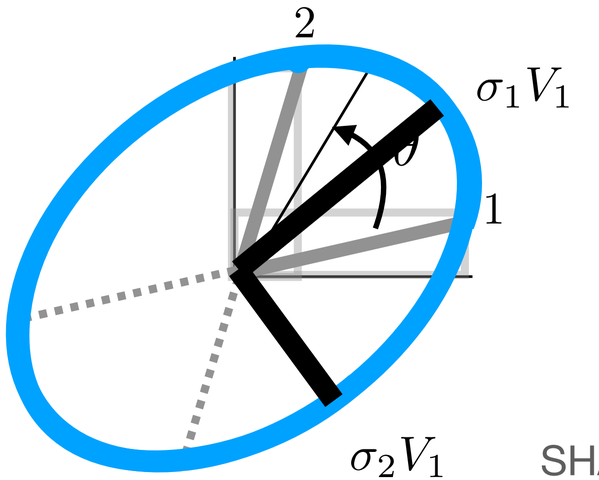
Unit circle

SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

θ ↓

[cos(6.2), sin(6.2)]]

CRDS = [[1.0, 0.2],
[0.2, 1.0]]

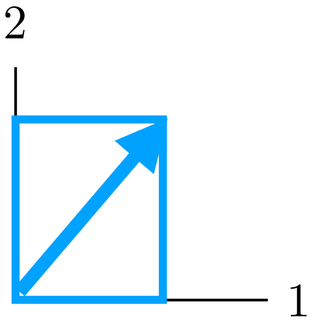


SHAPE @ CRDS @ AXES

Axes & Coordinates - 2D Shapes

x = [0.5, 0.75] AXES = [[1 , 0],
[0 , 1]]

x @ AXES

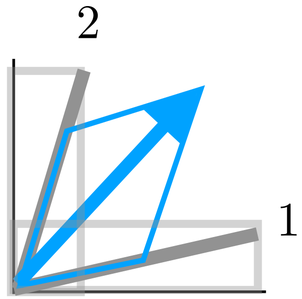


Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{32} \\ x_{31} & x_{32} \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \end{bmatrix}}_{A} = \begin{bmatrix} x_{11}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{12}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{21}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{22}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{31}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{32}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \end{bmatrix}$$

x = [0.5, 0.75] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

x @ CRDS @ AXES



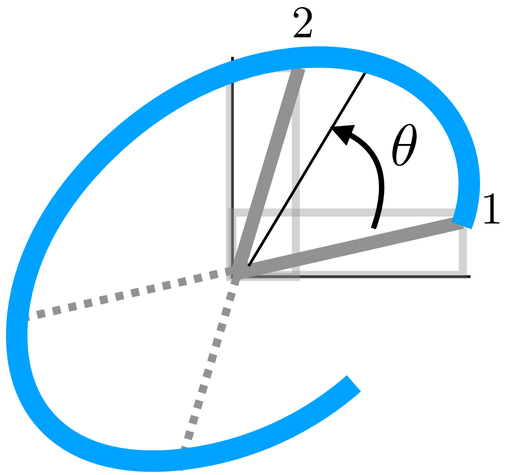
Circle arc

SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

θ

[cos(5.5), sin(5.5)]]

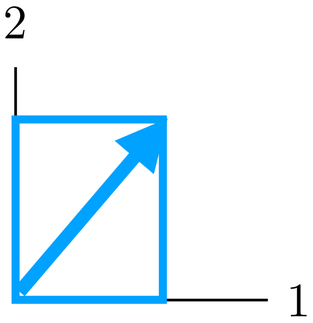
CRDS = [[1.0, 0.2],
[0.2, 1.0]]



Axes & Coordinates - 2D Shapes

x = [0.5, 0.75] AXES = [[1 , 0],
[0 , 1]]

x @ AXES

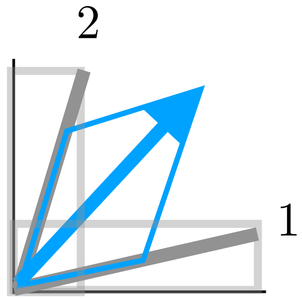


Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{32} \\ x_{31} & x_{32} \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \end{bmatrix}}_{A} = \begin{bmatrix} x_{11}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{12}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{21}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{22}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{31}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{32}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \end{bmatrix}$$

x = [0.5, 0.75] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

x @ CRDS @ AXES



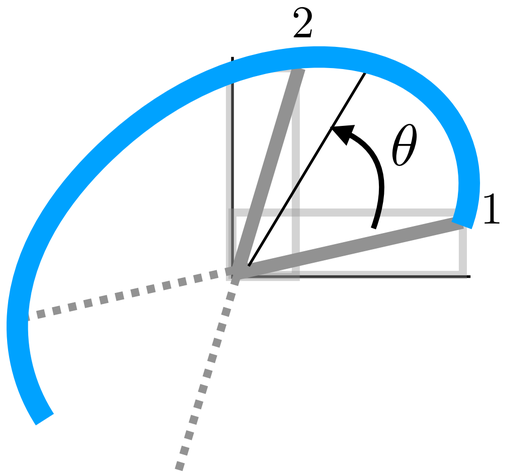
Circle arc

SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

θ

[cos(3.9), sin(3.9)]]

CRDS = [[1.0, 0.2],
[0.2, 1.0]]

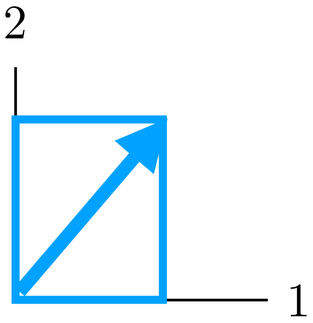


SHAPE @ CRDS @ AXES

Axes & Coordinates - 2D Shapes

x = [0.5, 0.75] AXES = [[1 , 0],
[0 , 1]]

x @ AXES

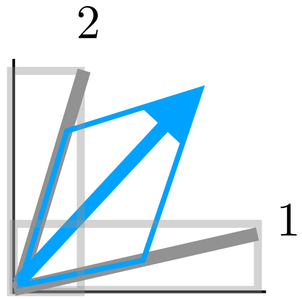


Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{32} \\ x_{31} & x_{32} \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \end{bmatrix}}_{A} = \begin{bmatrix} x_{11}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{12}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{21}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{22}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{31}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{32}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \end{bmatrix}$$

x = [0.5, 0.75] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

x @ CRDS @ AXES



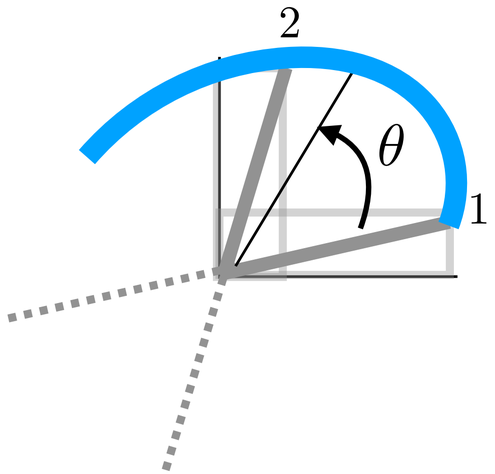
Circle arc

SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

θ

[cos(2.4), sin(2.4)]]

CRDS = [[1.0, 0.2],
[0.2, 1.0]]

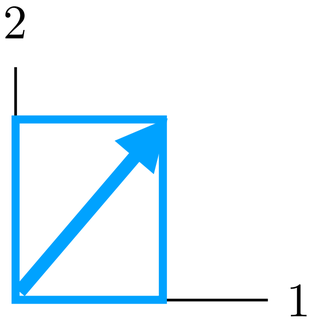


SHAPE @ CRDS @ AXES

Axes & Coordinates - 2D Shapes

x = [0.5, 0.75] AXES = [[1 , 0],
[0 , 1]]

x @ AXES

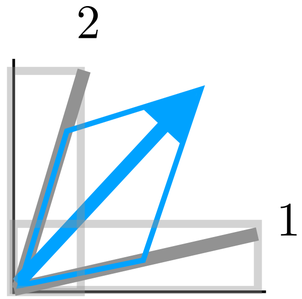


Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{32} \\ x_{31} & x_{32} \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \end{bmatrix}}_{A} = \begin{bmatrix} x_{11}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{12}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{21}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{22}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{31}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{32}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \end{bmatrix}$$

x = [0.5, 0.75] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

x @ CRDS @ AXES



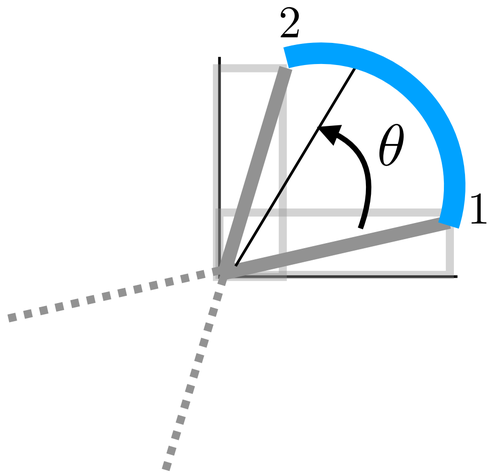
Circle arc

SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

CRDS = [[1.0, 0.2],
[0.2, 1.0]]

θ

[cos(1.6), sin(1.6)]]

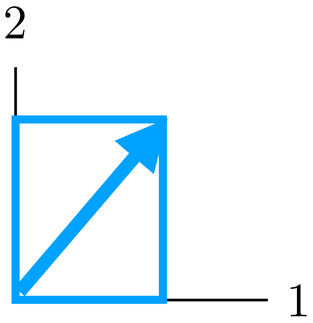


SHAPE @ CRDS @ AXES

Axes & Coordinates - 2D Shapes

x = [0.5, 0.75] AXES = [[1 , 0],
[0 , 1]]

x @ AXES

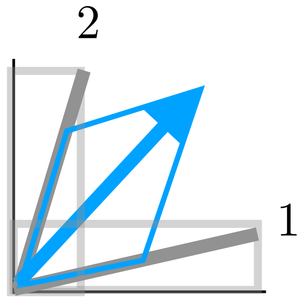


Matrix Multiplication

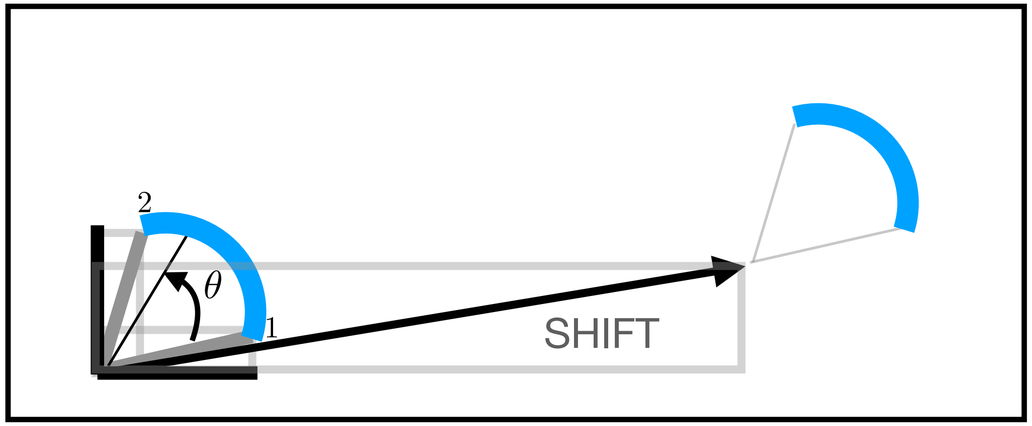
$$\underbrace{\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{32} \\ x_{31} & x_{32} \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \end{bmatrix}}_{A} = \begin{bmatrix} x_{11}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{12}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{21}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{22}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{31}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{32}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \end{bmatrix}$$

x = [0.5, 0.75] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

x @ CRDS @ AXES



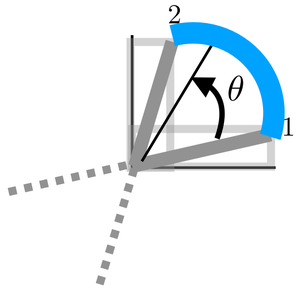
Drawing



Circle arc

SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

CRDS = [[1.0, 0.2],
[0.2, 1.0]]



θ

[cos(1.6), sin(1.6)]]

Code

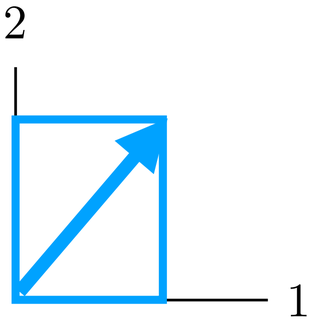
PTS = SHAPE @ CRDS @ AXES + SHIFT @ AXES

plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 2D Shapes

x = [0.5, 0.75] AXES = [[1 , 0],
[0 , 1]]

x @ AXES

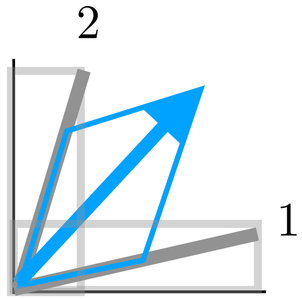


Matrix Multiplication

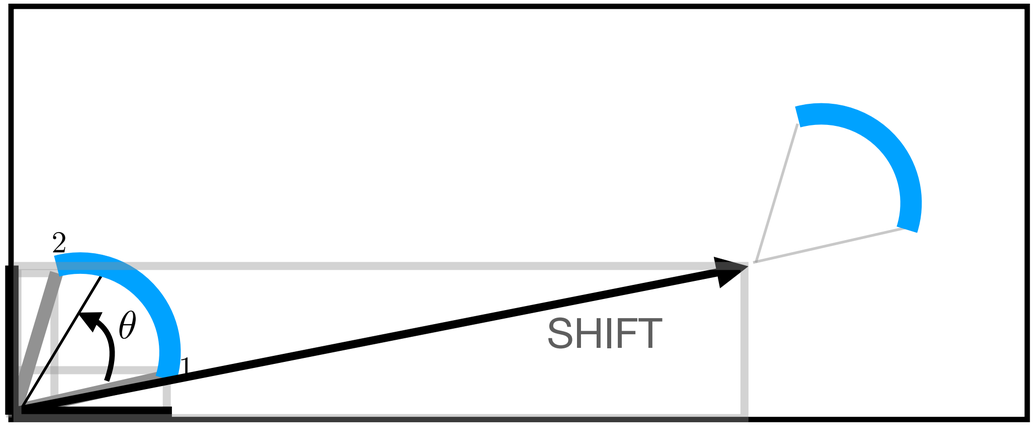
$$\underbrace{\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{32} \\ x_{31} & x_{32} \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \end{bmatrix}}_{A} = \begin{bmatrix} x_{11}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{12}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{21}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{22}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{31}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{32}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \end{bmatrix}$$

x = [0.5, 0.75] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

x @ CRDS @ AXES



Drawing



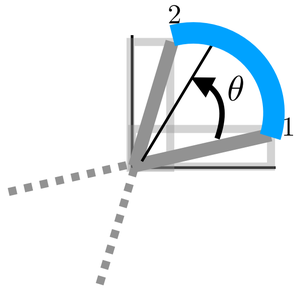
Circle arc

SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

CRDS = [[1.0, 0.2],
[0.2, 1.0]]

θ

[cos(1.6), sin(1.6)]]



Code

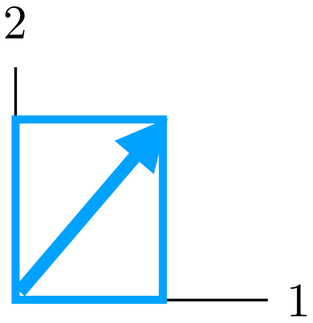
PTS = SHAPE @ CRDS @ AXES + SHIFT @ AXES

plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 2D Shapes

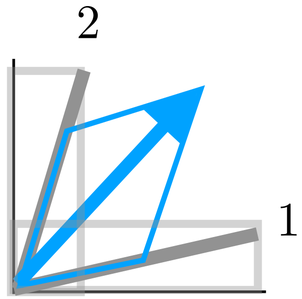
x = [0.5, 0.75] AXES = [[1 , 0],
[0 , 1]]

x @ AXES



x = [0.5, 0.75] CRDS = [[1.0, 0.2],
[0.2, 1.0]]

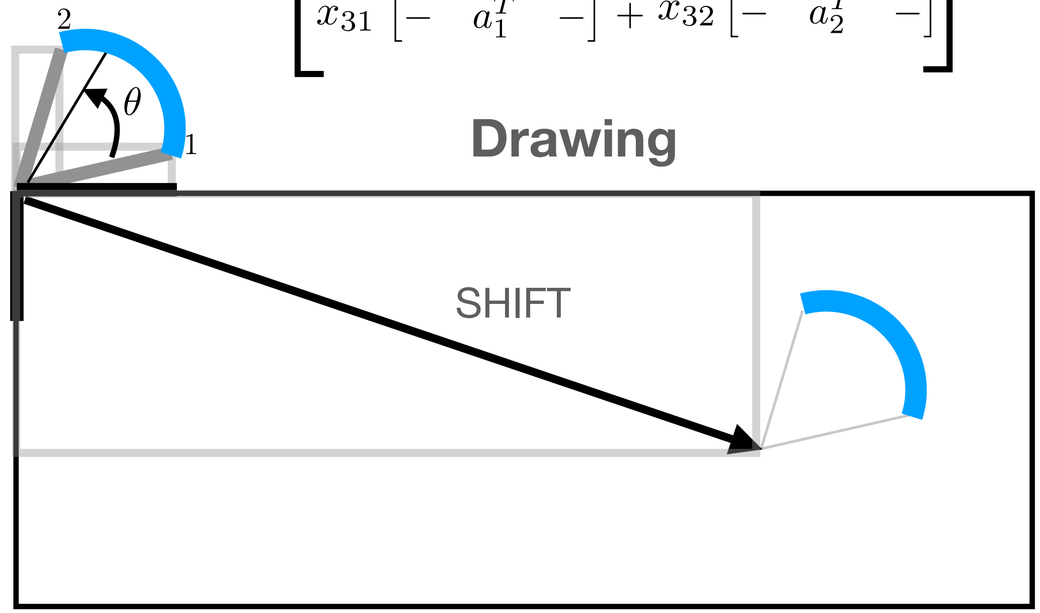
x @ CRDS @ AXES



Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{32} \\ x_{31} & x_{32} \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \end{bmatrix}}_{A} = \begin{bmatrix} x_{11}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{12}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{21}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{22}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \\ x_{31}\begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_{32}\begin{bmatrix} - & a_2^T & - \end{bmatrix} \end{bmatrix}$$

Drawing

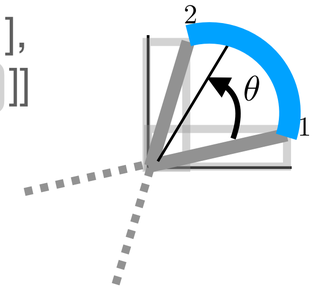


Circle arc

SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

CRDS = [[-1.0, 0.2],
[-0.2, 1.0]]

θ



[cos(1.6), sin(1.6)]]

Code

PTS = SHAPE @ CRDS @ AXES + SHIFT @ AXES

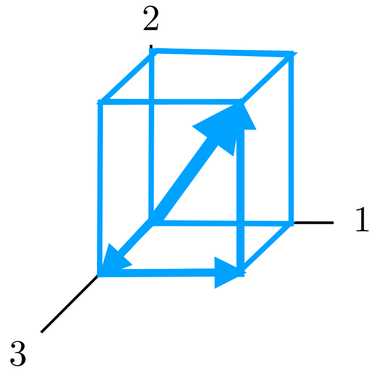
plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Shapes

x = [0.8, 1.0, 0.5]

AXES = [[1.0, 0.0],
[0.0, 1.0],
[-.7, -.7]]

x @ AXES



Matrix Multiplication

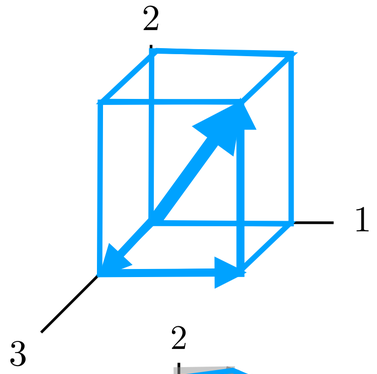
$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing

Axes & Coordinates - 3D Shapes

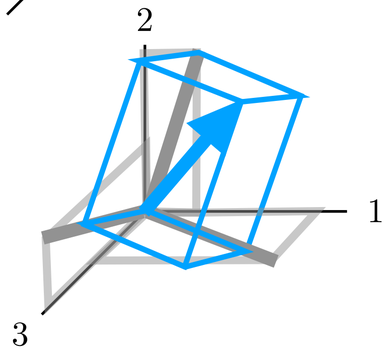
x = [0.8, 1.0, 0.5] AXES = [[1.0, 0.0],
[0.0, 1.0],
[-.7, -.7]]

x @ AXES



x = [0.8, 1.0, 0.5] CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

x @ CRDS @ AXES



Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

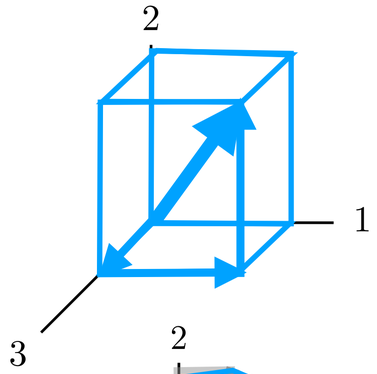
Drawing

Axes & Coordinates - 3D Shapes

x = [0.8, 1.0, 0.5]

AXES = [[1.0, 0.0],
[0.0, 1.0],
[-.7, -.7]]

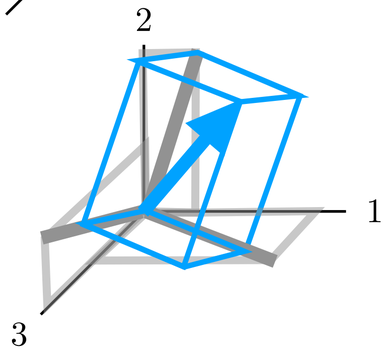
x @ AXES



x = [0.8, 1.0, 0.5]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

x @ CRDS @ AXES

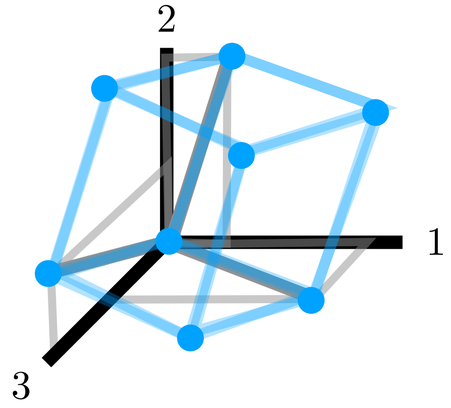


Cube

SHAPE = [[0, 0, 0],
[1, 0, 0],
[1, 1, 0],
[0, 1, 0],
[0, 0, 1],
[1, 0, 1],
[1, 1, 1],
[0, 1, 1]]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

SHAPE @ CRDS @ AXES



Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

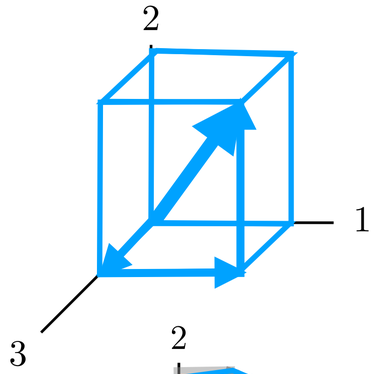
Drawing

Axes & Coordinates - 3D Shapes

x = [0.8, 1.0, 0.5]

AXES = [[1.0, 0.0],
[0.0, 1.0],
[-.7, -.7]]

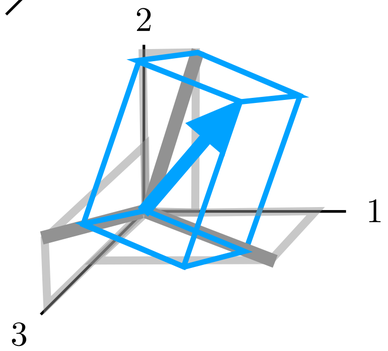
x @ AXES



x = [0.8, 1.0, 0.5]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

x @ CRDS @ AXES

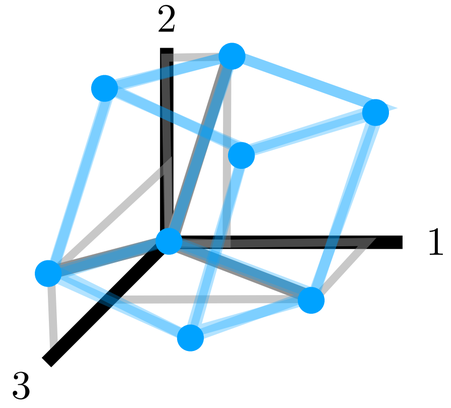


Cube

SHAPE = [[0, 0, 0],
[1, 0, 0],
[1, 1, 0],
[0, 1, 0],
[0, 0, 1],
[1, 0, 1],
[1, 1, 1],
[0, 1, 1]]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

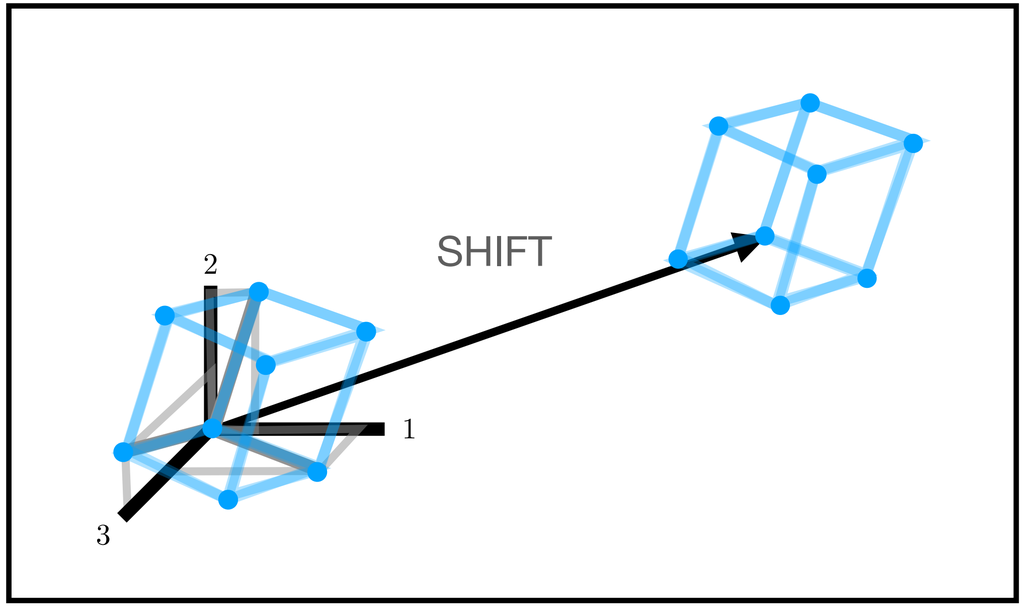
SHAPE @ CRDS @ AXES



Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Code

PTS = SHAPE @ CRDS @ AXES + SHIFT @ AXES

OR + SHIFT @ AXES2

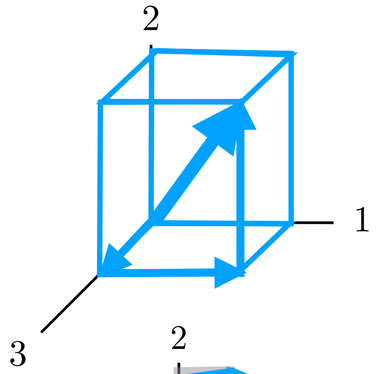
plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Shapes

x = [0.8, 1.0, 0.5]

AXES = [[1.0, 0.0],
[0.0, 1.0],
[-.7, -.7]]

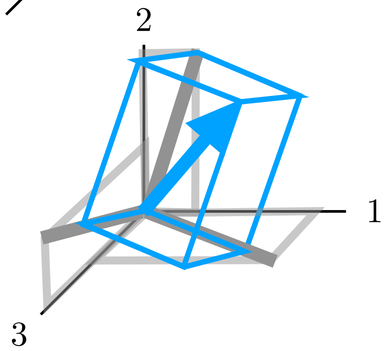
x @ AXES



x = [0.8, 1.0, 0.5]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

x @ CRDS @ AXES

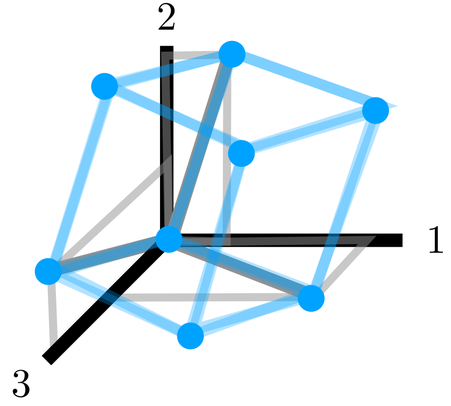


Cube

SHAPE = [[0, 0, 0],
[1, 0, 0],
[1, 1, 0],
[0, 1, 0],
[0, 0, 1],
[1, 0, 1],
[1, 1, 1],
[0, 1, 1]]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

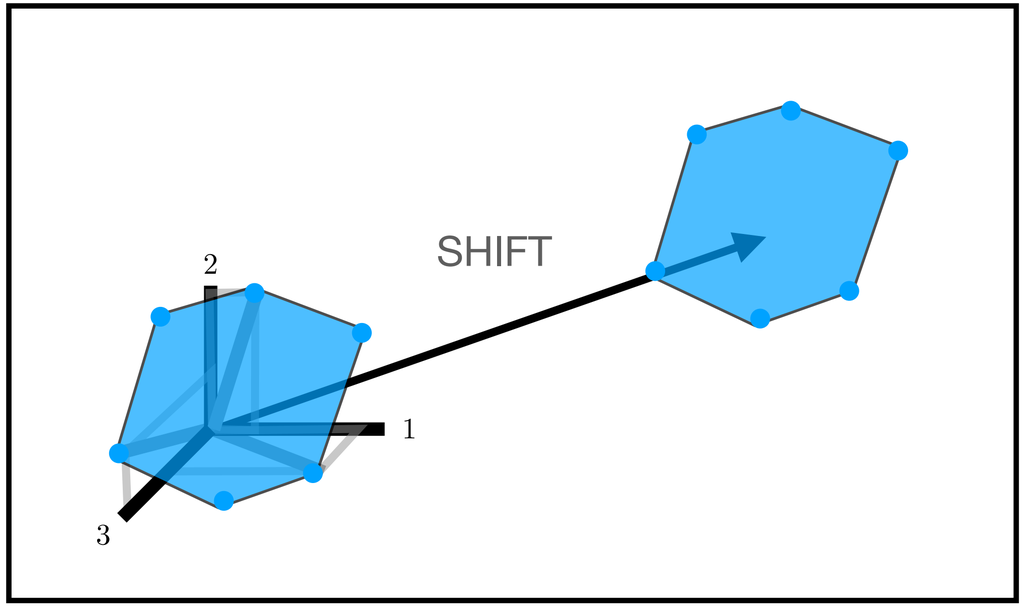
SHAPE @ CRDS @ AXES



Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Code

PTS = convexhull(SHAPE @ CRDS @ AXES)

PTS = PTS + SHIFT @ AXES **OR** + SHIFT @ AXES2

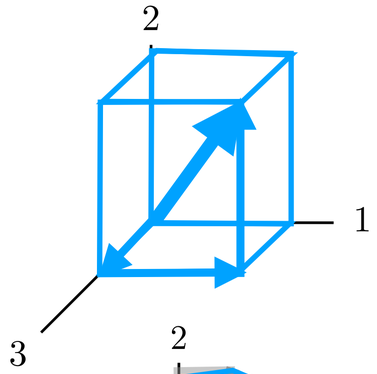
plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Shapes

x = [0.8, 1.0, 0.5]

AXES = [[1.0, 0.0],
[0.0, 1.0],
[-.7, -.7]]

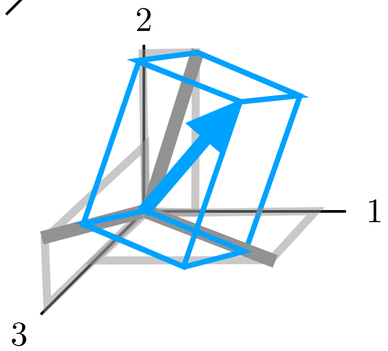
x @ AXES



x = [0.8, 1.0, 0.5]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

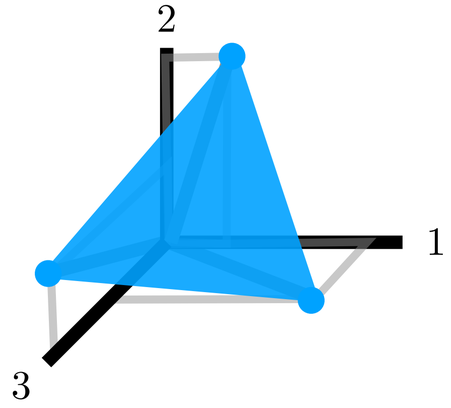
x @ CRDS @ AXES



Simplex

SHAPE = [[1, 0, 0],
[0, 1, 0],
[0, 0, 1]]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

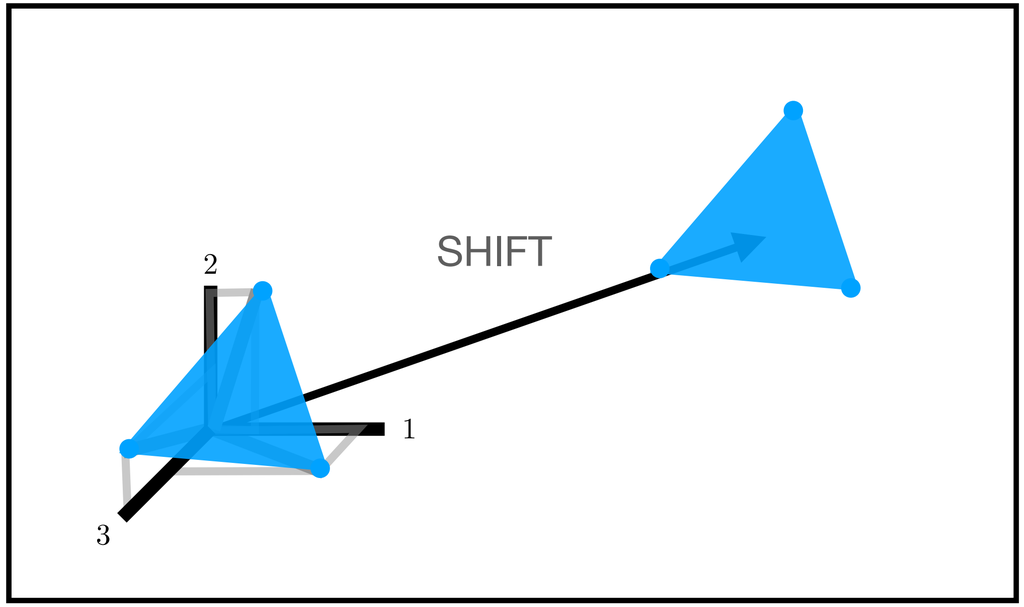


SHAPE @ CRDS @ AXES

Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Code

PTS = convexhull(SHAPE @ CRDS @ AXES)

PTS = PTS + SHIFT @ AXES **OR** + SHIFT @ AXES2

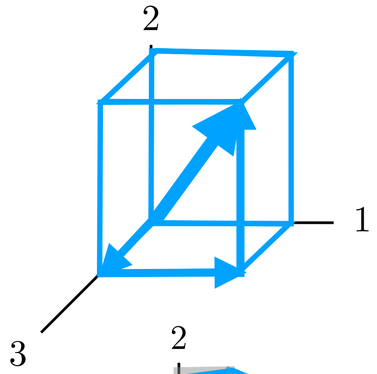
plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Shapes

x = [0.8, 1.0, 0.5]

AXES = [[1.0, 0.0],
[0.0, 1.0],
[-.7, -.7]]

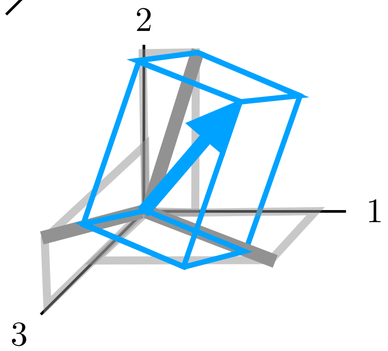
x @ AXES



x = [0.8, 1.0, 0.5]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

x @ CRDS @ AXES



Unit circle

SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

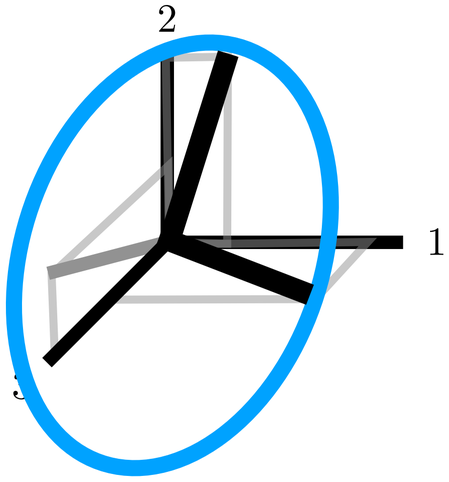
θ

[cos(6.2), sin(6.2)]]

PLANE = [[1, 0, 0],
[0, 1, 0],

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

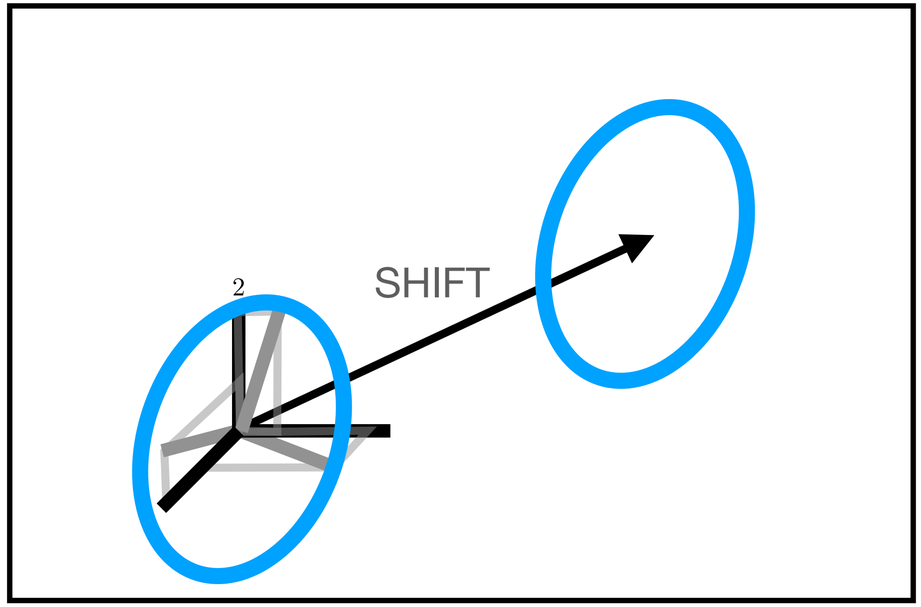
SHAPE @ PLANE @ CRDS @ AXES



Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Code

PTS = SHAPE @ PLANE @ CRDS @ AXES

PTS = PTS + SHIFT @ AXES **OR** + SHIFT @ AXES2

plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Sh

for Axis-Length Representation: **take SVD of** PLANE @ CRDS @ AXES **2 x 2 matrix** **(see above for details)**

$$x \quad \begin{bmatrix} - & a_3^T & - \end{bmatrix} \quad A$$

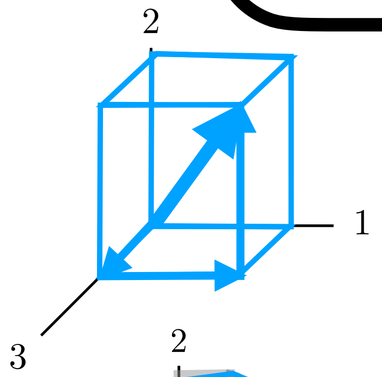
$$+ x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix}$$

$$+ x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

x = [0.8, 1.0, 0.5]

AXES = [[1.0, 0.0],
[0.0, 1.0],
[-.7, -.7]]

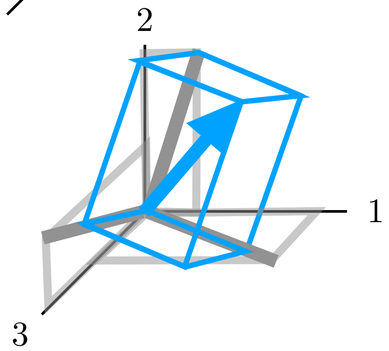
x @ AXES



x = [0.8, 1.0, 0.5]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

x @ CRDS @ AXES



Unit circle

SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

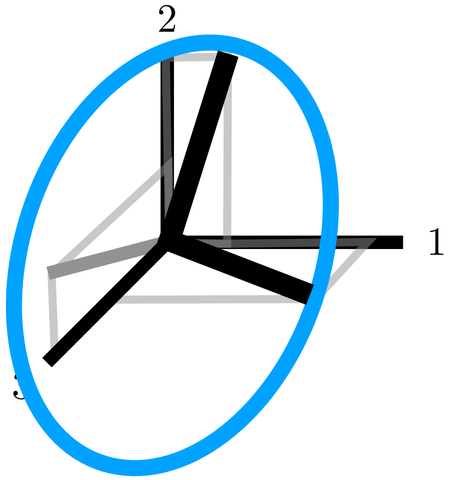
θ

[cos(6.2), sin(6.2)]]

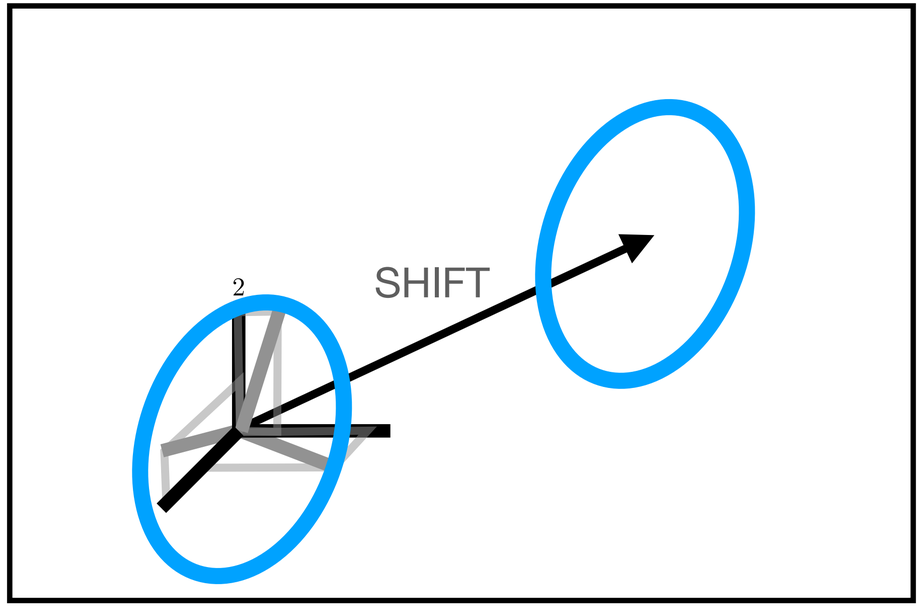
PLANE = [[1, 0, 0],
[0, 1, 0],

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

SHAPE @ PLANE @ CRDS @ AXES



Drawing



Code

PTS = SHAPE @ PLANE @ CRDS @ AXES

PTS = PTS + SHIFT @ AXES **OR** + SHIFT @ AXES2

plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Sh

for Axis-Length Representation: **take SVD of** PLANE @ CRDS @ AXES **2 x 2 matrix** **(see above for details)**

$$x \quad \begin{bmatrix} - & a_3^T & - \end{bmatrix} \quad A$$

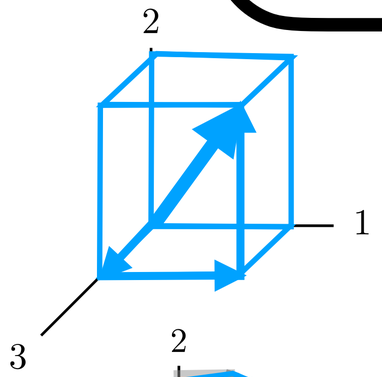
$$+ x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix}$$

$$+ x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

x = [0.8, 1.0, 0.5]

AXES = [[1.0, 0.0],
[0.0, 1.0],
[-.7, -.7]]

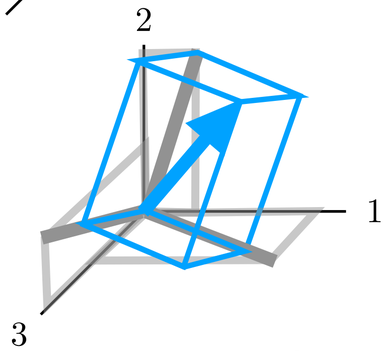
x @ AXES



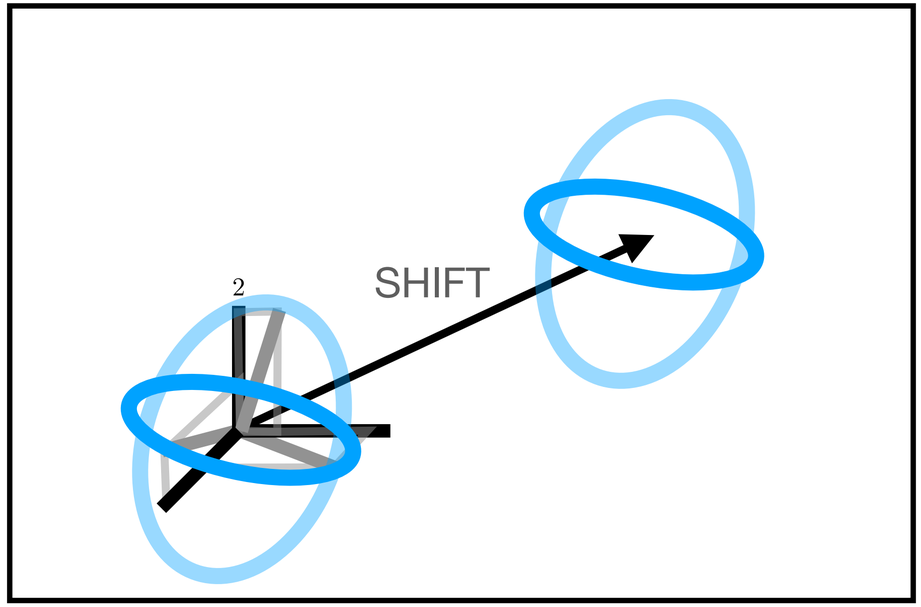
x = [0.8, 1.0, 0.5]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

x @ CRDS @ AXES



Drawing



Unit circle

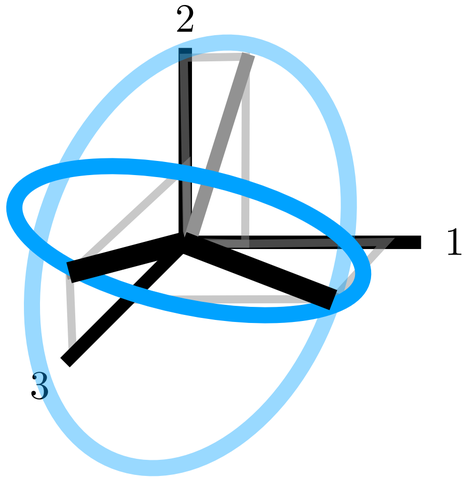
SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

θ

[cos(6.2), sin(6.2)]]

PLANE = [[1, 0, 0],
[0, 0, 1],

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]



SHAPE @ PLANE @ CRDS @ AXES

Code

PTS = SHAPE @ PLANE @ CRDS @ AXES

PTS = PTS + SHIFT @ AXES **OR** + SHIFT @ AXES2

plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Sh

for Axis-Length Representation: **take SVD of** PLANE @ CRDS @ AXES **2 x 2 matrix** **(see above for details)**

$$x \quad \begin{bmatrix} - & a_3^T & - \end{bmatrix} \quad A$$

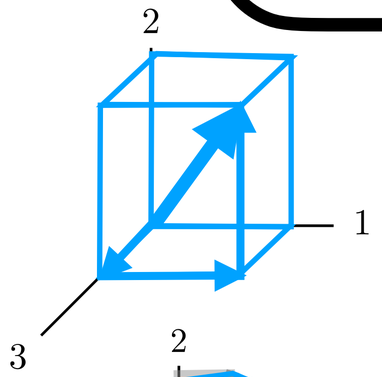
$$+ x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix}$$

$$+ x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

x = [0.8, 1.0, 0.5]

AXES = [[1.0, 0.0],
[0.0, 1.0],
[-.7, -.7]]

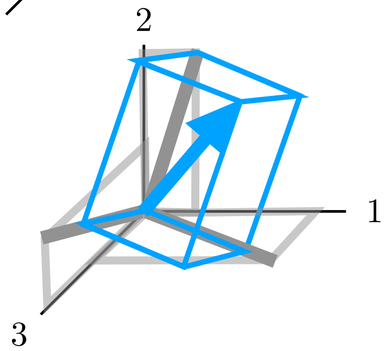
x @ AXES



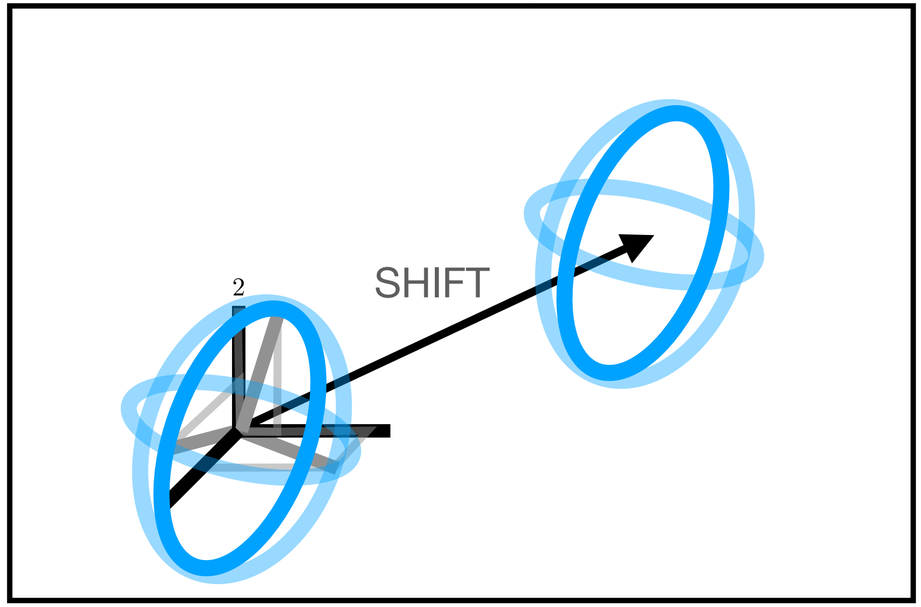
x = [0.8, 1.0, 0.5]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

x @ CRDS @ AXES



Drawing



Unit circle

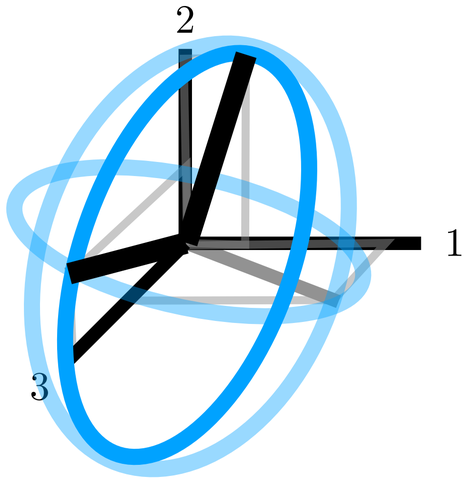
SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

θ

[cos(6.2), sin(6.2)]]

PLANE = [[0, 1, 0],
[0, 0, 1],

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]



SHAPE @ PLANE @ CRDS @ AXES

Code

PTS = SHAPE @ PLANE @ CRDS @ AXES

PTS = PTS + SHIFT @ AXES **OR** + SHIFT @ AXES2

plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Shapes

3 x 2 matrix | **3 x 3 rotation** | **2 x 2 rotation**

$$\text{CRDS @ AXES} = \begin{bmatrix} & U & \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} & V^T & \end{bmatrix}$$

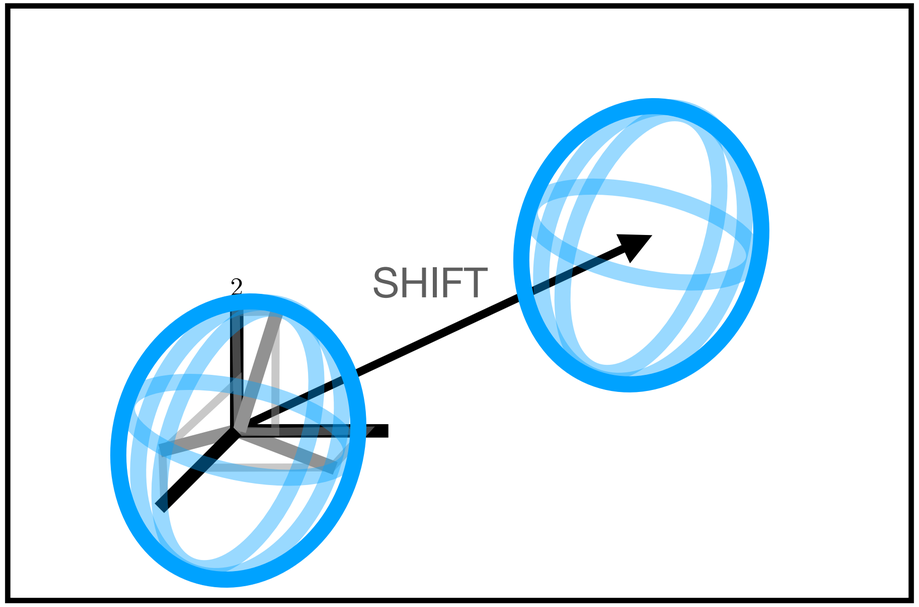
Singular Value Decomposition

$$= \begin{bmatrix} | & | & | \\ U_1 & U_2 & U_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} - & V_1^T & - \\ - & V_2^T & - \end{bmatrix}$$

Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Unit circle

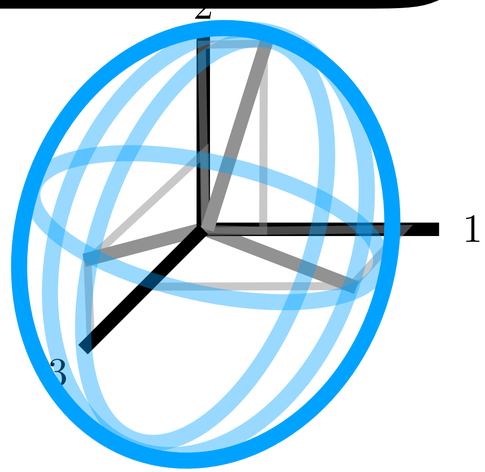
SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

θ

[cos(6.2), sin(6.2)]]

PLANE = [[0, 1, 0],
[0, 0, 1],

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]



SHAPE @ PLANE @ CRDS @ AXES

Code

PTS = SHAPE @ PLANE @ CRDS @ AXES

PTS = PTS + SHIFT @ AXES **OR** + SHIFT @ AXES2

plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Shapes

3 x 2 matrix

3 x 3 rotation

2 x 2 rotation

$$\text{CRDS @ AXES} = \begin{bmatrix} \\ U \\ \\ \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \\ V^T \\ \\ \end{bmatrix}$$

Singular Value Decomposition

$$= \begin{bmatrix} | & | & | \\ U_1 & U_2 & U_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} - & V_1^T & - \\ - & V_2^T & - \end{bmatrix}$$

← 2D Axis1

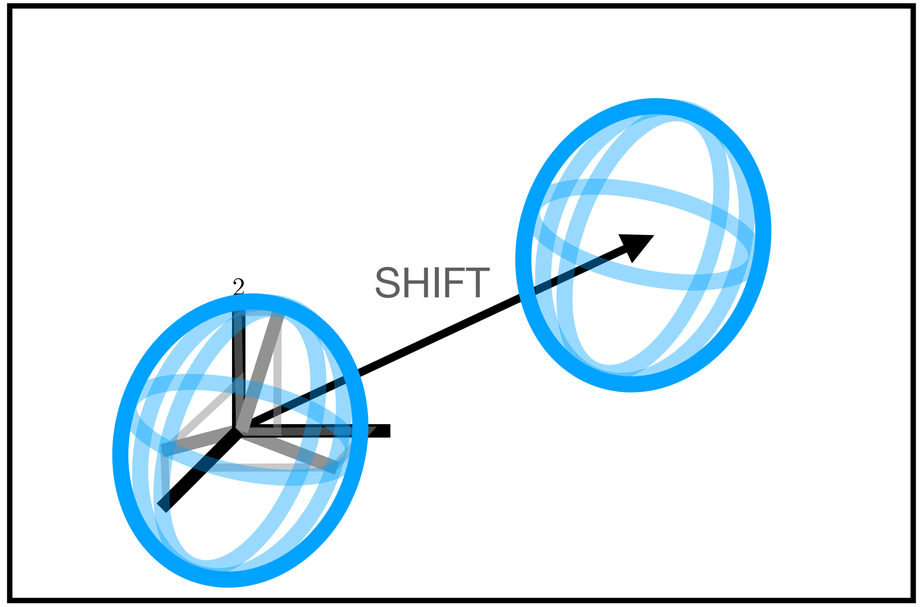
3D Axis1

Length Axis1

Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Unit circle

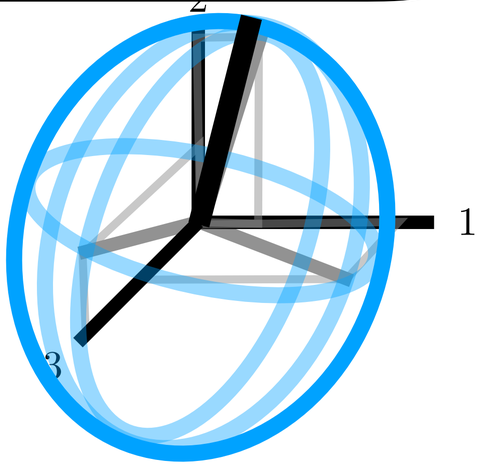
SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

θ

[cos(6.2), sin(6.2)]]

PLANE = [[0, 1, 0],
[0, 0, 1],

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]



SHAPE @ PLANE @ CRDS @ AXES

Code

```
PTS = SHAPE @ PLANE @ CRDS @ AXES
PTS = PTS + SHIFT @ AXES   OR   + SHIFT @ AXES2

plot( PTS[:,0] , PTS[:,1] )
```

Axes & Coordinates - 3D Shapes

3 x 2 matrix

3 x 3 rotation

2 x 2 rotation

$$\text{CRDS @ AXES} = \begin{bmatrix} & U & \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} & V^T & \end{bmatrix}$$

Singular Value Decomposition

$$= \begin{bmatrix} | & | & | \\ U_1 & U_2 & U_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} - & V_1^T & - \\ - & V_2^T & - \end{bmatrix}$$

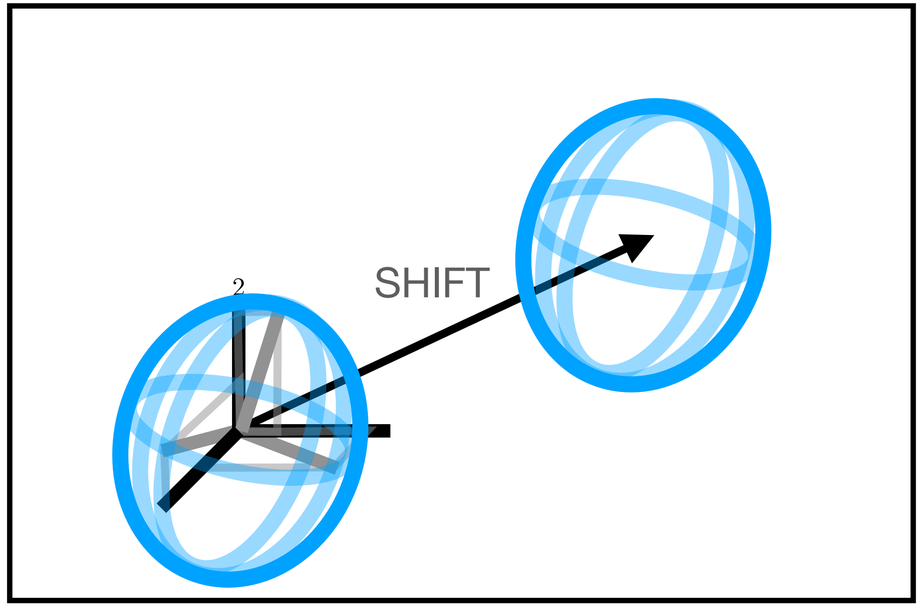
← 2D Axis2

3D Axis2 — Length Axis2

Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Unit circle

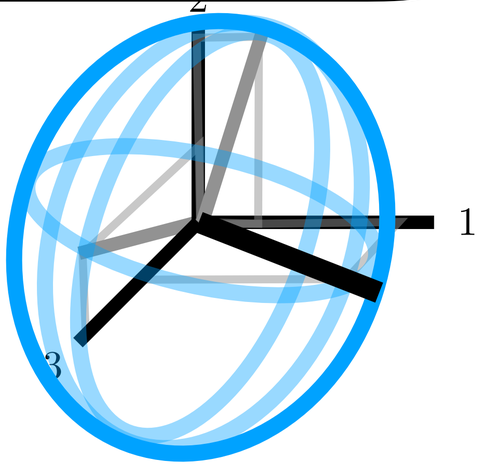
SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

θ

[cos(6.2), sin(6.2)]]

PLANE = [[0, 1, 0],
[0, 0, 1],

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]



SHAPE @ PLANE @ CRDS @ AXES

Code

PTS = SHAPE @ PLANE @ CRDS @ AXES

PTS = PTS + SHIFT @ AXES **OR** + SHIFT @ AXES2

plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Shapes

3 x 2 matrix | **3 x 3 rotation** | **2 x 2 rotation**

$$\text{CRDS @ AXES} = \begin{bmatrix} & U & \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} & V^T & \end{bmatrix}$$

Singular Value Decomposition

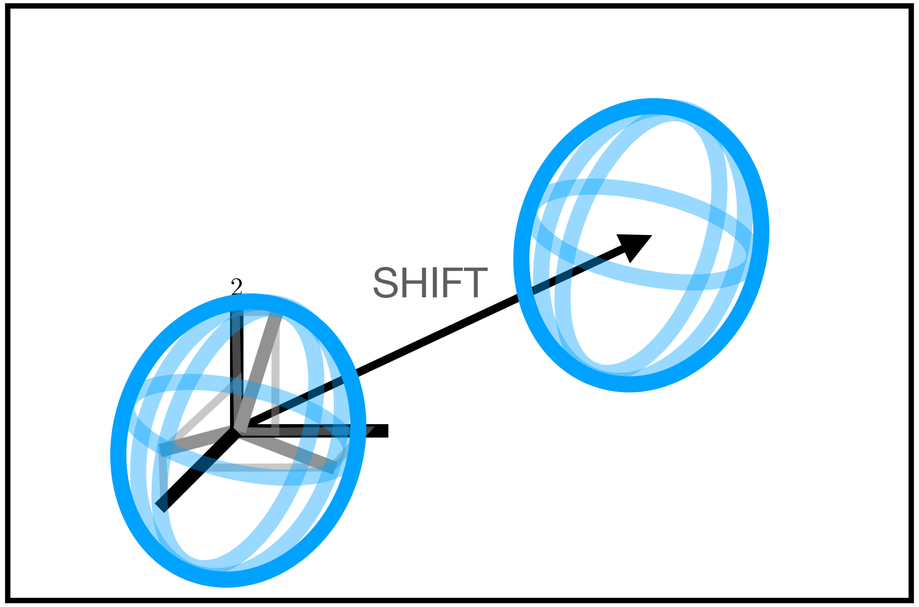
$$= \begin{bmatrix} | & | & | \\ U_1 & U_2 & U_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} - & V_1^T & - \\ - & V_2^T & - \end{bmatrix}$$

3D Depth Direction

Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Unit circle

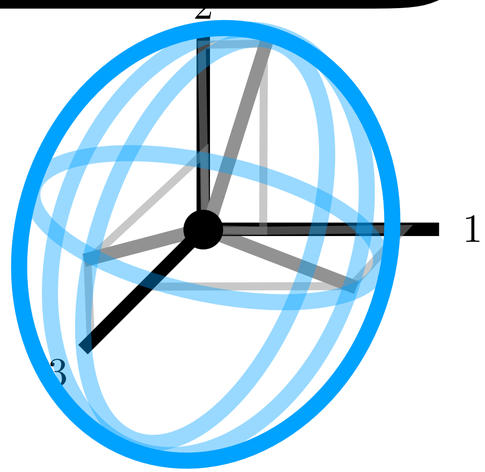
SHAPE = [[cos(0.0), sin(0.0)],
[cos(0.1), sin(0.1)],
[cos(0.2), sin(0.2)],
[cos(0.3), sin(0.3)],

θ

[cos(6.2), sin(6.2)]]

PLANE = [[0, 1, 0],
[0, 0, 1],

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]



SHAPE @ PLANE @ CRDS @ AXES

Code

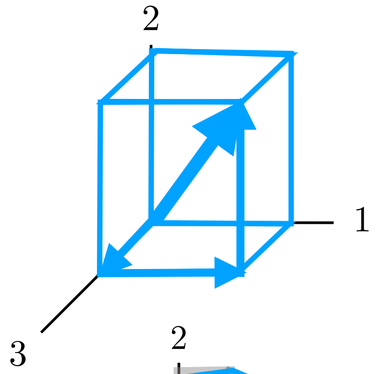
```
PTS = SHAPE @ PLANE @ CRDS @ AXES
PTS = PTS + SHIFT @ AXES   OR   + SHIFT @ AXES2

plot( PTS[:,0] , PTS[:,1] )
```

Axes & Coordinates - 3D Shapes

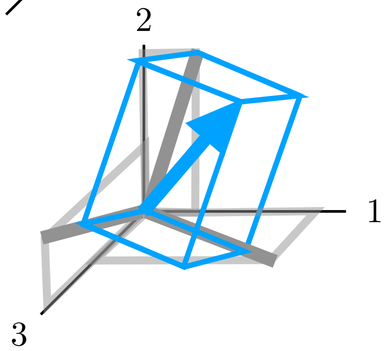
x = [0.8, 1.0, 0.5] AXES = [[1.0, 0.0],
[0.0, 1.0],
[-.7, -.7]]

x @ AXES



x = [0.8, 1.0, 0.5] CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

x @ CRDS @ AXES

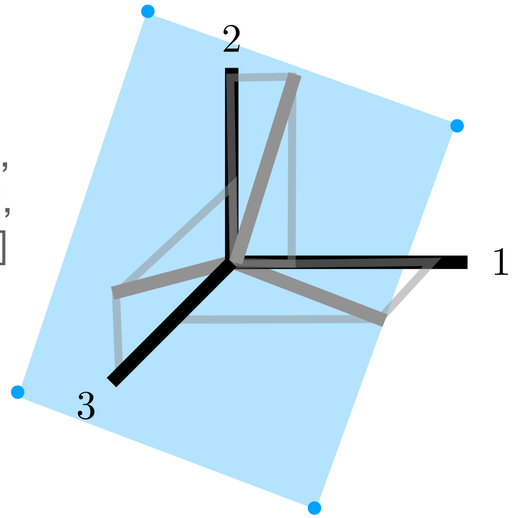


Plane - 2D

SHAPE = [[-1, -1, 0],
[1, -1, 0],
[1, 1, 0],
[-1, 1, 0]]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

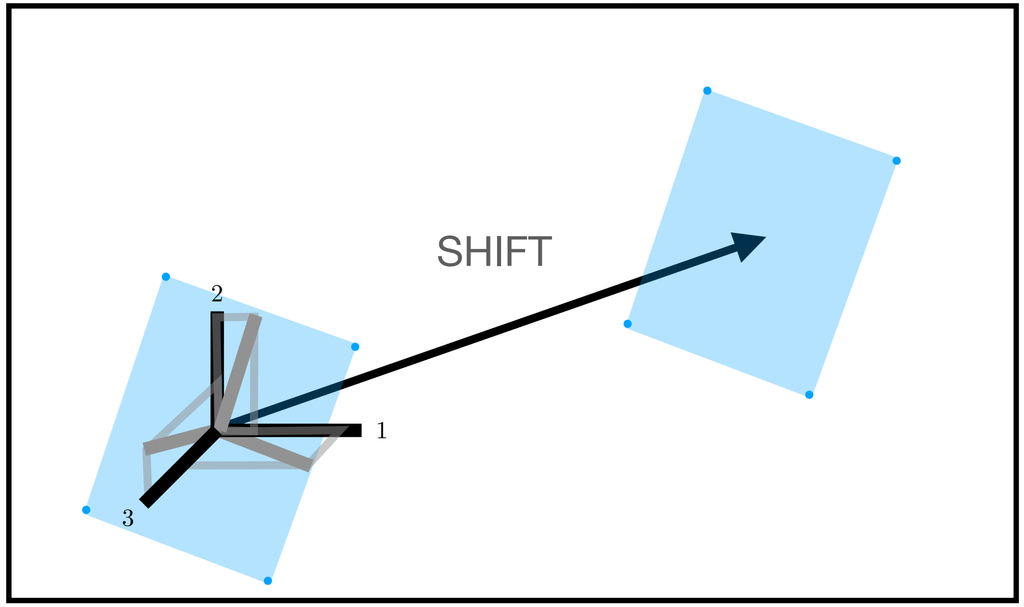
SHAPE @ CRDS @ AXES



Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Code

PTS = SHAPE @ CRDS @ AXES + SHIFT @ AXES

OR + SHIFT @ AXES2

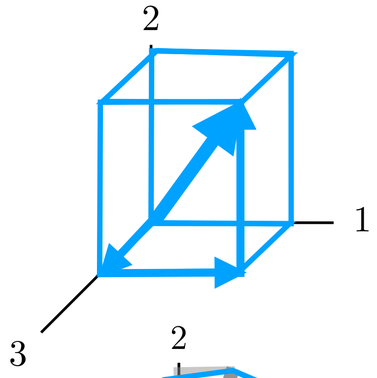
plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Shapes

x = [0.8, 1.0, 0.5]

AXES = [[1.0, 0.0],
[0.0, 1.0],
[-.7, -.7]]

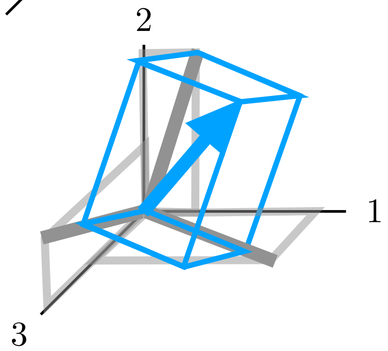
x @ AXES



x = [0.8, 1.0, 0.5]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

x @ CRDS @ AXES

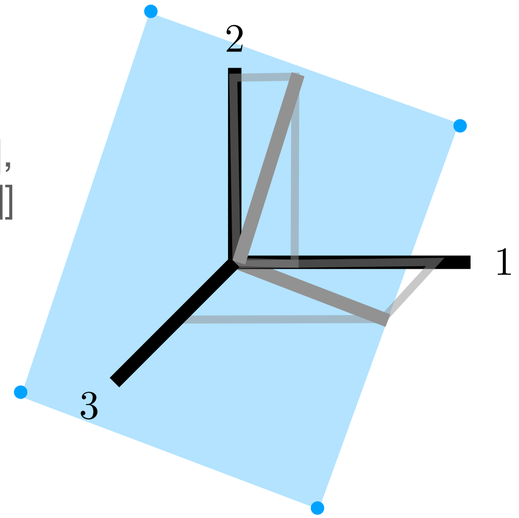


Plane - 2D

SHAPE = [[-1, -1],
[1, -1],
[1, 1],
[-1, 1]]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0]]

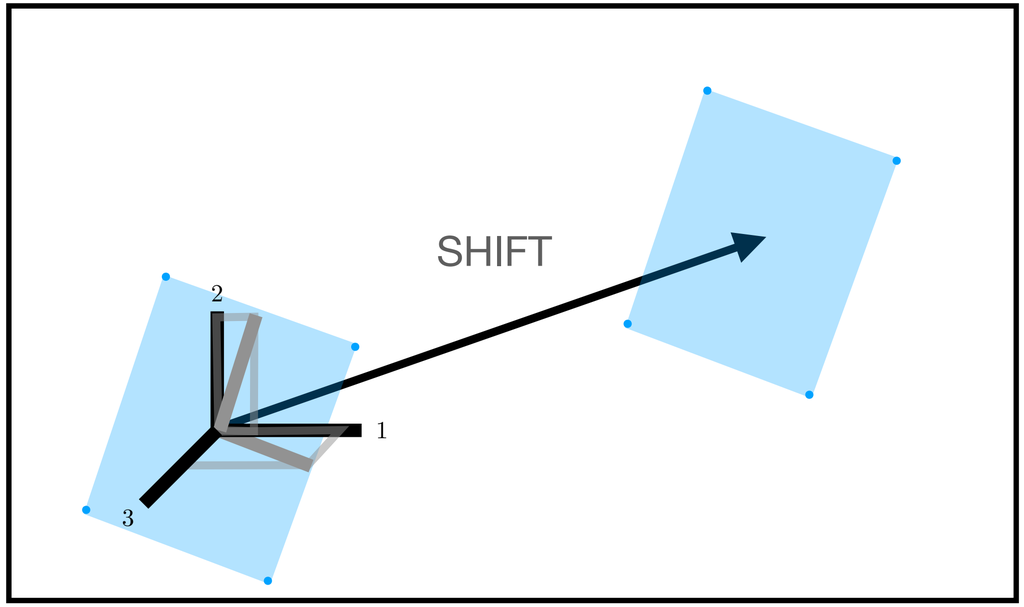
SHAPE @ CRDS @ AXES



Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Code

PTS = SHAPE @ CRDS @ AXES + SHIFT @ AXES

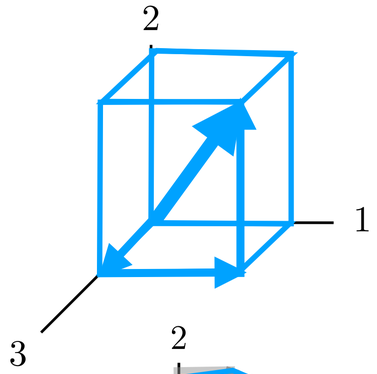
OR + SHIFT @ AXES2

plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Shapes

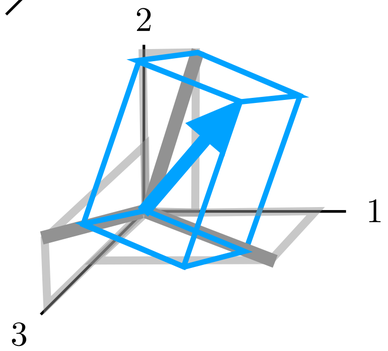
x = [0.8, 1.0, 0.5] AXES = [[1.0, 0.0],
[0.0, 1.0],
[-.7, -.7]]

x @ AXES



x = [0.8, 1.0, 0.5] CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0],
[0.0, 0.3, 1.0]]

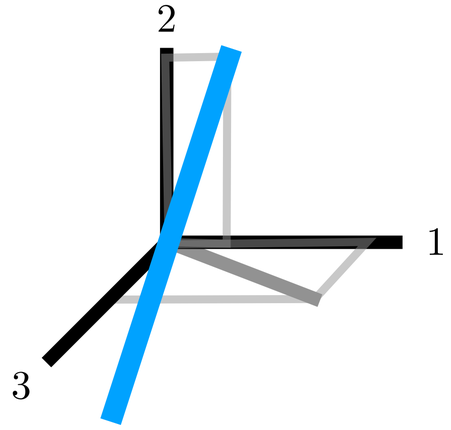
x @ CRDS @ AXES



Plane - 1D

SHAPE = [[-1], CRDS = [[1.0, 0.0, 0.3]]
[1],

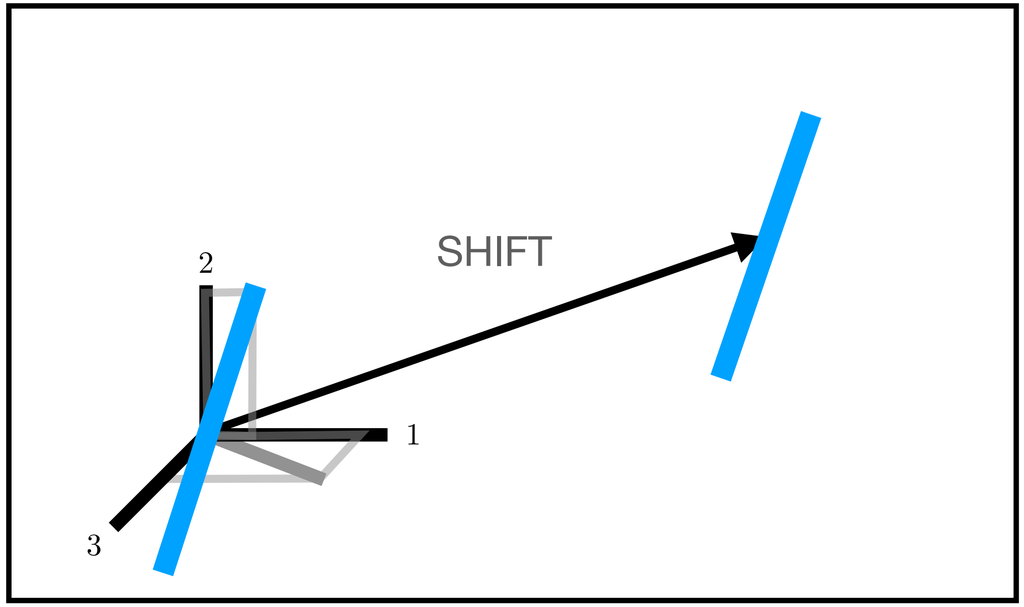
SHAPE @ CRDS @ AXES



Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Code

PTS = SHAPE @ CRDS @ AXES + SHIFT @ AXES

OR + SHIFT @ AXES2

plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Shapes

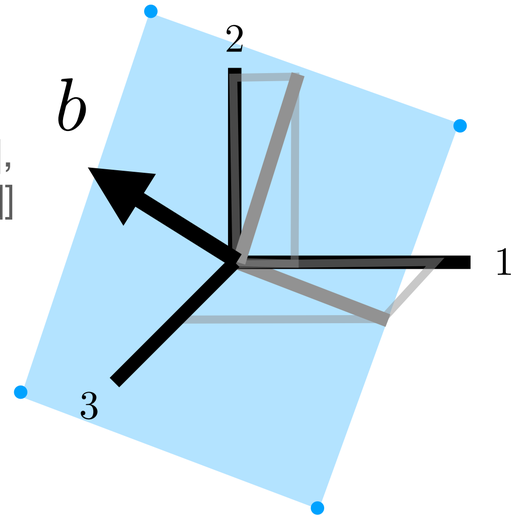
b **perpendicular to plane**

Plane - 2D, normal vector

SHAPE = [[-1, -1],
[1, -1],
[1, 1],
[-1, 1]]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0]]

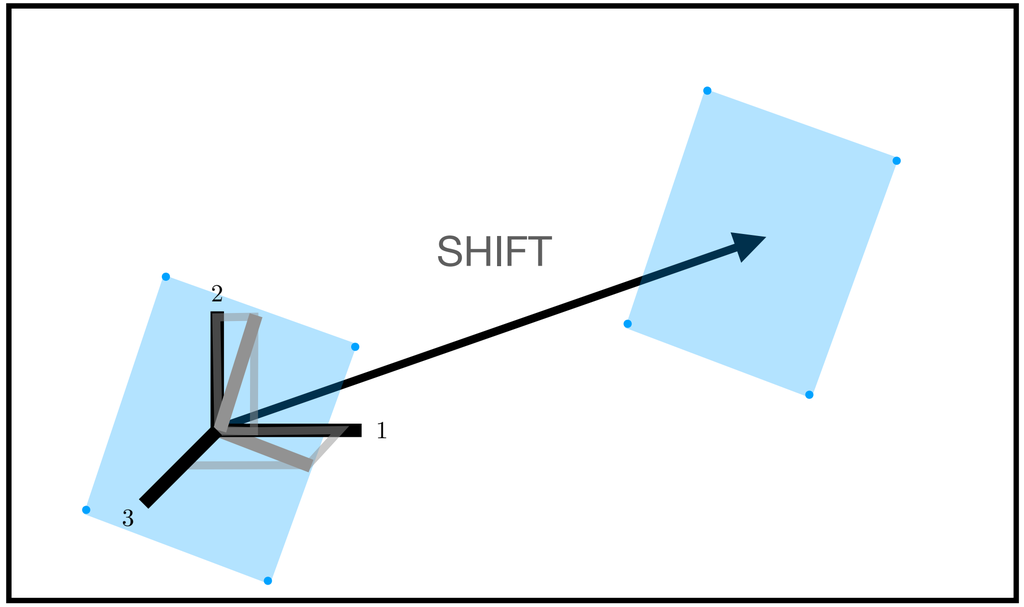
SHAPE @ CRDS @ AXES



Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Code

PTS = SHAPE @ CRDS @ AXES + SHIFT @ AXES

OR + SHIFT @ AXES2

plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Shapes

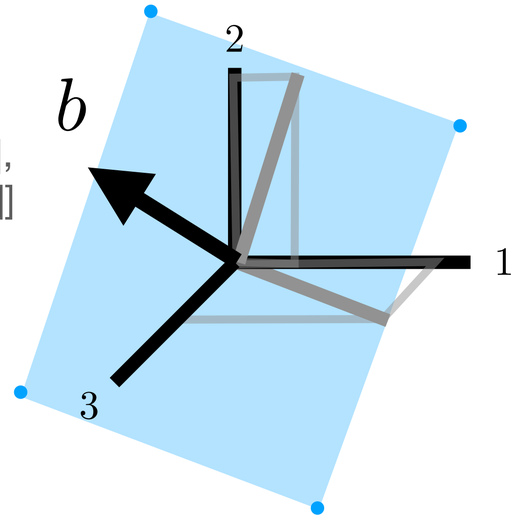
b **perpendicular to plane** **SVD**

$$b^T = [U_1] \begin{bmatrix} \sigma_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} - & V_1^T & - \\ - & V_2^T & - \\ - & V_3^T & - \end{bmatrix}$$

Plane - 2D, normal vector

SHAPE = [[-1, -1],
[1, -1],
[1, 1],
[-1, 1]]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0]]

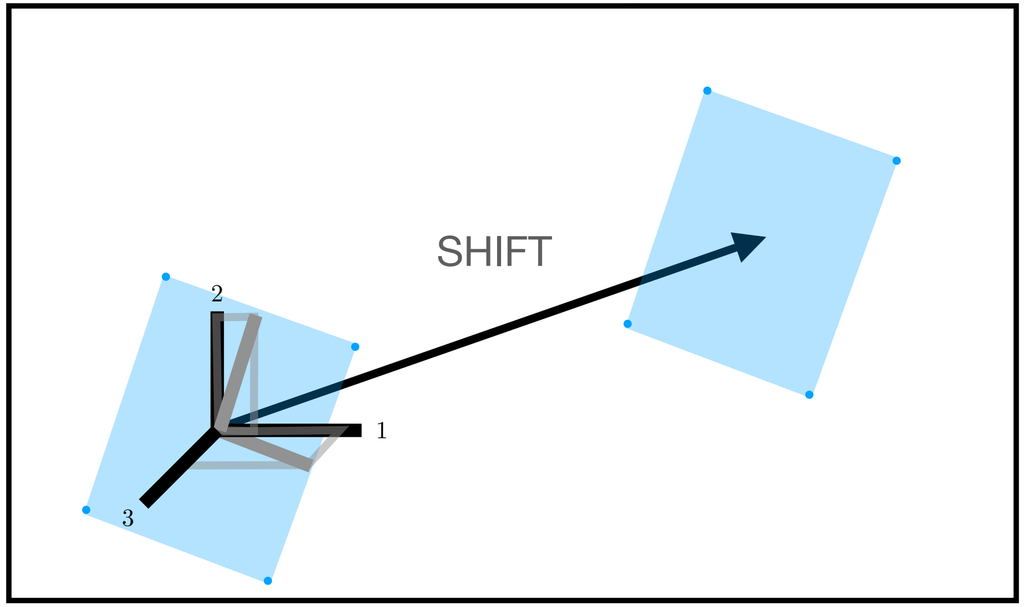


SHAPE @ CRDS @ AXES

Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Code

PTS = SHAPE @ CRDS @ AXES + SHIFT @ AXES

OR + SHIFT @ AXES2

plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Shapes

b **perpendicular to plane** **SVD**

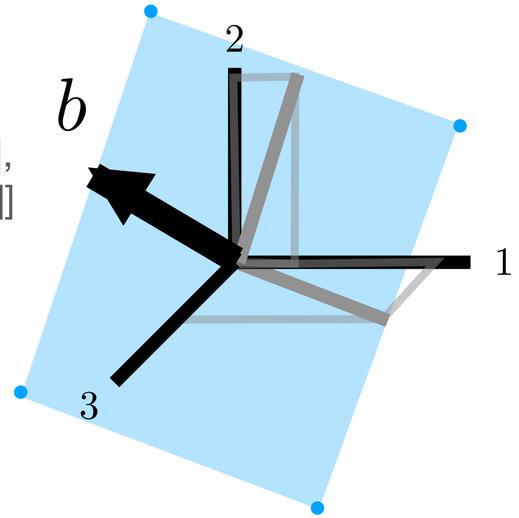
$$b^T = [U_1] \; [\sigma_1 \quad 0 \quad 0] \begin{bmatrix} - & V_1^T & - \\ - & V_2^T & - \\ - & V_3^T & - \end{bmatrix}$$

← **3D orthogonal direction**

Plane - 2D, normal vector

SHAPE = [[-1, -1],
[1, -1],
[1, 1],
[-1, 1]]

CRDS = [[1.0, 0.0, 0.3],
[0.3, 1.0, 0.0]]

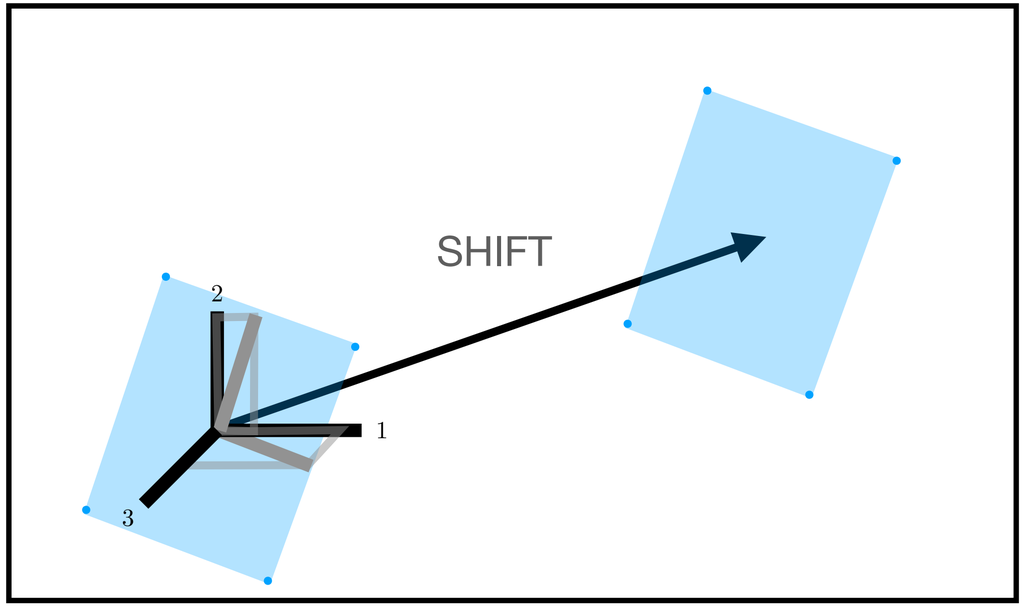


SHAPE @ CRDS @ AXES

Matrix Multiplication

$$\underbrace{[x_1 \quad x_2 \quad x_3]}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Code

PTS = SHAPE @ CRDS @ AXES + SHIFT @ AXES

OR + SHIFT @ AXES2

plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Shapes

b **perpendicular to plane** **SVD**

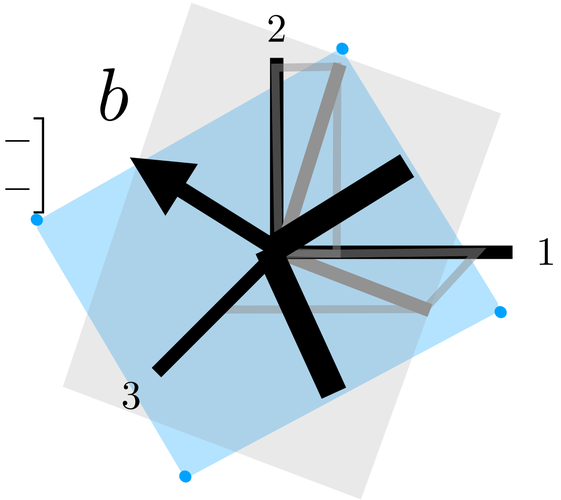
$$b^T = [U_1] \; [\sigma_1 \quad 0 \quad 0] \begin{bmatrix} - & V_1^T & - \\ - & V_2^T & - \\ - & V_3^T & - \end{bmatrix}$$

← **3D in plane directions**

Plane - 2D, normal vector

SHAPE = [[-1, -1],
[1, -1],
[1, 1],
[-1, 1]]

CRDS = $\begin{bmatrix} - & V_2^T & - \\ - & V_3^T & - \end{bmatrix}$

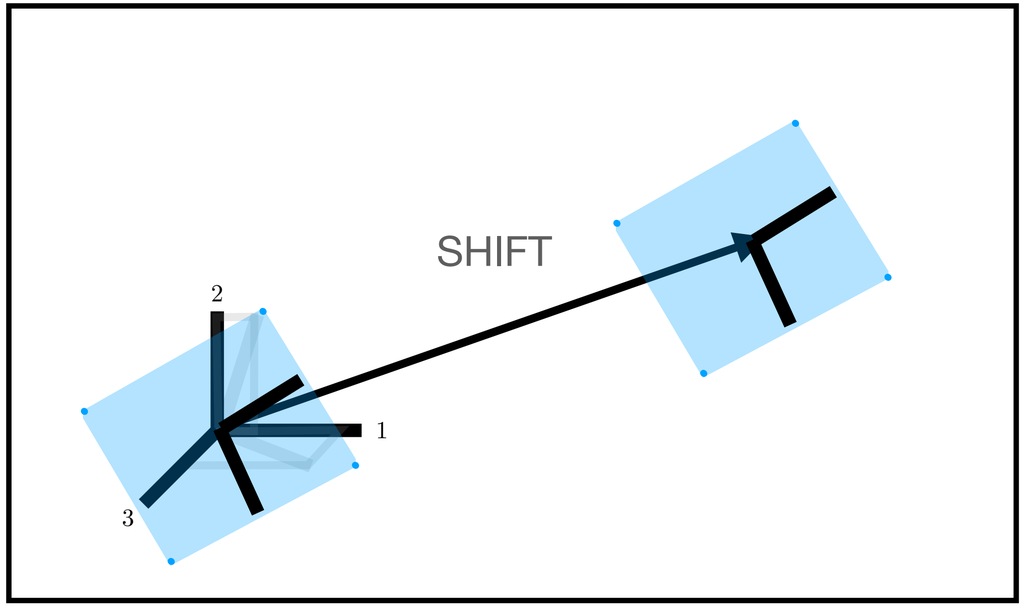


SHAPE @ CRDS @ AXES

Matrix Multiplication

$$\underbrace{[x_1 \quad x_2 \quad x_3]}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Code

PTS = SHAPE @ CRDS @ AXES + SHIFT @ AXES

OR + SHIFT @ AXES2

plot(PTS[:,0] , PTS[:,1])

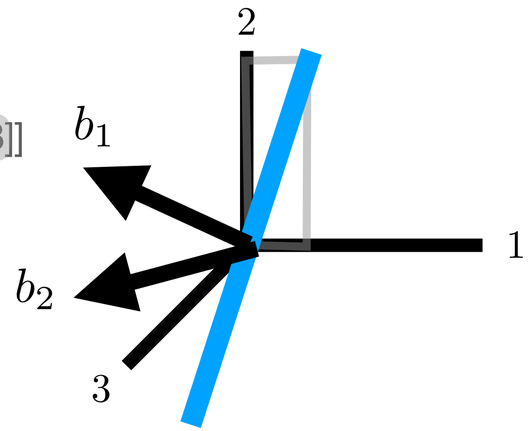
Axes & Coordinates - 3D Shapes

b_1, b_2 **perpendicular to line**

Plane - 1D, normal plane

SHAPE = [[-1],
[1],

CRDS = [[1.0, 0.0, 0.3]]

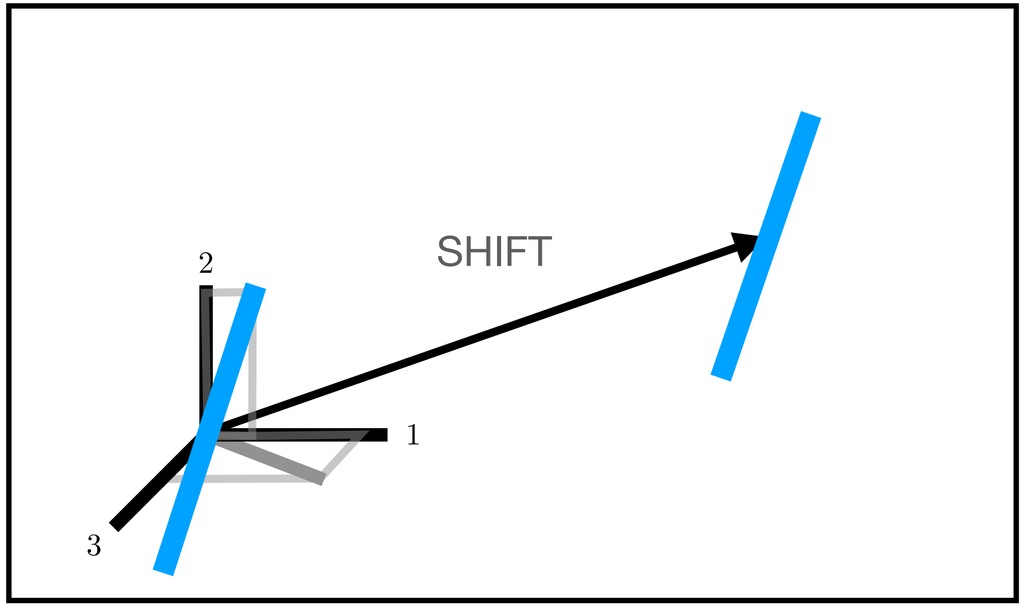


SHAPE @ CRDS @ AXES

Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Code

PTS = SHAPE @ CRDS @ AXES + SHIFT @ AXES

OR + SHIFT @ AXES2

plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Shapes

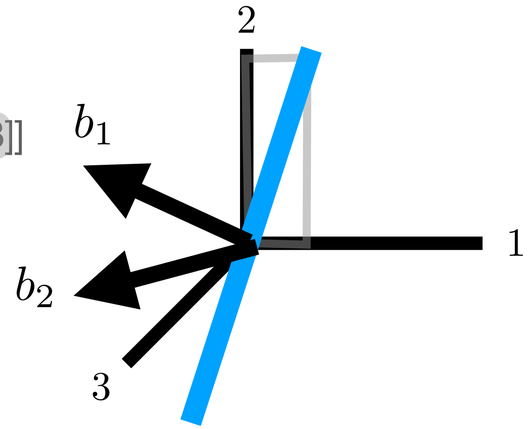
b_1, b_2 **perpendicular to line**

$$\begin{bmatrix} - & b_1^T & - \\ - & b_2^T & - \end{bmatrix} = \begin{bmatrix} | & | \\ U_1 & U_2 \\ | & | \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \end{bmatrix} \begin{bmatrix} - & V_1^T & - \\ - & V_2^T & - \\ - & V_3^T & - \end{bmatrix}$$

Plane - 1D, normal plane

SHAPE = [[-1],
[1],

CRDS = [[1.0, 0.0, 0.3]]

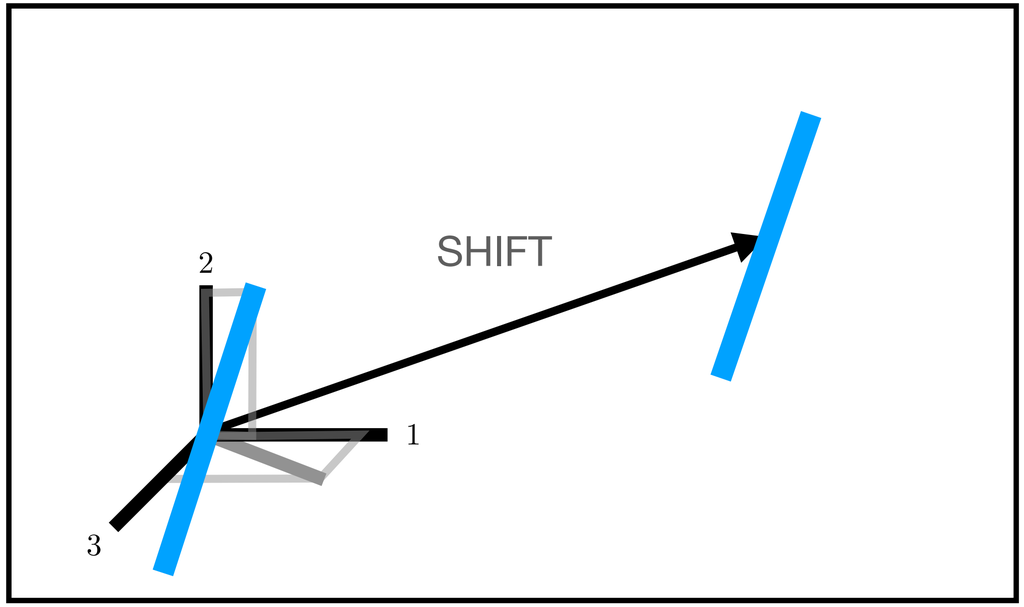


SHAPE @ CRDS @ AXES

Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Code

PTS = SHAPE @ CRDS @ AXES + SHIFT @ AXES

OR + SHIFT @ AXES2

plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Shapes

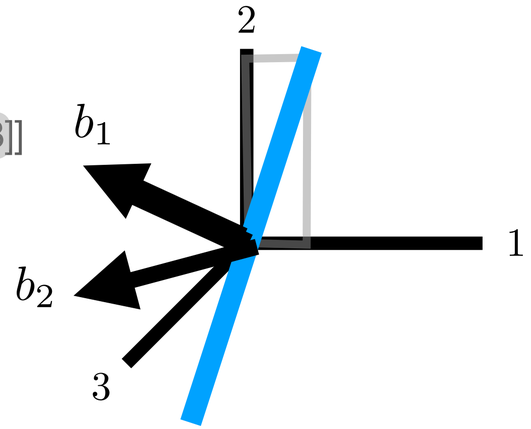
b_1, b_2 **perpendicular to line**

$$\begin{bmatrix} - & b_1^T & - \\ - & b_2^T & - \end{bmatrix} = \begin{bmatrix} | & | \\ U_1 & U_2 \\ | & | \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \end{bmatrix} \begin{bmatrix} - & V_1^T & - \\ - & V_2^T & - \\ - & V_3^T & - \end{bmatrix}$$

Plane - 1D, normal plane

SHAPE = [[-1],
[1],

CRDS = [[1.0, 0.0, 0.3]]

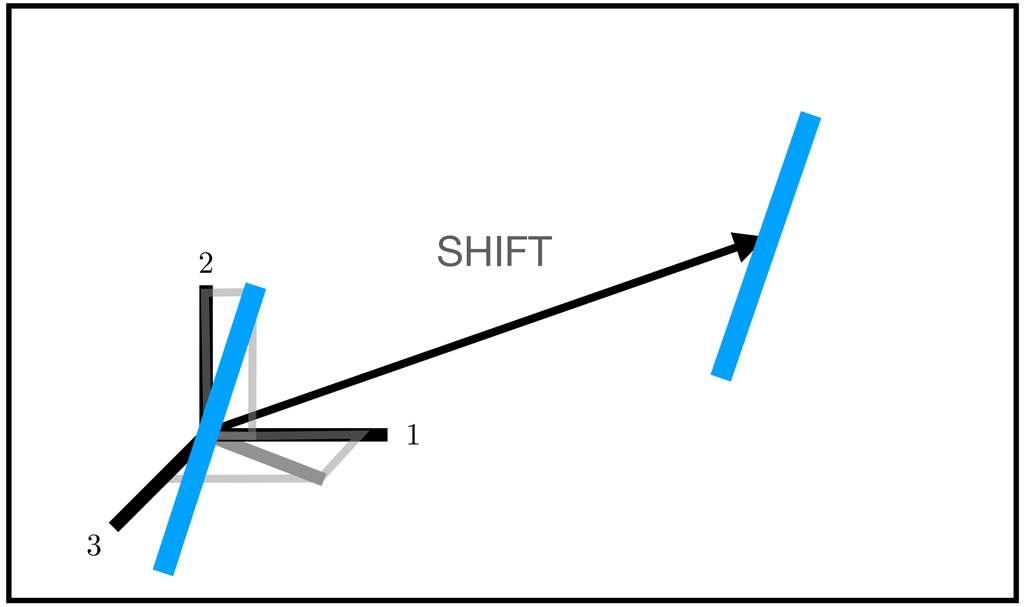


SHAPE @ CRDS @ AXES

Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Code

PTS = SHAPE @ CRDS @ AXES + SHIFT @ AXES

OR + SHIFT @ AXES2

plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Shapes

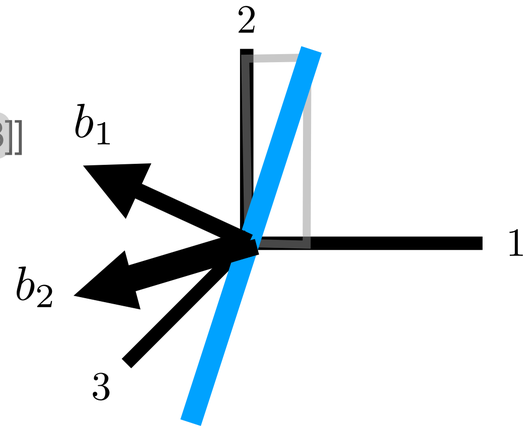
b_1, b_2 **perpendicular to line**

$$\begin{bmatrix} - & b_1^T & - \\ - & b_2^T & - \end{bmatrix} = \begin{bmatrix} | & | \\ U_1 & U_2 \\ | & | \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \end{bmatrix} \begin{bmatrix} - & V_1^T & - \\ - & V_2^T & - \\ - & V_3^T & - \end{bmatrix}$$

Plane - 1D, normal plane

SHAPE = [[-1],
[1],

CRDS = [[1.0, 0.0, 0.3]]

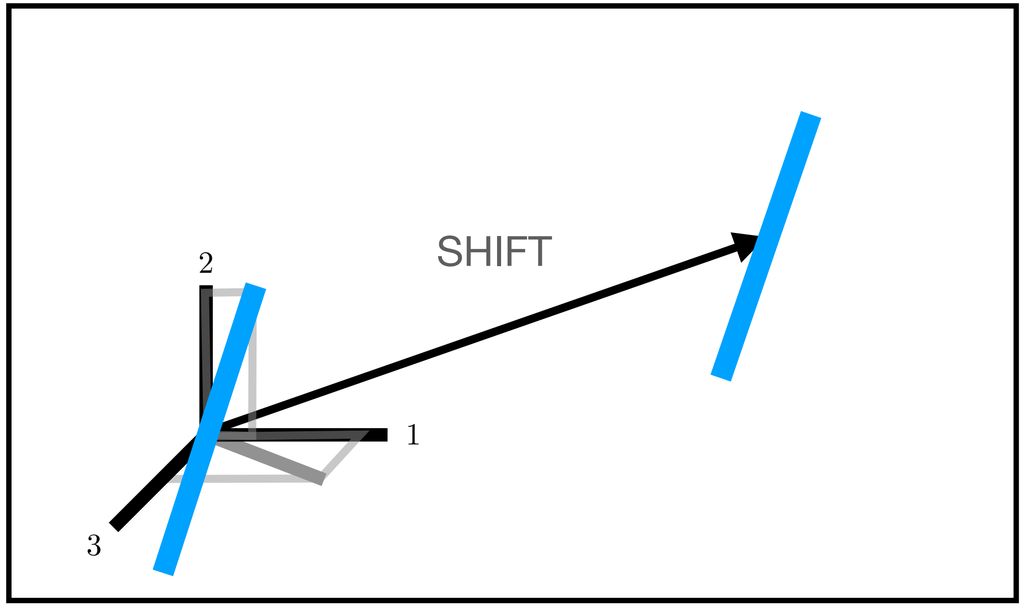


SHAPE @ CRDS @ AXES

Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Code

PTS = SHAPE @ CRDS @ AXES + SHIFT @ AXES

OR + SHIFT @ AXES2

plot(PTS[:,0] , PTS[:,1])

Axes & Coordinates - 3D Shapes

b_1, b_2 **perpendicular to line**

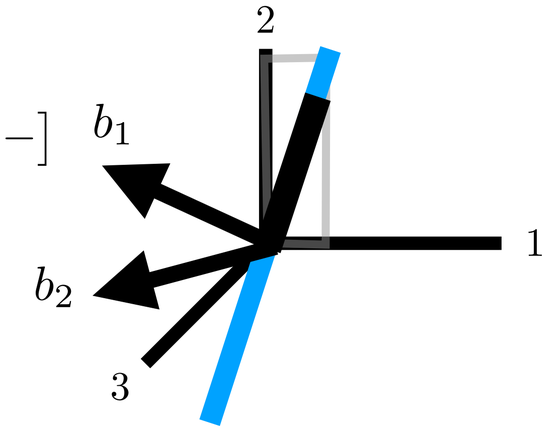
$$\begin{bmatrix} - & b_1^T & - \\ - & b_2^T & - \end{bmatrix} = \begin{bmatrix} | & | \\ U_1 & U_2 \\ | & | \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \end{bmatrix} \begin{bmatrix} - & V_1^T & - \\ - & V_2^T & - \\ - & V_3^T & - \end{bmatrix}$$

CRDS = $\begin{bmatrix} - & V_3^T & - \end{bmatrix}$

Plane - 1D, normal plane

SHAPE = [[-1],
[1],

CRDS = $\begin{bmatrix} - & V_3^T & - \end{bmatrix}$

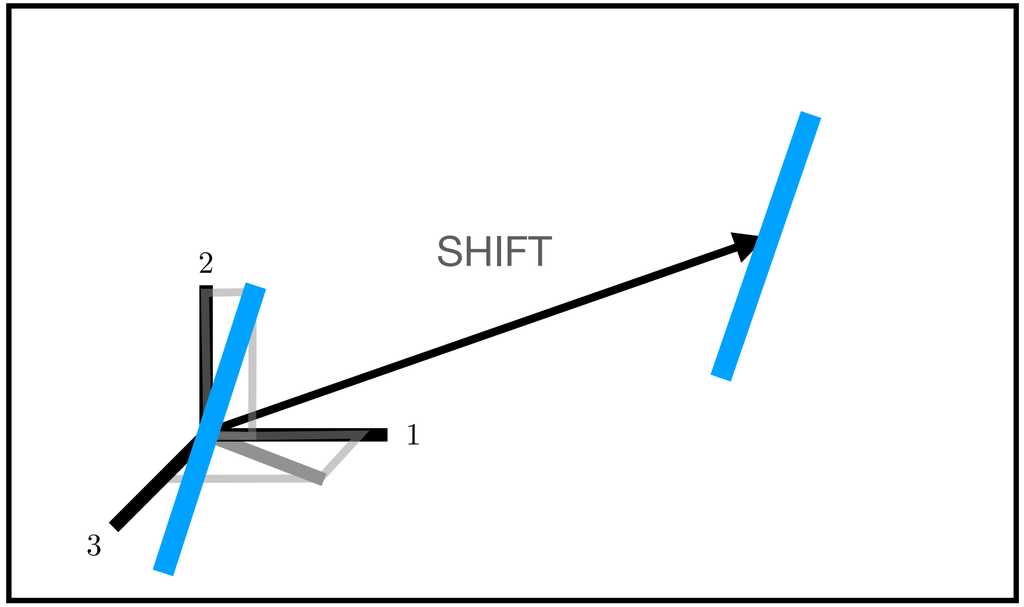


SHAPE @ CRDS @ AXES

Matrix Multiplication

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}}_{x} \underbrace{\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \\ - & a_3^T & - \end{bmatrix}}_{A} = x_1 \begin{bmatrix} - & a_1^T & - \end{bmatrix} + x_2 \begin{bmatrix} - & a_2^T & - \end{bmatrix} + x_3 \begin{bmatrix} - & a_3^T & - \end{bmatrix}$$

Drawing



Code

PTS = SHAPE @ CRDS @ AXES + SHIFT @ AXES

OR + SHIFT @ AXES2

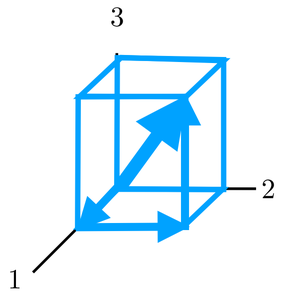
plot(PTS[:,0] , PTS[:,1])

Camera Views - 3D

x = [0.8, 1.0, 0.5]

AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

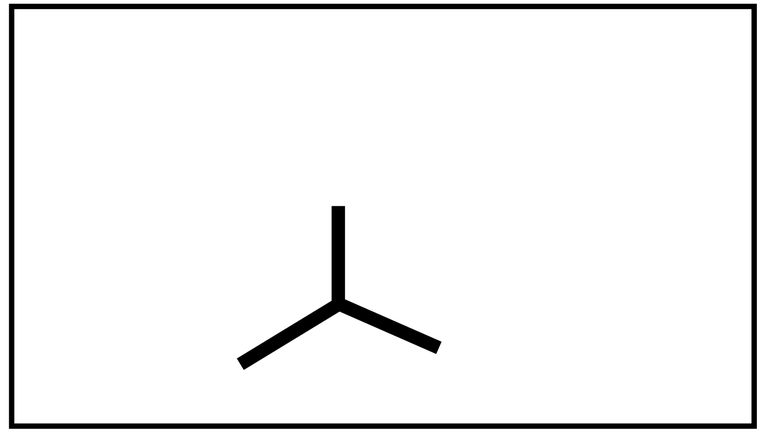
x @ AXES

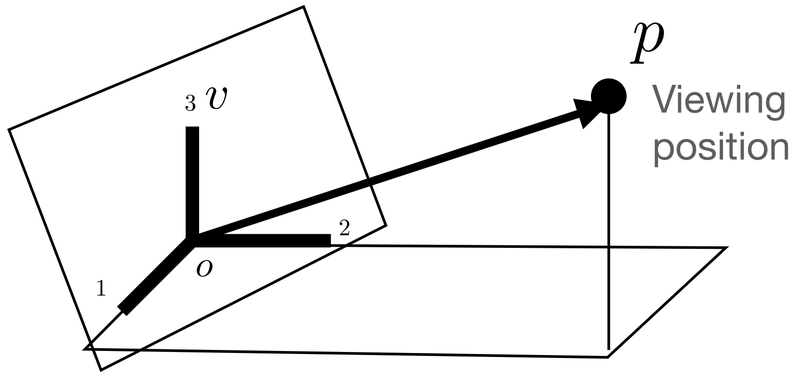


Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

Viewing position

Drawing - 2D Projection



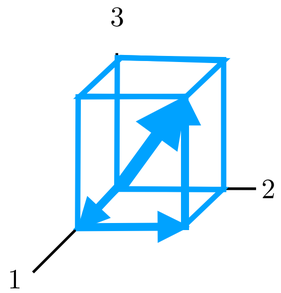


Camera Views - 3D

x = [0.8, 1.0, 0.5]

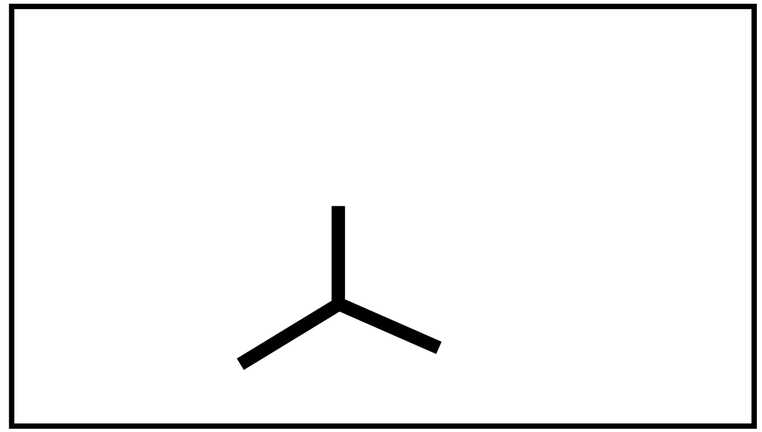
AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

x @ AXES

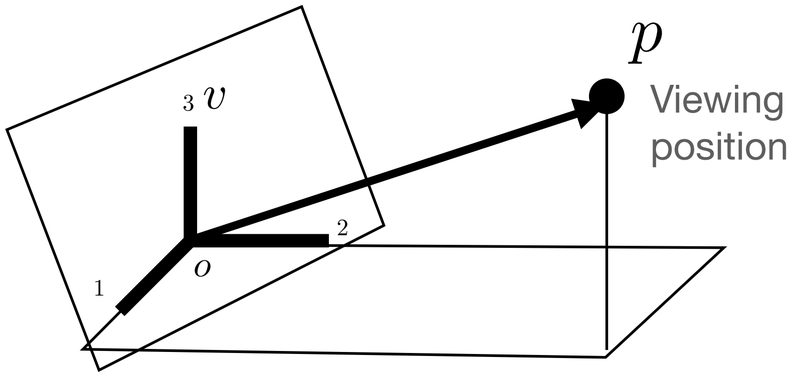


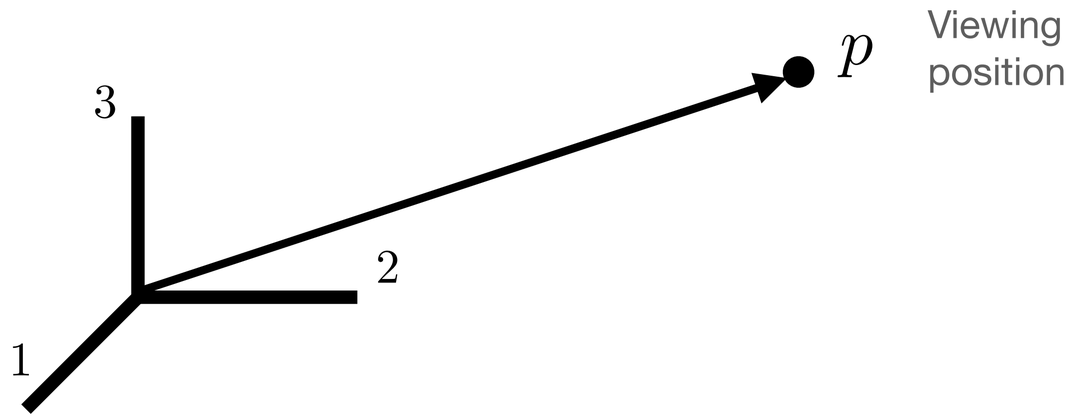
Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

Drawing - 2D Projection



Viewing position



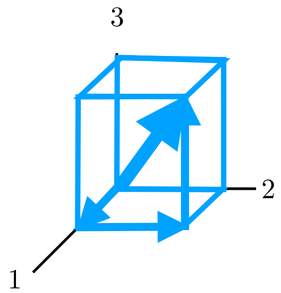


Camera Views - 3D

x = [0.8, 1.0, 0.5]

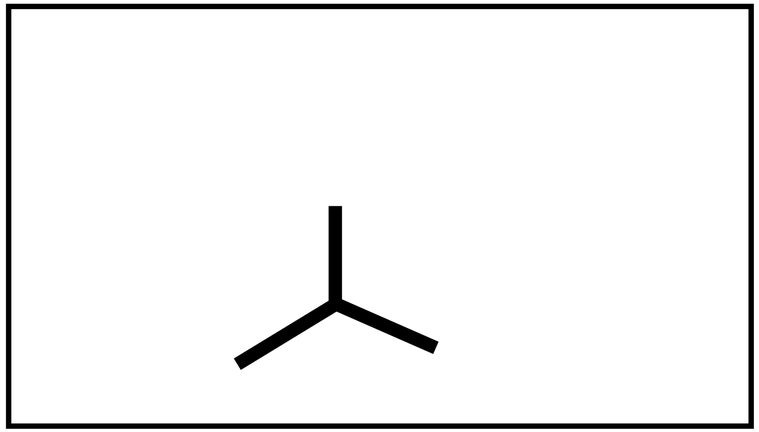
AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

x @ AXES



Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

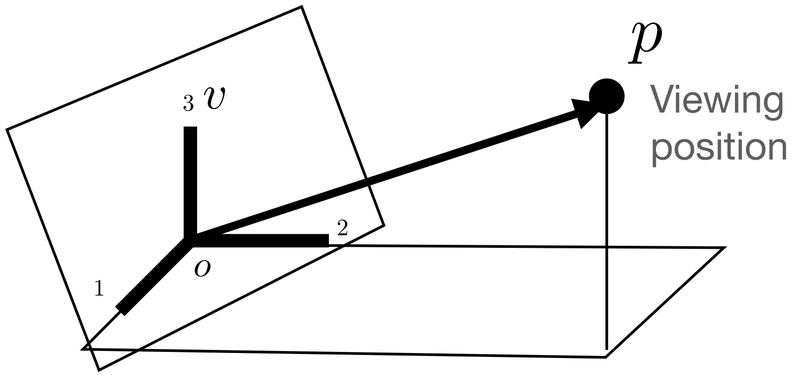
Drawing - 2D Projection

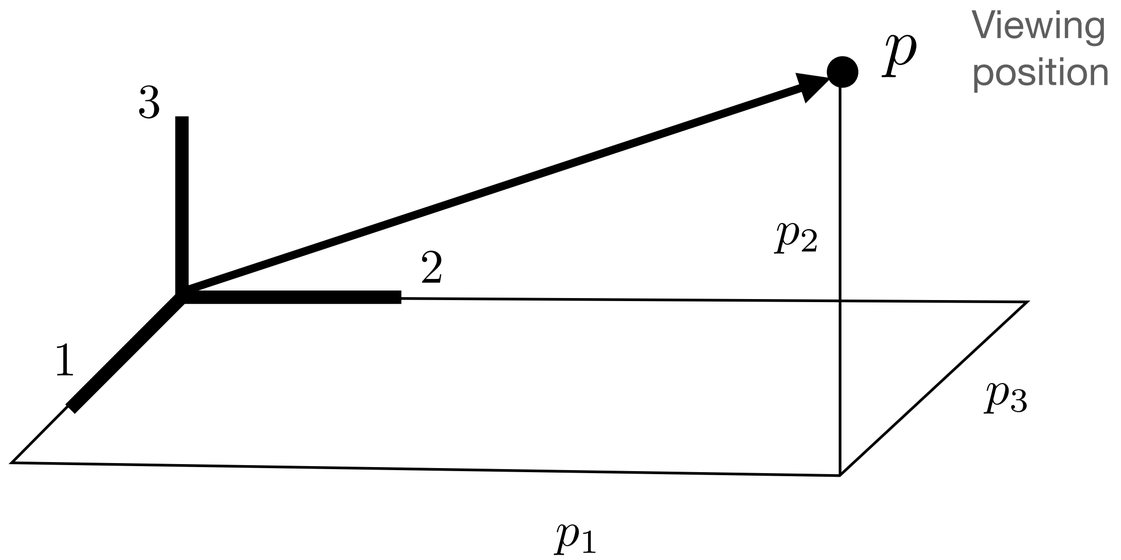


Viewing position

Cartesian coordinates

$$p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$$



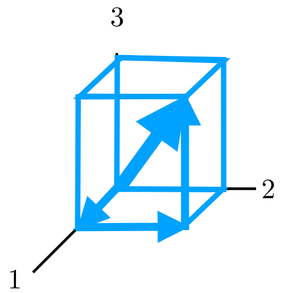


Camera Views - 3D

x = [0.8, 1.0, 0.5]

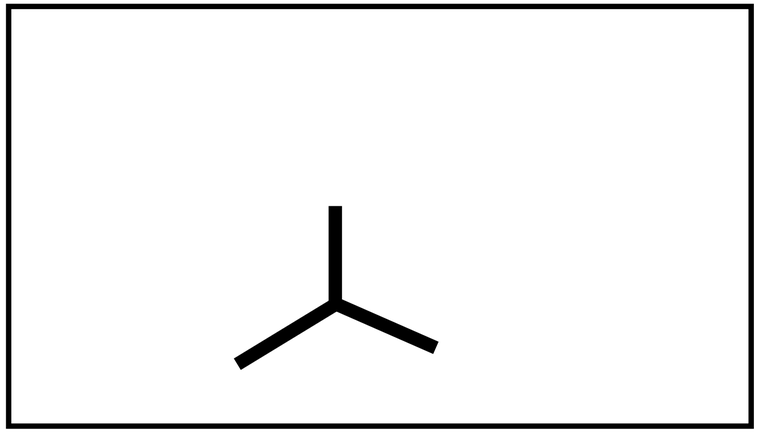
AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

x @ AXES



Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

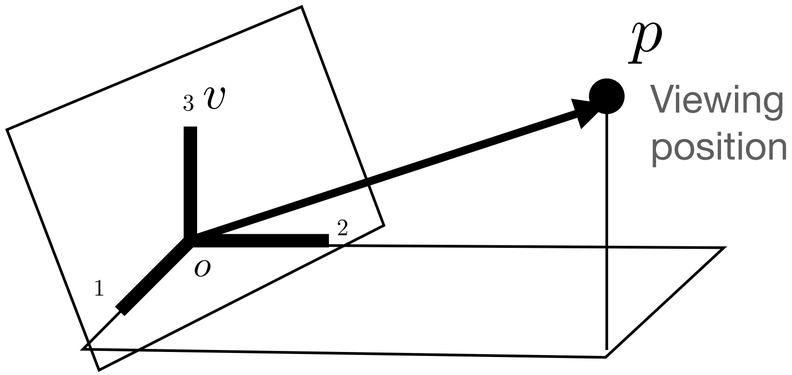
Drawing - 2D Projection

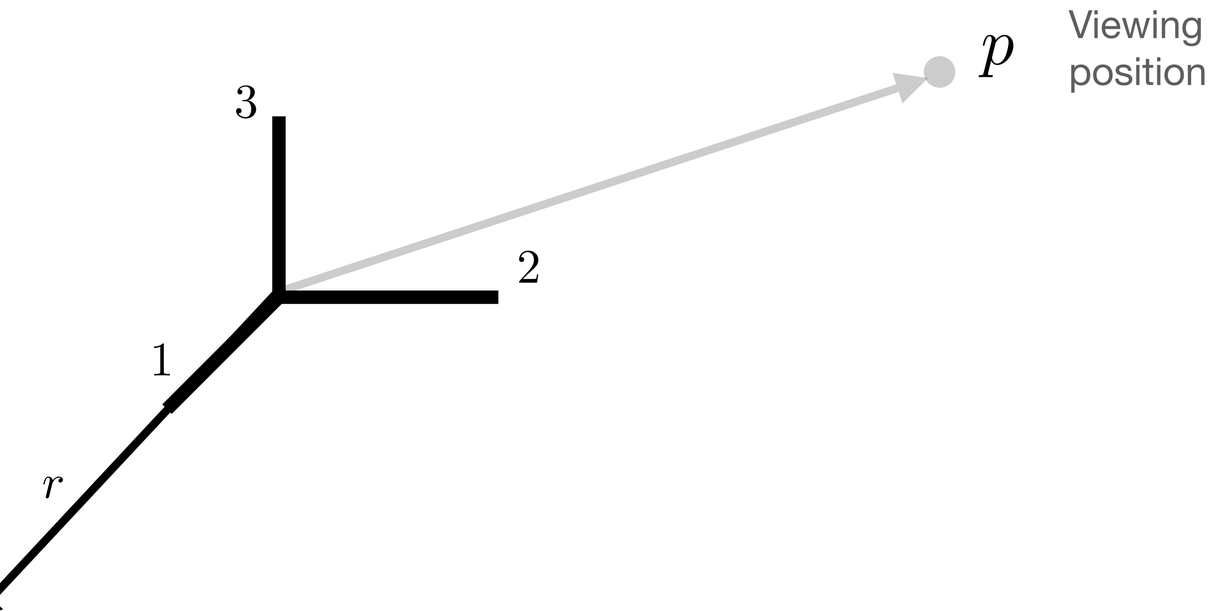


Viewing position

Polar coordinates

$$\begin{bmatrix} r & 0 & 0 \end{bmatrix}$$



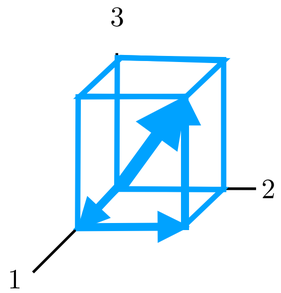


Camera Views - 3D

x = [0.8, 1.0, 0.5]

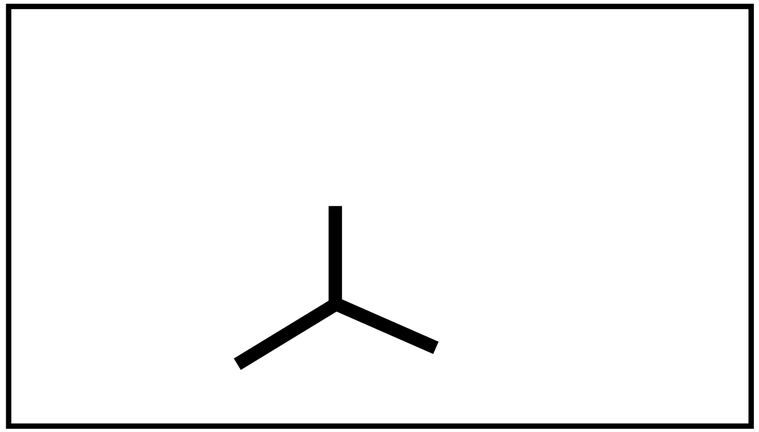
AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

x @ AXES



Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

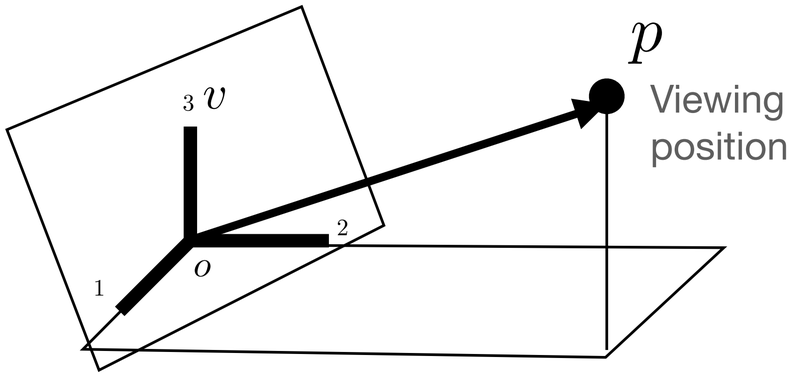
Drawing - 2D Projection

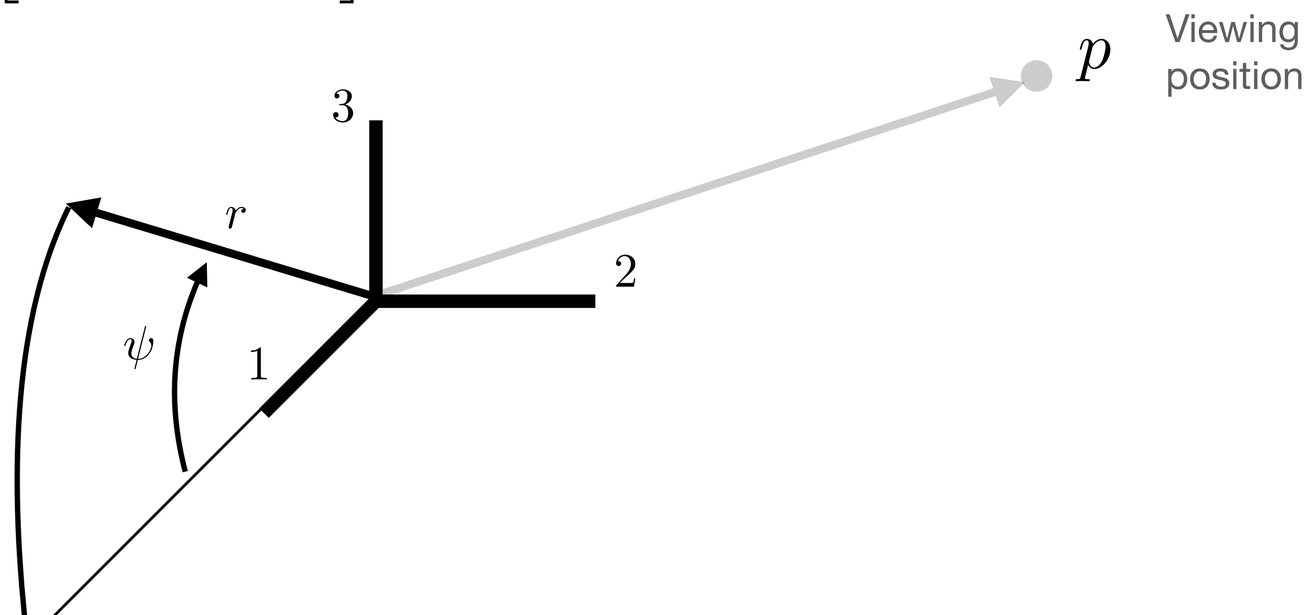


Viewing position

Polar coordinates

$$\begin{bmatrix} r & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\psi & 0 & -\sin\psi \\ 0 & 1 & 0 \\ \sin\psi & 0 & \cos\psi \end{bmatrix}$$



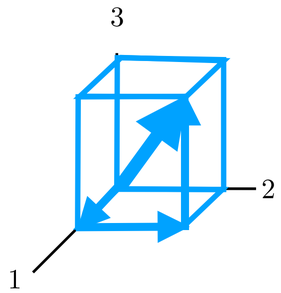


Camera Views - 3D

x = [0.8, 1.0, 0.5]

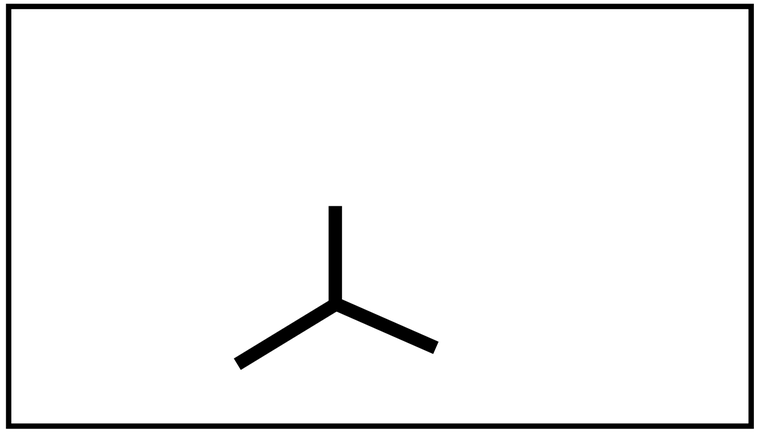
AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

x @ AXES



Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

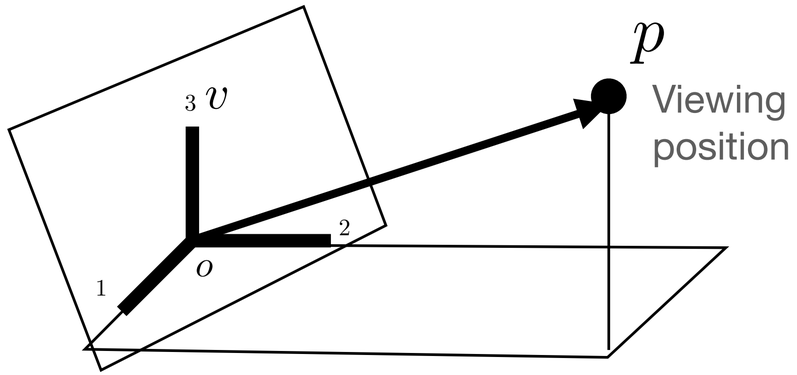
Drawing - 2D Projection

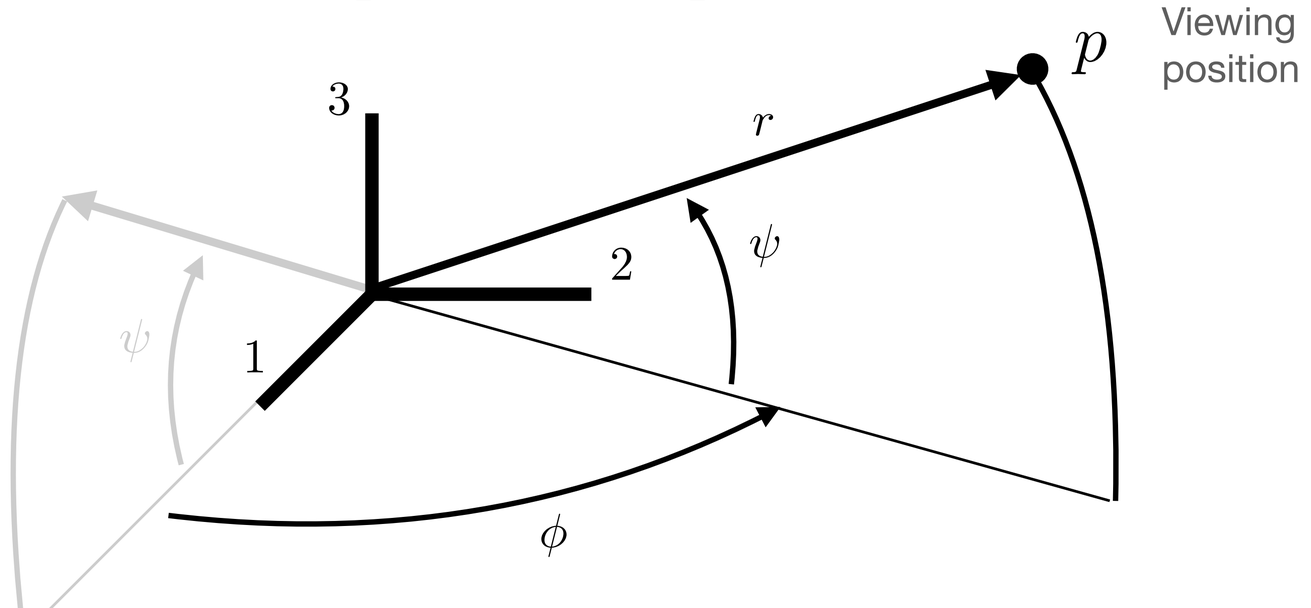


Viewing position

Polar coordinates

$$\begin{bmatrix} r & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\psi & 0 & -\sin\psi \\ 0 & 1 & 0 \\ \sin\psi & 0 & \cos\psi \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



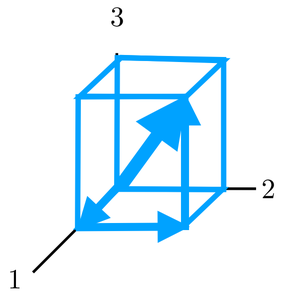


Camera Views - 3D

x = [0.8, 1.0, 0.5]

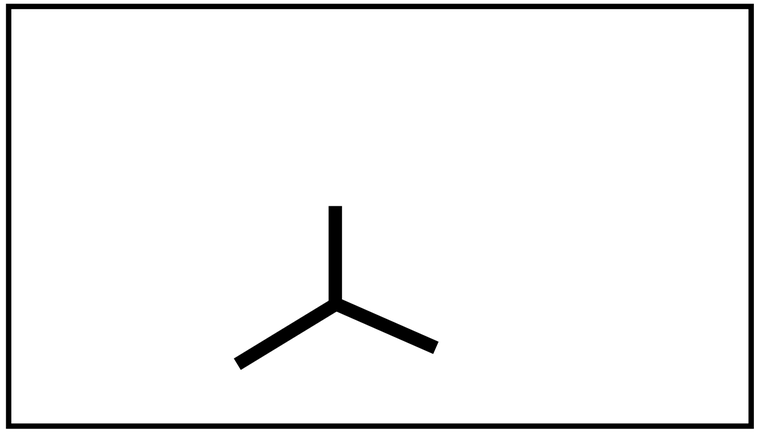
AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

x @ AXES



Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

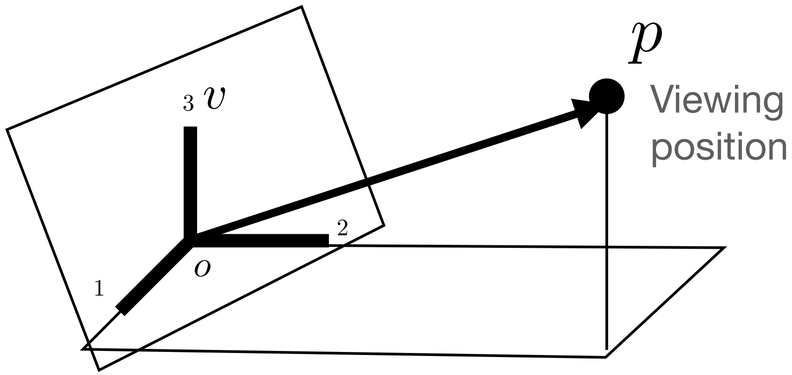
Drawing - 2D Projection

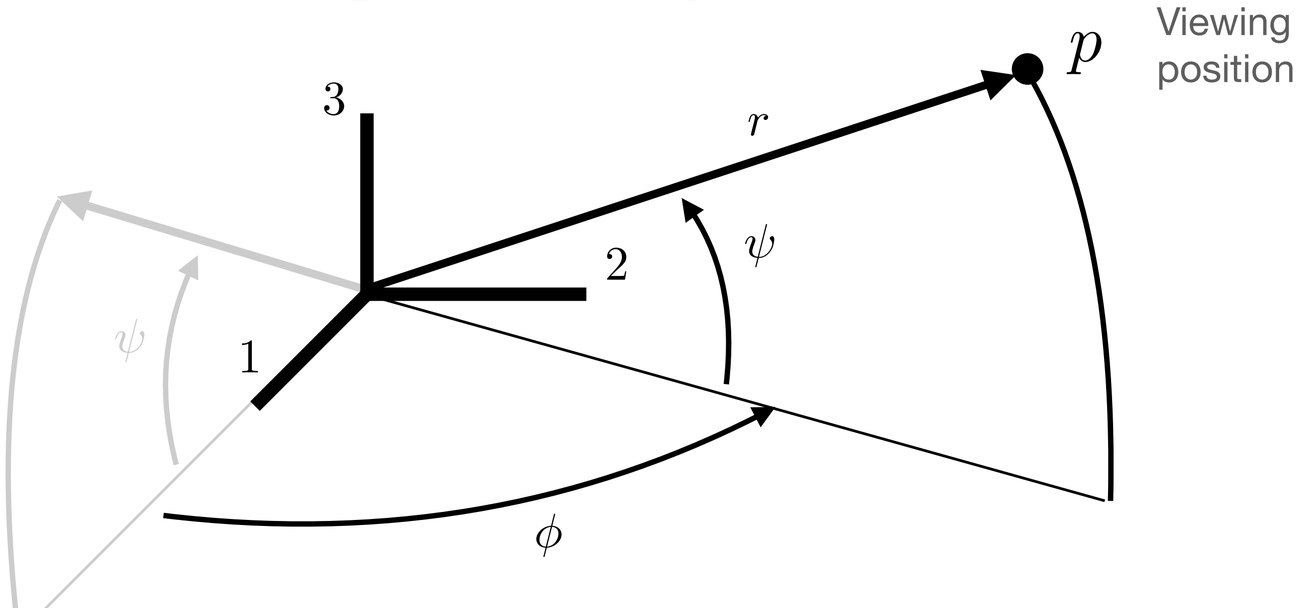


Viewing position

Polar coordinates

$$\begin{bmatrix} r & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\psi & 0 & -\sin\psi \\ 0 & 1 & 0 \\ \sin\psi & 0 & \cos\psi \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} = r \begin{bmatrix} \cos\psi\cos\phi & \cos\psi\sin\phi & -\sin\psi \end{bmatrix}$$



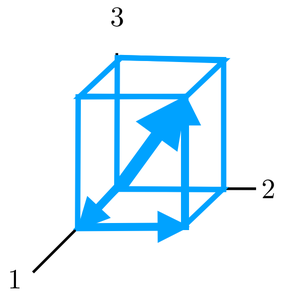


Camera Views - 3D

x = [0.8, 1.0, 0.5]

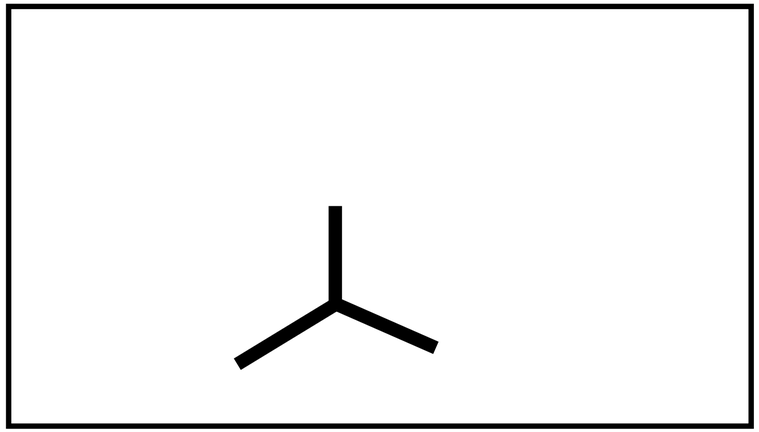
AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

x @ AXES



Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

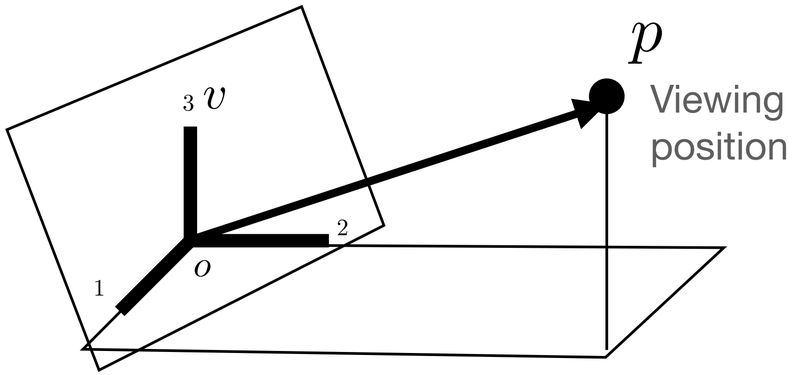
Drawing - 2D Projection

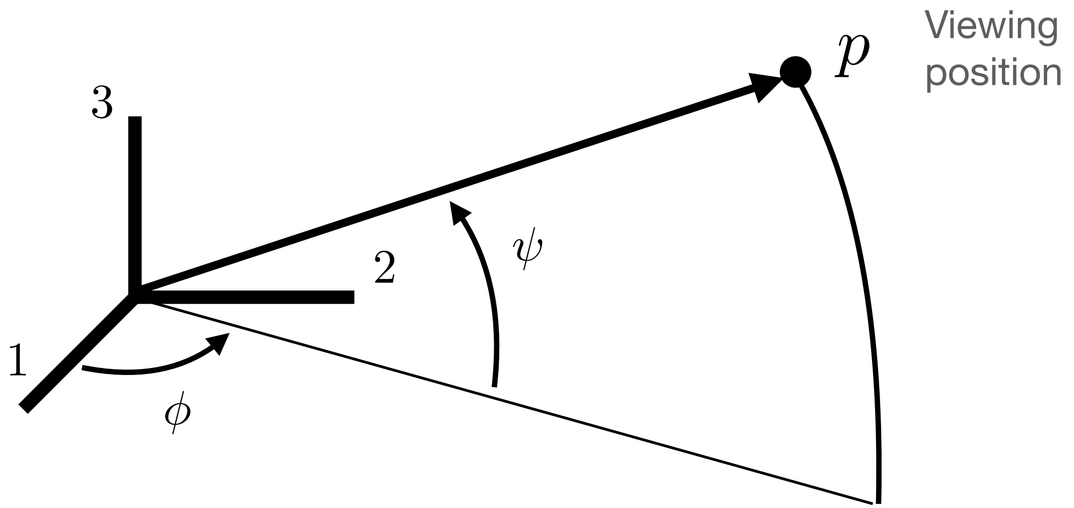


Viewing position

Polar coordinates

$$p = \begin{bmatrix} r\cos\psi\cos\phi & r\cos\psi\sin\phi & -r\sin\psi \end{bmatrix}$$



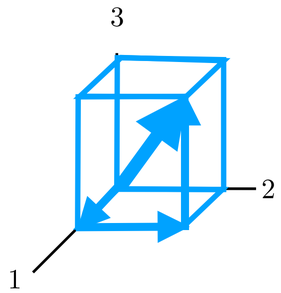


Camera Views - 3D

x = [0.8, 1.0, 0.5]

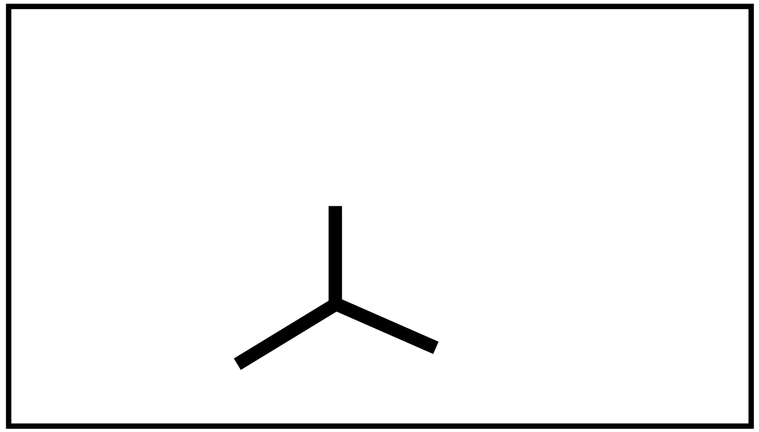
AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

x @ AXES



Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

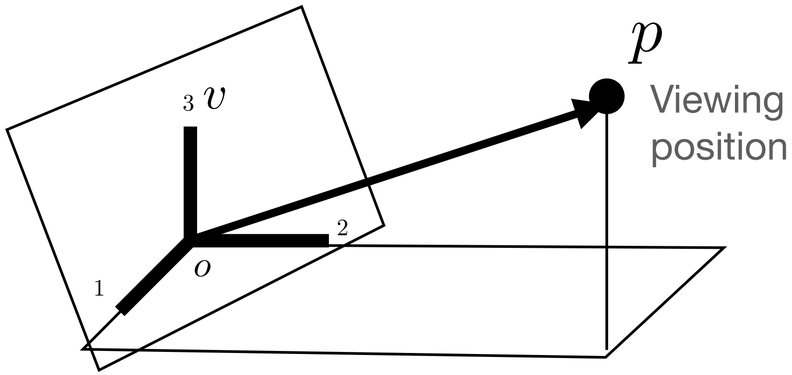
Drawing - 2D Projection

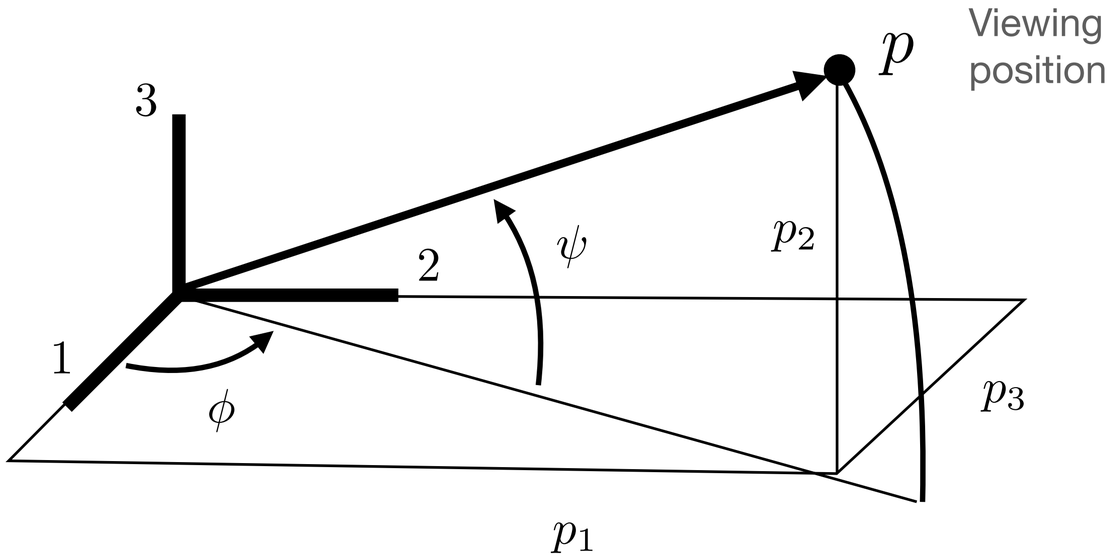


Viewing position

Polar coordinates

$$p = \begin{bmatrix} r\cos\psi\cos\phi & r\cos\psi\sin\phi & -r\sin\psi \end{bmatrix}$$
$$= \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$$



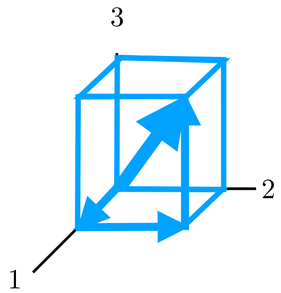


Camera Views - 3D

x = [0.8, 1.0, 0.5]

AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

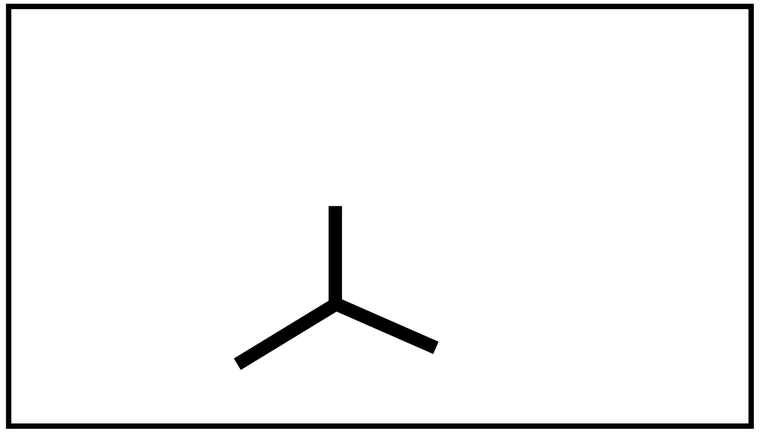
x @ AXES



Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

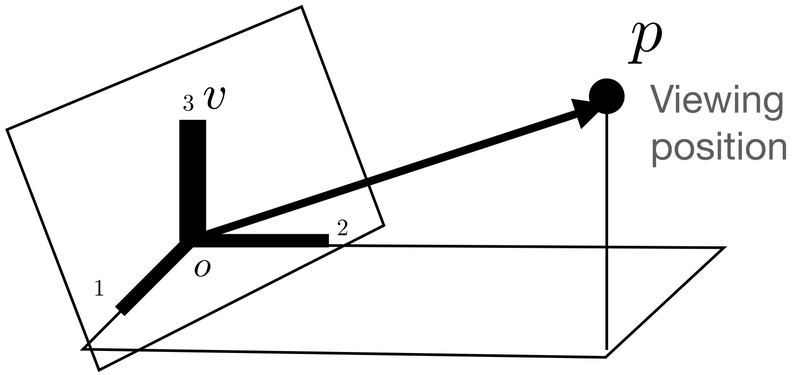
Vertical direction $v = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

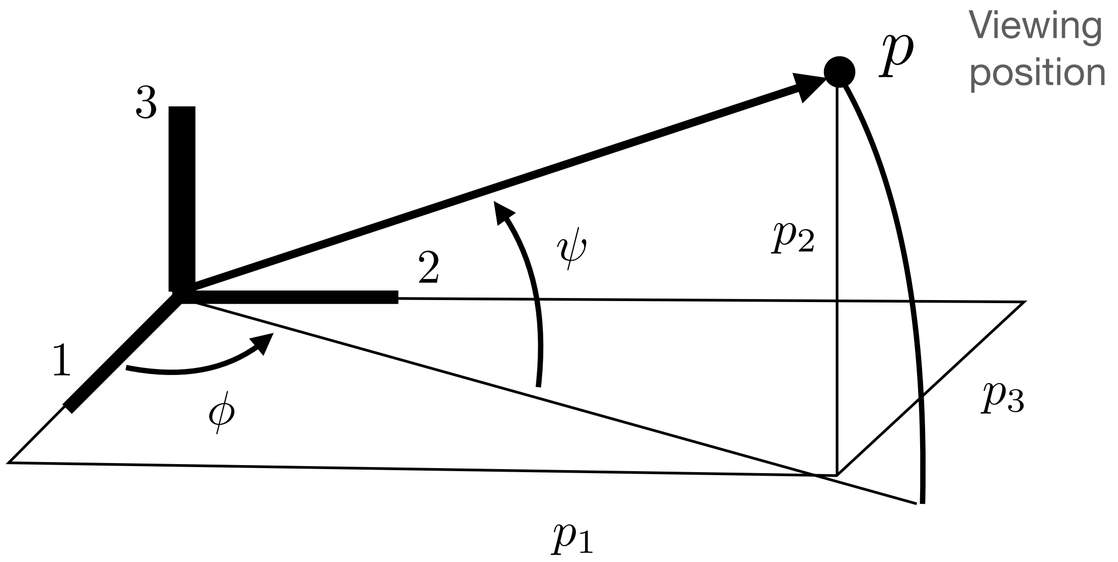
Drawing - 2D Projection



Vertical Direction

$$p = \begin{bmatrix} r\cos\psi\cos\phi & r\cos\psi\sin\phi & -r\sin\psi \end{bmatrix}$$
$$= \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$$



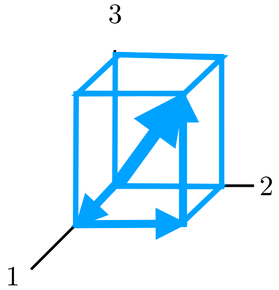


Camera Views - 3D

x = [0.8, 1.0, 0.5]

AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

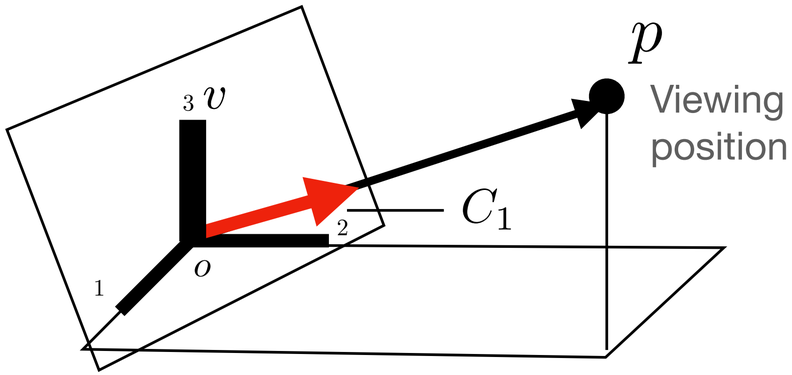
x @ AXES



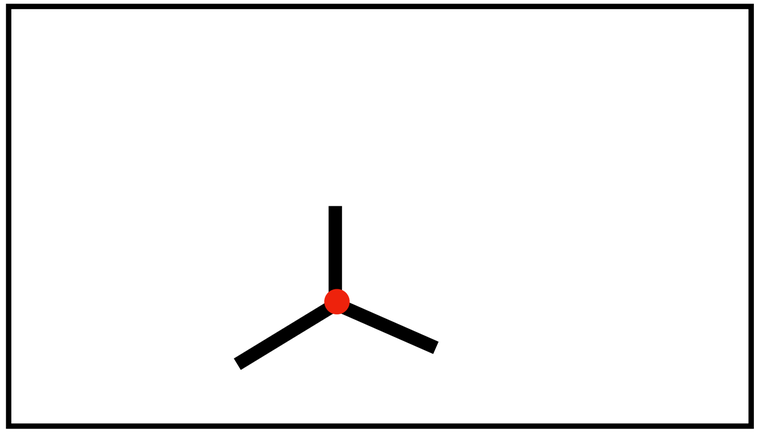
Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

Vertical direction $v = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

$C_1 = \text{normalize}(p)$

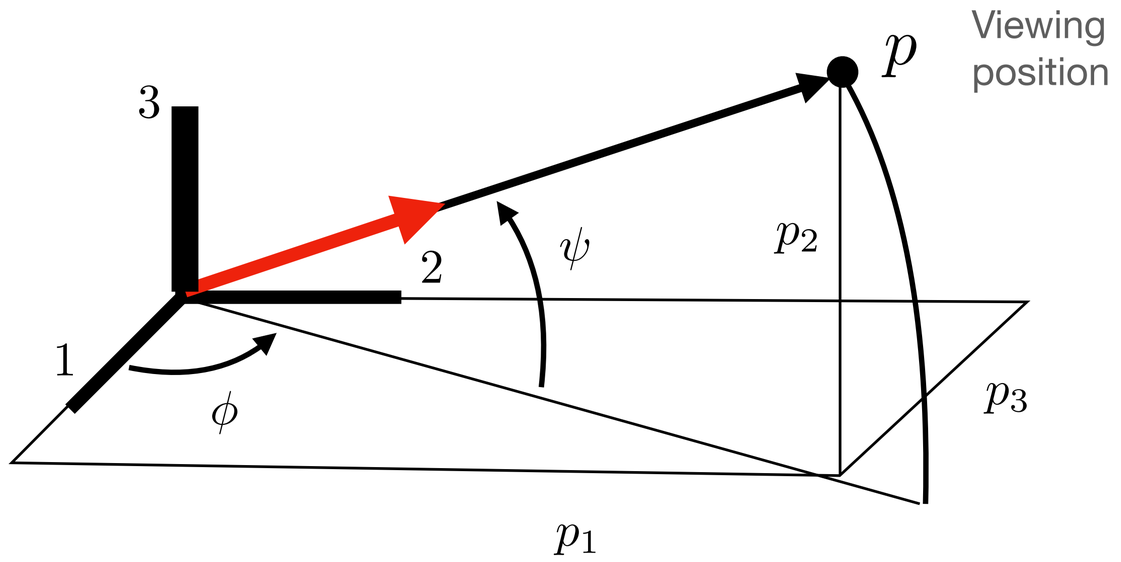


Drawing - 2D Projection



Constructing Camera Coordinates

$$p = \begin{bmatrix} r\cos\psi\cos\phi & r\cos\psi\sin\phi & -r\sin\psi \end{bmatrix}$$
$$= \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$$

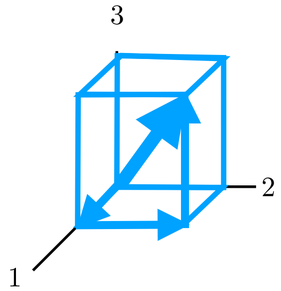


Camera Views - 3D

x = [0.8, 1.0, 0.5]

AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

x @ AXES



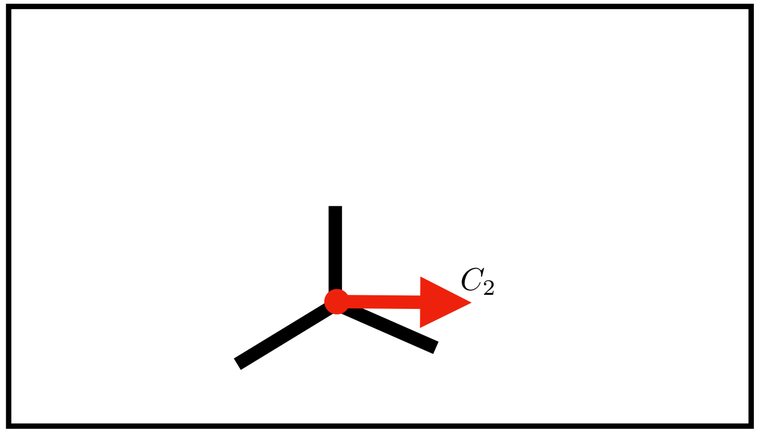
Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

Vertical direction $v = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

$C_1 = \text{normalize}(p)$

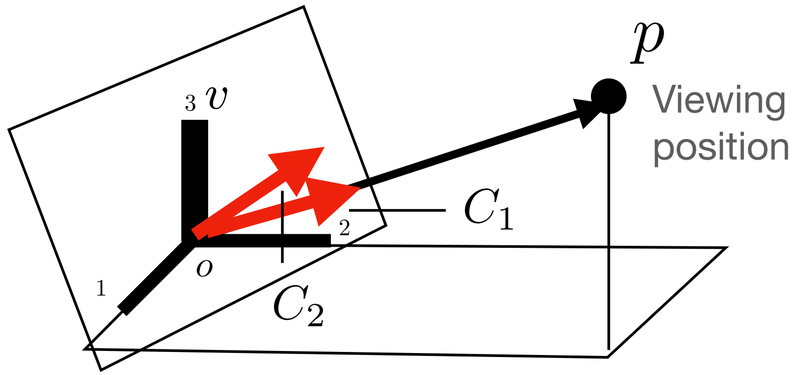
$C_2 = \text{normalize}(v \times C_1)$

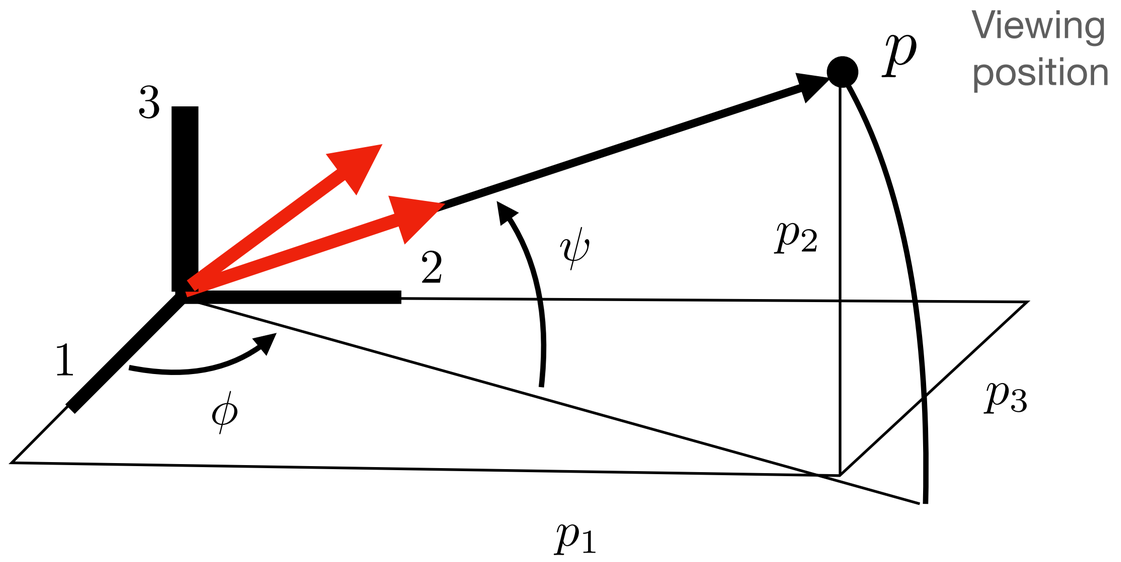
Drawing - 2D Projection



Constructing Camera Coordinates

$$p = \begin{bmatrix} r\cos\psi\cos\phi & r\cos\psi\sin\phi & -r\sin\psi \end{bmatrix}$$
$$= \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$$



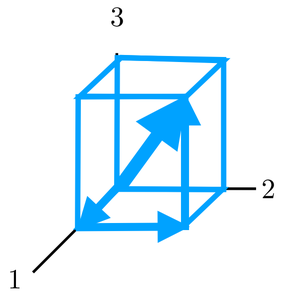


Camera Views - 3D

x = [0.8, 1.0, 0.5]

AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

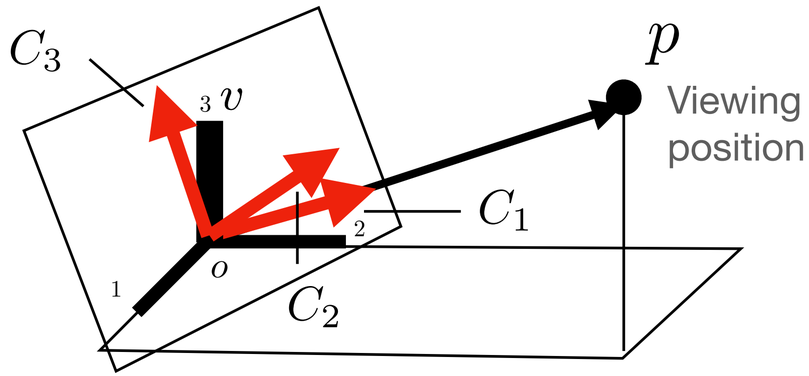
x @ AXES



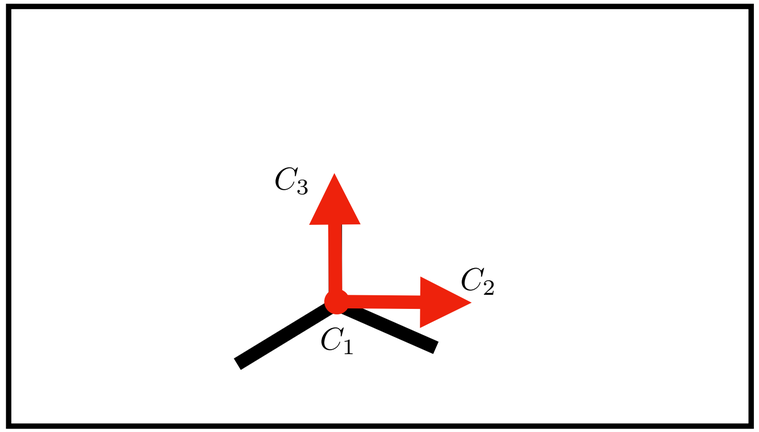
Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

Vertical direction $v = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

$C_1 = \text{normalize}(p)$
$C_2 = \text{normalize}(v \times C_1)$
$C_3 = \text{normalize}(C_1 \times C_2)$

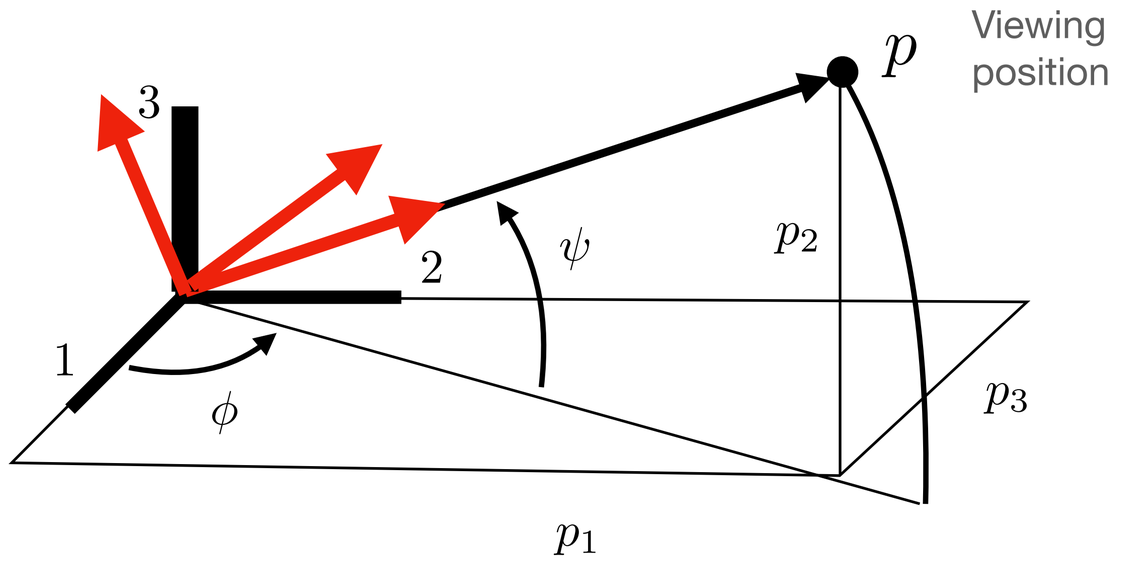


Drawing - 2D Projection



Constructing Camera Coordinates

$$p = \begin{bmatrix} r\cos\psi\cos\phi & r\cos\psi\sin\phi & -r\sin\psi \end{bmatrix}$$
$$= \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$$

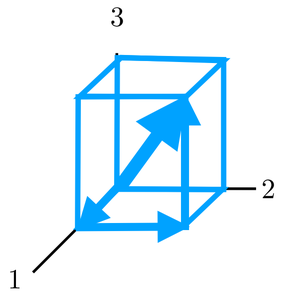


Camera Views - 3D

x = [0.8, 1.0, 0.5]

AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

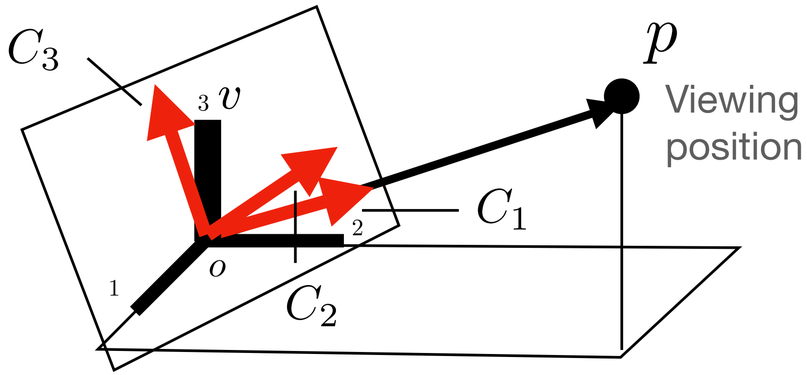
x @ AXES



Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

Vertical direction $v = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

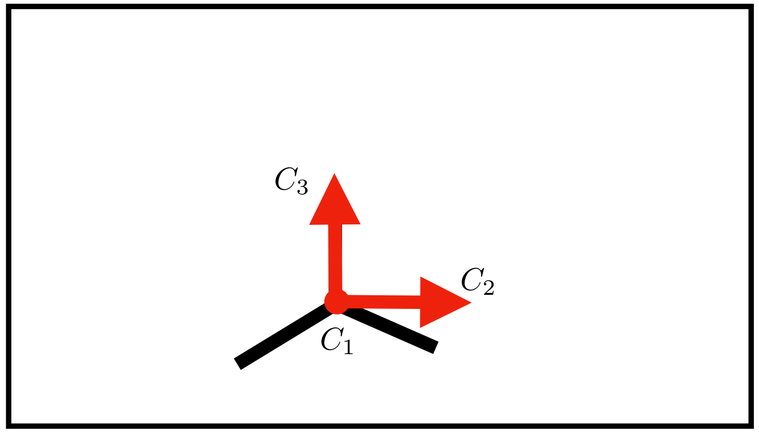
$$C_1 = \text{normalize}(p)$$
$$C_2 = \text{normalize}(v \times C_1)$$
$$C_3 = \text{normalize}(C_1 \times C_2)$$



3D Coordinate Transform

$$C = \begin{bmatrix} - & C_1^T & - \\ - & C_2^T & - \\ - & C_3^T & - \end{bmatrix}$$

Drawing - 2D Projection

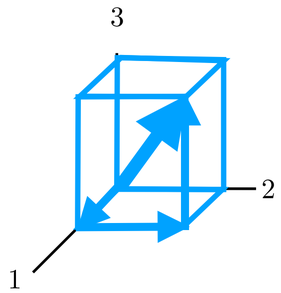


Camera Views - 3D

x = [0.8, 1.0, 0.5]

AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

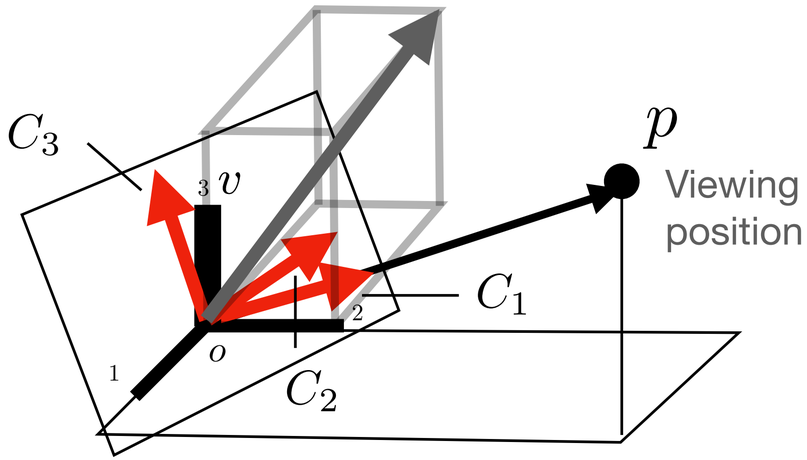
x @ AXES



Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

Vertical direction $v = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

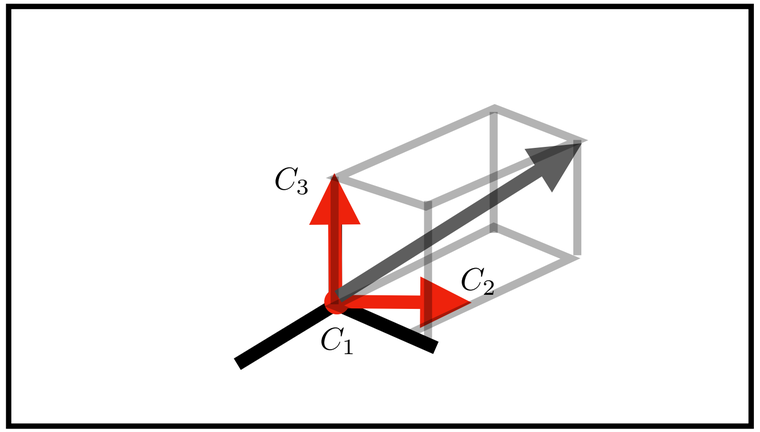
$$C_1 = \text{normalize}(p)$$
$$C_2 = \text{normalize}(v \times C_1)$$
$$C_3 = \text{normalize}(C_1 \times C_2)$$



3D Coordinate Transform

$$C = \begin{bmatrix} - & C_1^T & - \\ - & C_2^T & - \\ - & C_3^T & - \end{bmatrix}$$

Drawing - 2D Projection



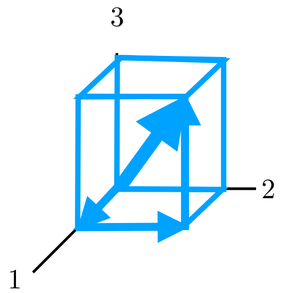
World Coords $x^w = \begin{bmatrix} x_1^w & x_2^w & x_3^w \end{bmatrix}$

Camera Views - 3D

x = [0.8, 1.0, 0.5]

AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

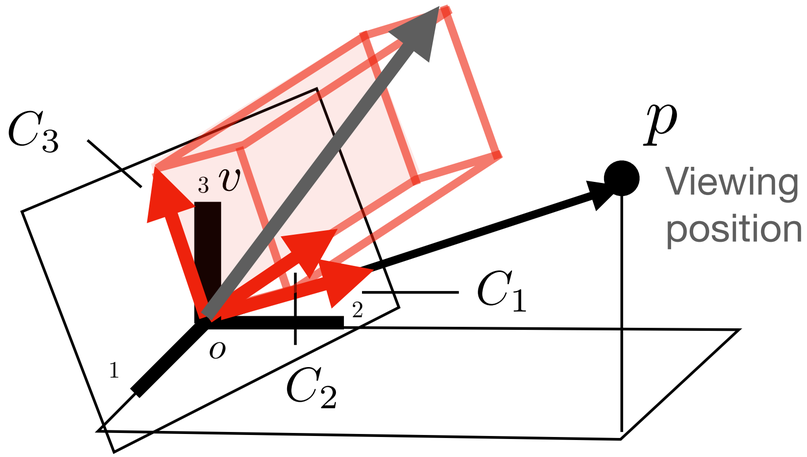
x @ AXES



Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

Vertical direction $v = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

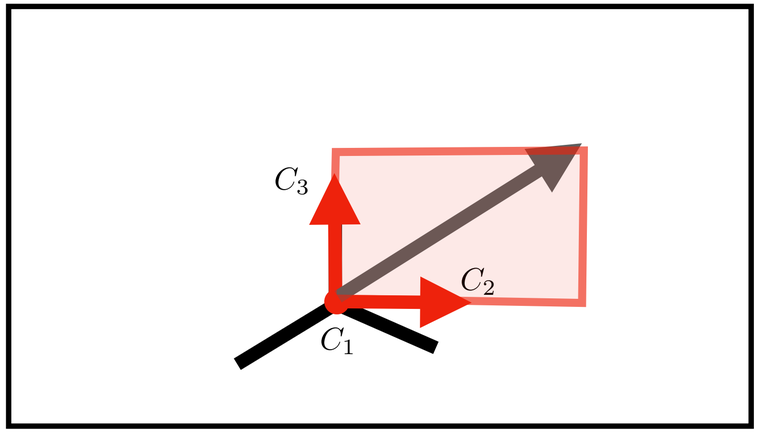
$$C_1 = \text{normalize}(p)$$
$$C_2 = \text{normalize}(v \times C_1)$$
$$C_3 = \text{normalize}(C_1 \times C_2)$$



3D Coordinate Transform

$$C = \begin{bmatrix} - & C_1^T & - \\ - & C_2^T & - \\ - & C_3^T & - \end{bmatrix}$$

Drawing - 2D Projection



World Coords $x^w = \begin{bmatrix} x_1^w & x_2^w & x_3^w \end{bmatrix}$

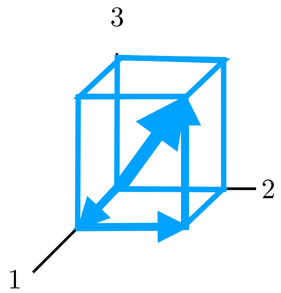
Camera Coords $x^c = \begin{bmatrix} x_1^c & x_2^c & x_3^c \end{bmatrix}$

Camera Views - 3D

x = [0.8, 1.0, 0.5]

AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

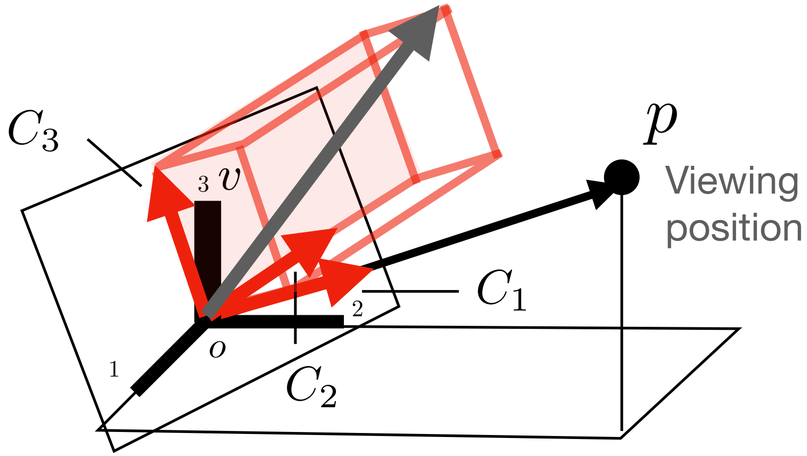
x @ AXES



Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

Vertical direction $v = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

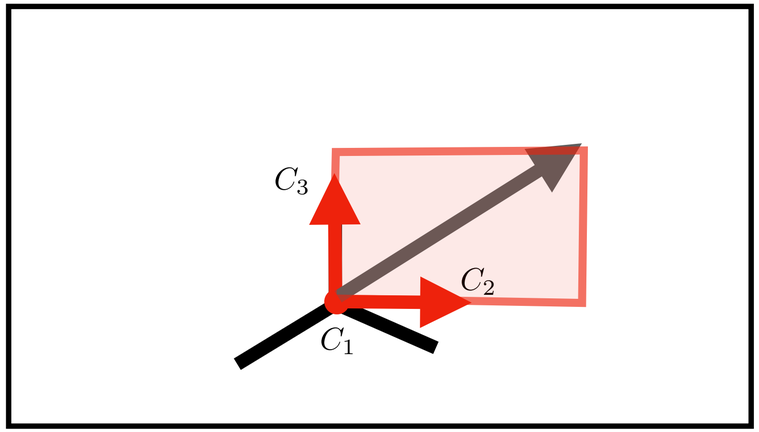
$C_1 = \text{normalize}(p)$
$C_2 = \text{normalize}(v \times C_1)$
$C_3 = \text{normalize}(C_1 \times C_2)$



3D Coordinate Transform

$$C = \begin{bmatrix} - & C_1^T & - \\ - & C_2^T & - \\ - & C_3^T & - \end{bmatrix}$$

Drawing - 2D Projection



World Coords $x^w = \begin{bmatrix} x_1^w & x_2^w & x_3^w \end{bmatrix}$

Camera Coords $x^c = \begin{bmatrix} x_1^c & x_2^c & x_3^c \end{bmatrix}$

$$\begin{bmatrix} x_1^c & x_2^c & x_3^c \end{bmatrix} \begin{bmatrix} - & C_1^T & - \\ - & C_2^T & - \\ - & C_3^T & - \end{bmatrix} = \begin{bmatrix} x_1^w & x_2^w & x_3^w \end{bmatrix}$$

$$x^c C = x^w$$

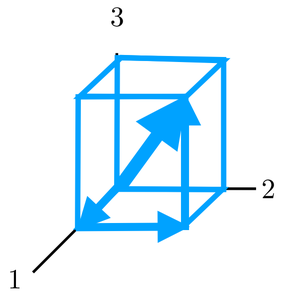
$$\Rightarrow \quad x^c = x^w C^{-1}$$

Camera Views - 3D

x = [0.8, 1.0, 0.5]

AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

x @ AXES



Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

Vertical direction $v = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

$$C_1 = \text{normalize}(p)$$
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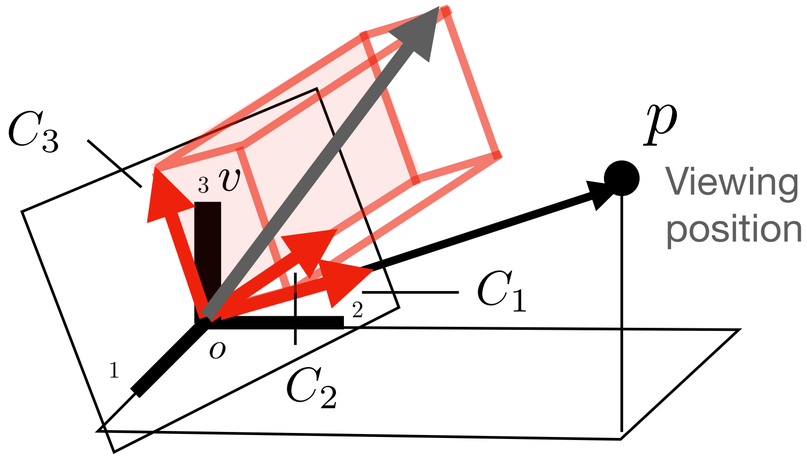
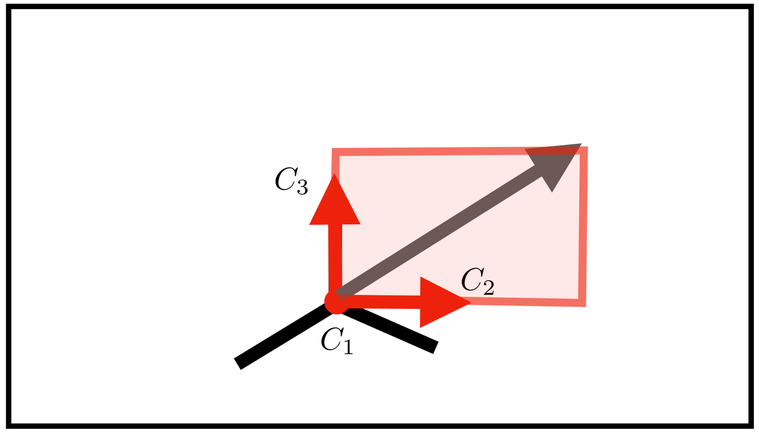
$$C_3 = \text{normalize}(C_1 \times C_2)$$



3D Coordinate Transform

$$C = \begin{bmatrix} - & C_1^T & - \\ - & C_2^T & - \\ - & C_3^T & - \end{bmatrix}$$

Drawing - 2D Projection



World Coords $x^w = \begin{bmatrix} x_1^w & x_2^w & x_3^w \end{bmatrix}$

Camera Coords $x^c = \begin{bmatrix} x_1^c & x_2^c & x_3^c \end{bmatrix}$

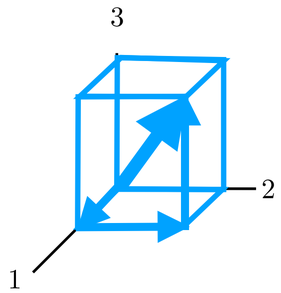
$$\Rightarrow \quad x^c = x^w C^{-1}$$

Camera Views - 3D

x = [0.8, 1.0, 0.5]

AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

x @ AXES



Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

Vertical direction $v = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

$$C_1 = \text{normalize}(p)$$
$$C_2 = \text{normalize}(v \times C_1)$$
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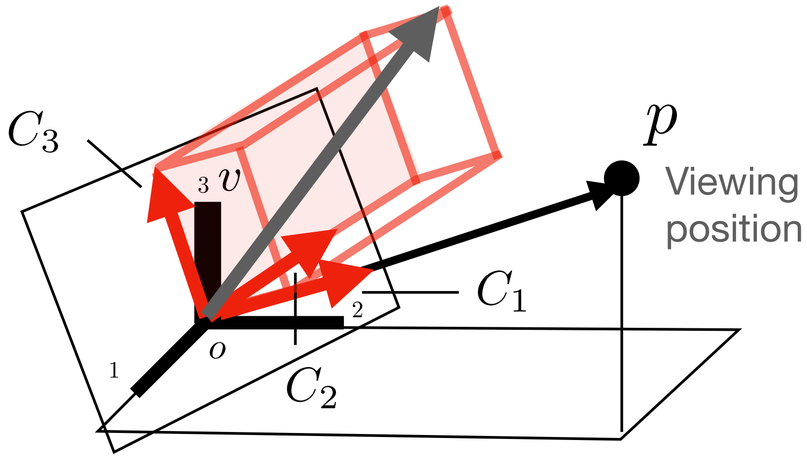


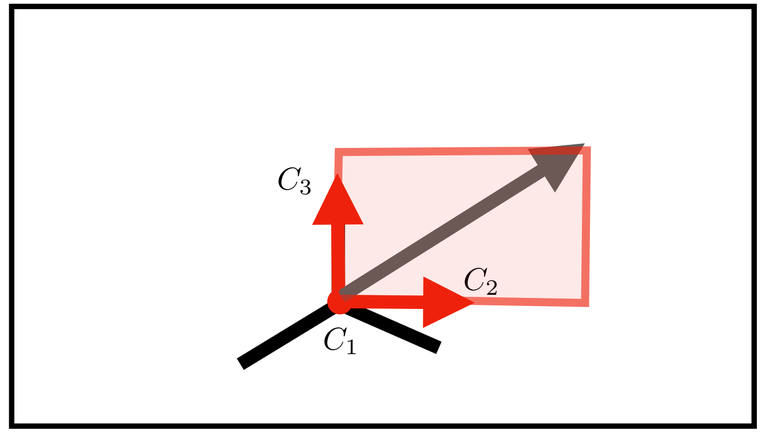
3D Coordinate Transform

$$C = \begin{bmatrix} - & C_1^T & - \\ - & C_2^T & - \\ - & C_3^T & - \end{bmatrix}$$

Code:

xc = xw @ inv(C)

Drawing - 2D Projection



World Coords $x^w = \begin{bmatrix} x_1^w & x_2^w & x_3^w \end{bmatrix}$

Camera Coords $x^c = \begin{bmatrix} x_1^c & x_2^c & x_3^c \end{bmatrix}$

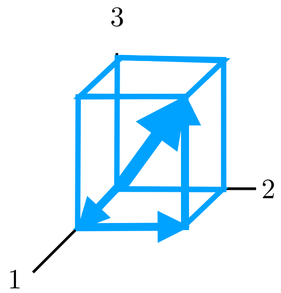
$$\Rightarrow \quad x^c = x^w C^{-1}$$

Camera Views - 3D

x = [0.8, 1.0, 0.5]

AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

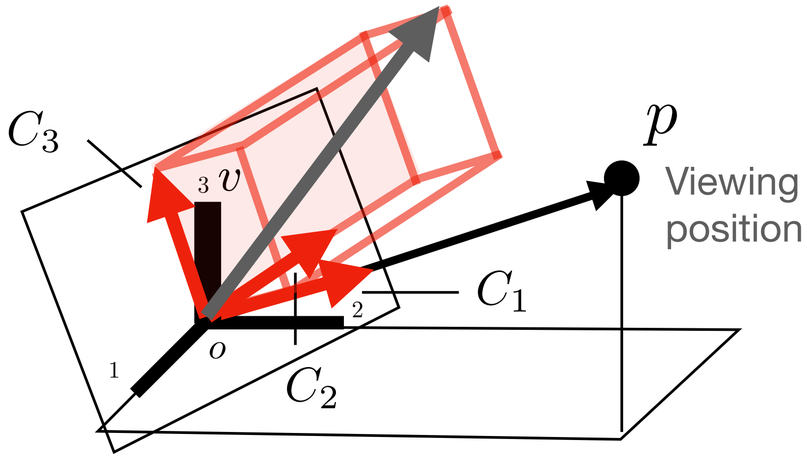
x @ AXES



Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

Vertical direction $v = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

$C_1 = \text{normalize}(p)$
$C_2 = \text{normalize}(v \times C_1)$
$C_3 = \text{normalize}(C_1 \times C_2)$



3D Coordinate Transform

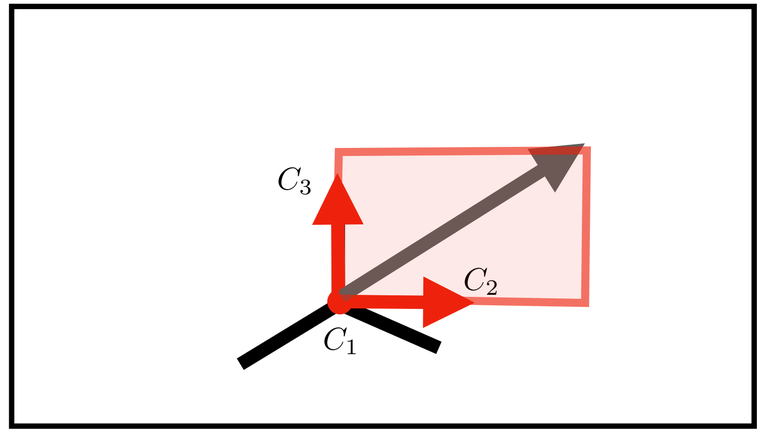
$$C = \begin{bmatrix} - & C_1^T & - \\ - & C_2^T & - \\ - & C_3^T & - \end{bmatrix}$$

Code:

xc = xw @ inv(C)

AXEScam3D = eyew @ inv(C)

Drawing - 2D Projection



World Coords $x^w = \begin{bmatrix} x_1^w & x_2^w & x_3^w \end{bmatrix}$

Camera Coords $x^c = \begin{bmatrix} x_1^c & x_2^c & x_3^c \end{bmatrix}$

$$\Rightarrow \quad x^c = x^w C^{-1}$$

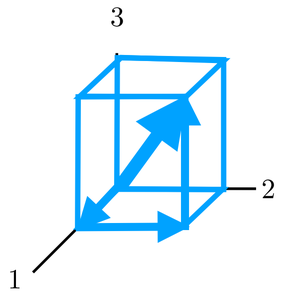
eyew = [[1. , 0. , 0.],
[0. , 1. , 0.],
[0. , 0. , 1.]]

Camera Views - 3D

x = [0.8, 1.0, 0.5]

AXES = [[-0.7,-0.7],
[1.0, 0.0],
[0.0, 1.0]]

x @ AXES



Viewing position $p = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$

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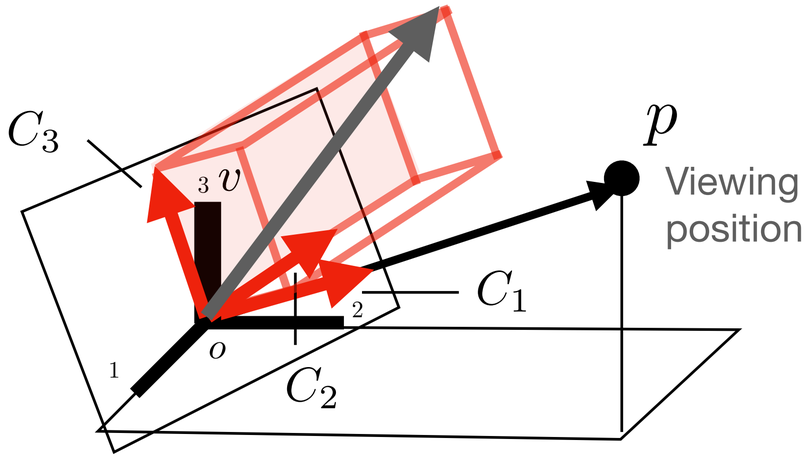
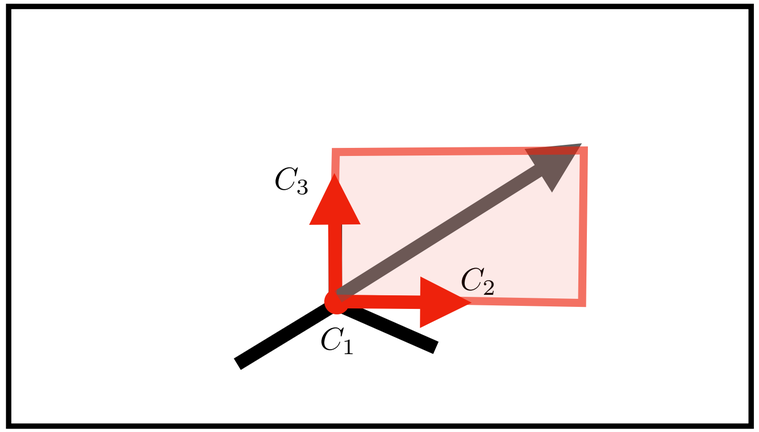


2D Projection Transform

$$C = \begin{bmatrix} - & C_1^T & - \\ - & C_2^T & - \\ - & C_3^T & - \end{bmatrix}$$

Drawing - 2D Projection



World Coords $x^w = \begin{bmatrix} x_1^w & x_2^w & x_3^w \end{bmatrix}$

Camera Coords $x^c = \begin{bmatrix} x_1^c & x_2^c & x_3^c \end{bmatrix}$

$$\Rightarrow \quad x^c = x^w C^{-1}$$

Code:

xc = xw @ inv(C)

AXEScam3D = eyew @ inv(C)

AXES = AXEScam3D[:,1:]

eyew = [[1. , 0. , 0.],
[0. , 1. , 0.],
[0. , 0. , 1.]]

Select second two columns