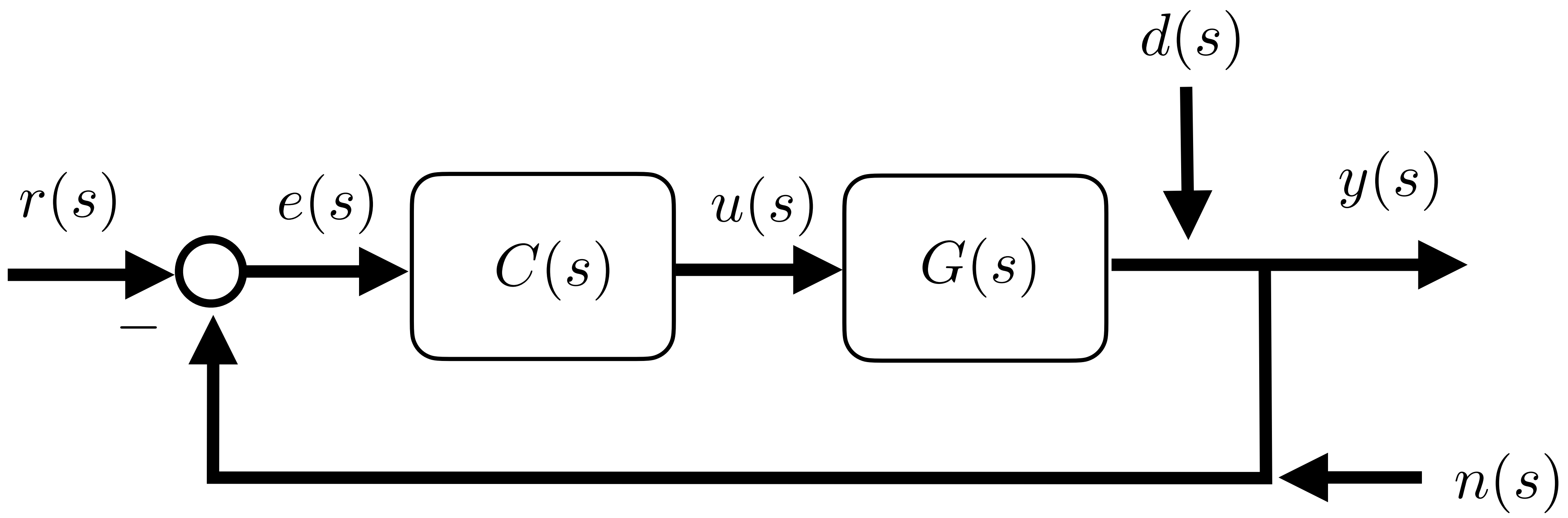
Control Design: Disturbance Rejection

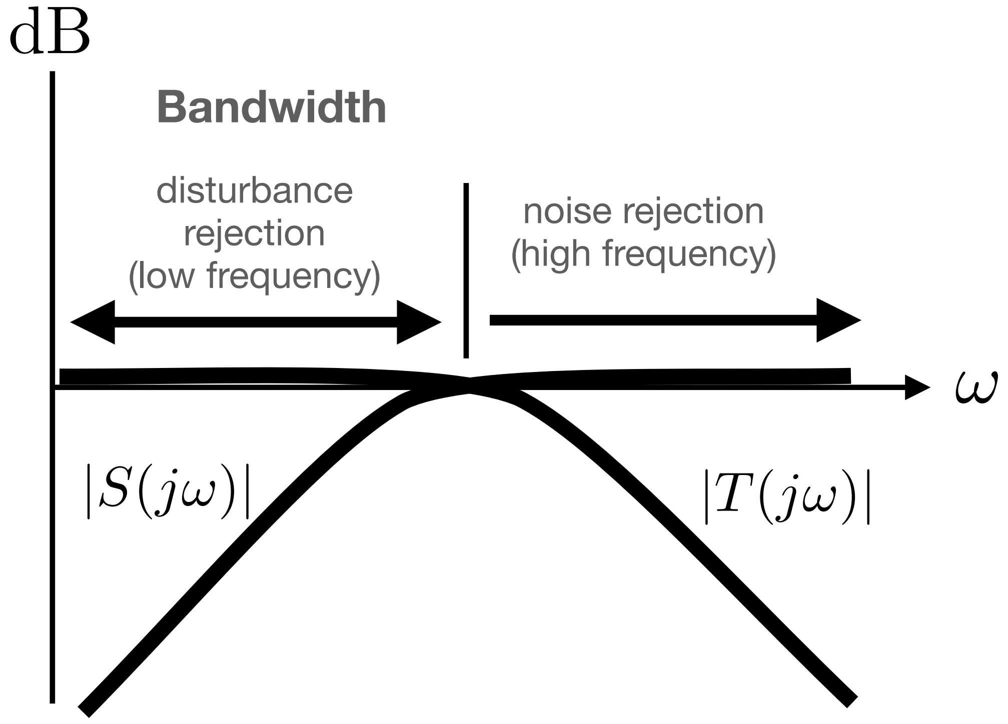
Control Theory

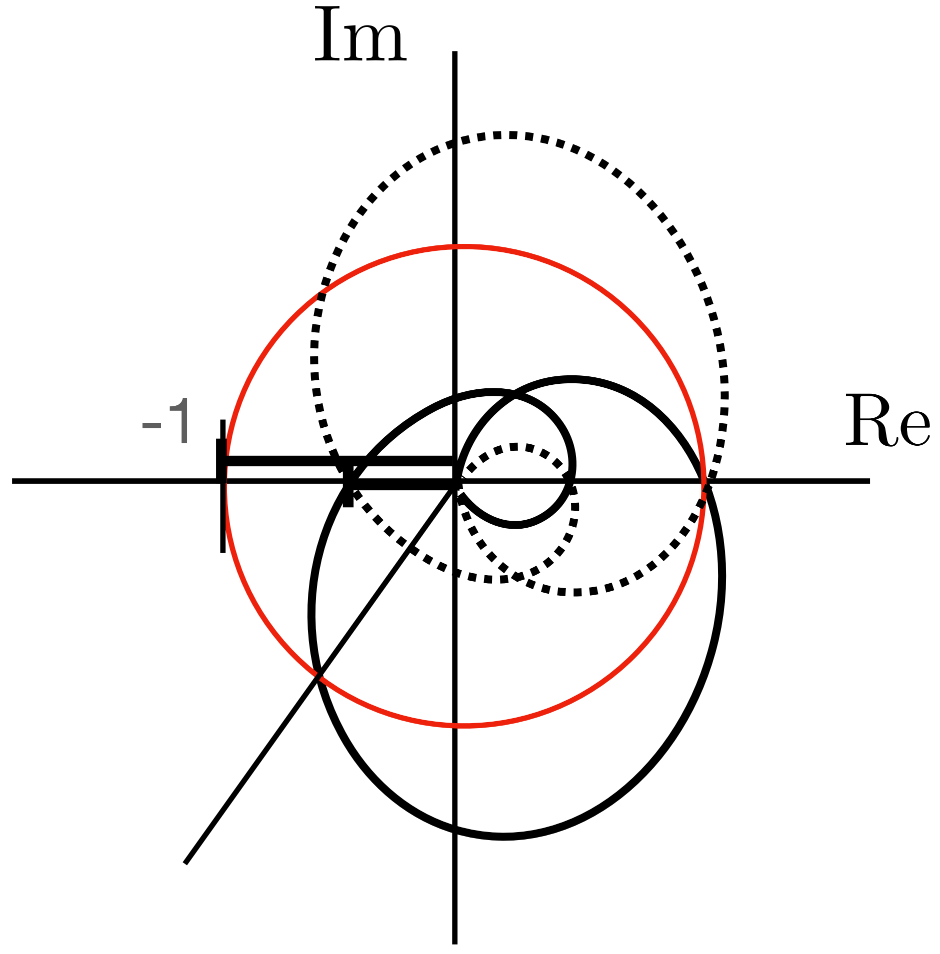
Major Contributions: Behcet Ackimese

Winter 2022 - Dan Calderone

SISO: Sensitivity and Robustness







$L(j\omega) = (GC)(j\omega)$

$$y = GC(r - y - n) + d \quad \rightarrow \quad (I + GC)y = GC(r - n) + d \quad \rightarrow \quad y = \underbrace{(I+GC)^{-1}GC}_{T}(r-n) + \underbrace{(I+GC)^{-1}}_{S}d$$

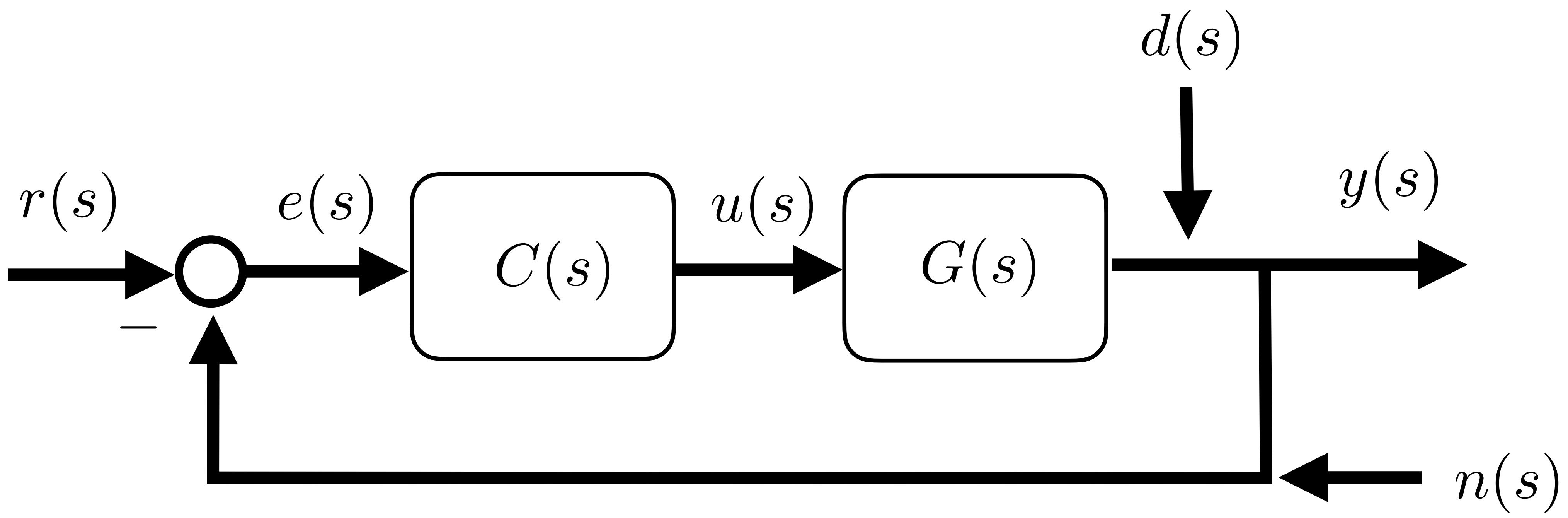
$$\begin{aligned}
e &= r - y \\
&= r - (I+GC)^{-1}GC(r-n) - (I+GC)^{-1}d \\
&= (I+GC)^{-1}(I+GC)r - (I+GC)^{-1}GC(r-n) - (I+GC)^{-1}d \\
&= (I+GC)^{-1}r + (I+GC)^{-1}GCn - (I+GC)^{-1}d \\
&= \underbrace{(I+GC)^{-1}}_{S}r + \underbrace{(I+GC)^{-1}GC}_{T}n - \underbrace{(I+GC)^{-1}}_{S}d
\end{aligned}$$

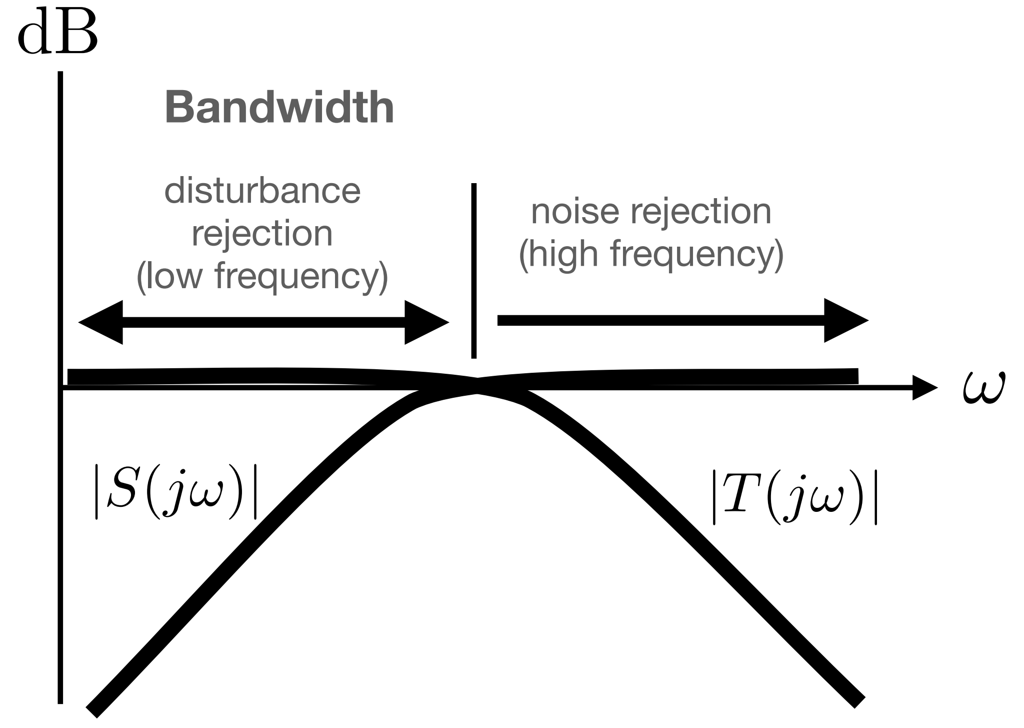
Sensitivity $\quad S = (I+GC)^{-1}$

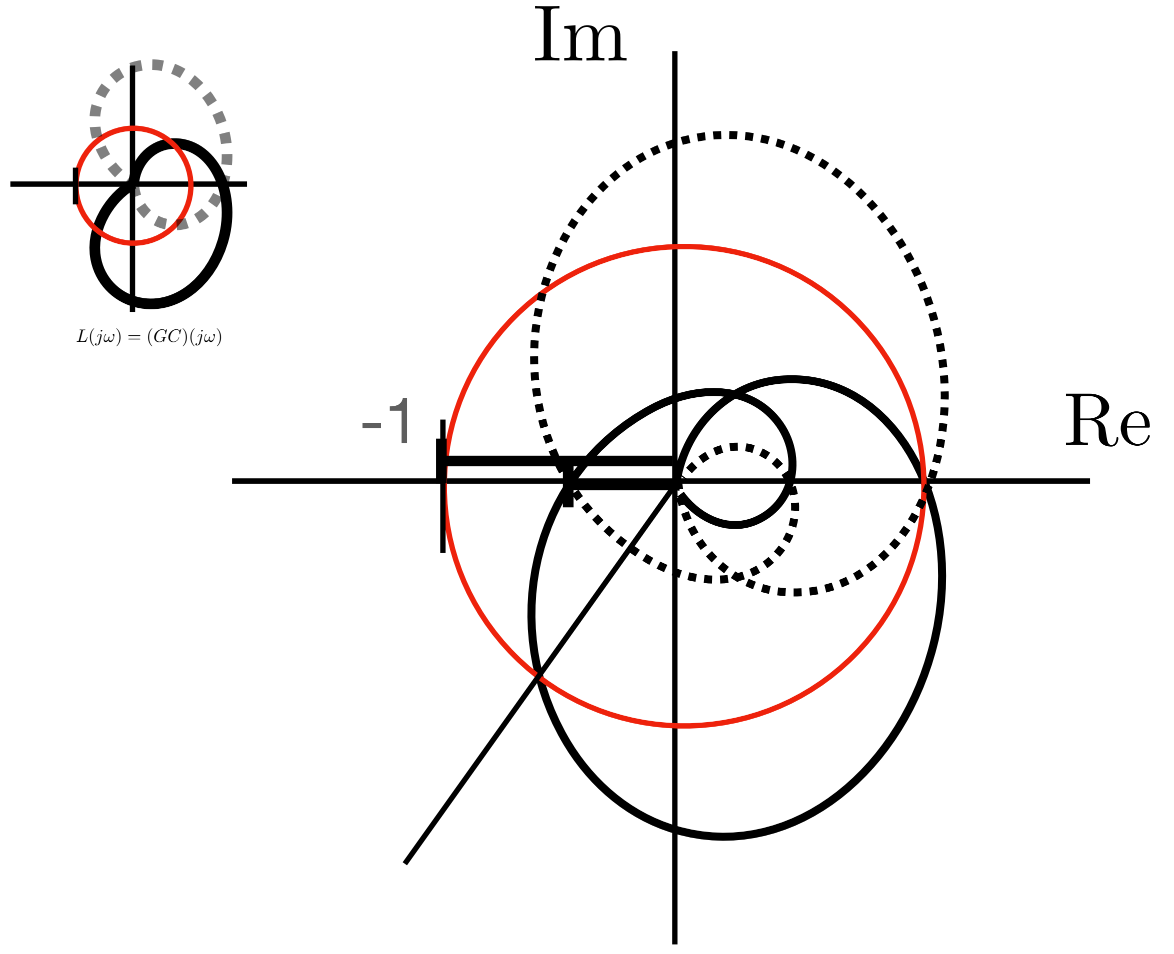
Complementary Sensitivity $\quad T = (I+GC)^{-1}GC$

… fundamental limitation $\quad S + T = I$

SISO: Sensitivity and Robustness







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$$y = GC(r - y - n) + d$$

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$$S = \frac{1}{|1 + L|}$$

$$e = r - y = \underbrace{(I + GC)^{-1}}_{S}r + \underbrace{(I + GC)^{-1}GC}_{T}n - \underbrace{(I + GC)^{-1}}_{S}d$$

Sensitivity $S = (I + GC)^{-1}$

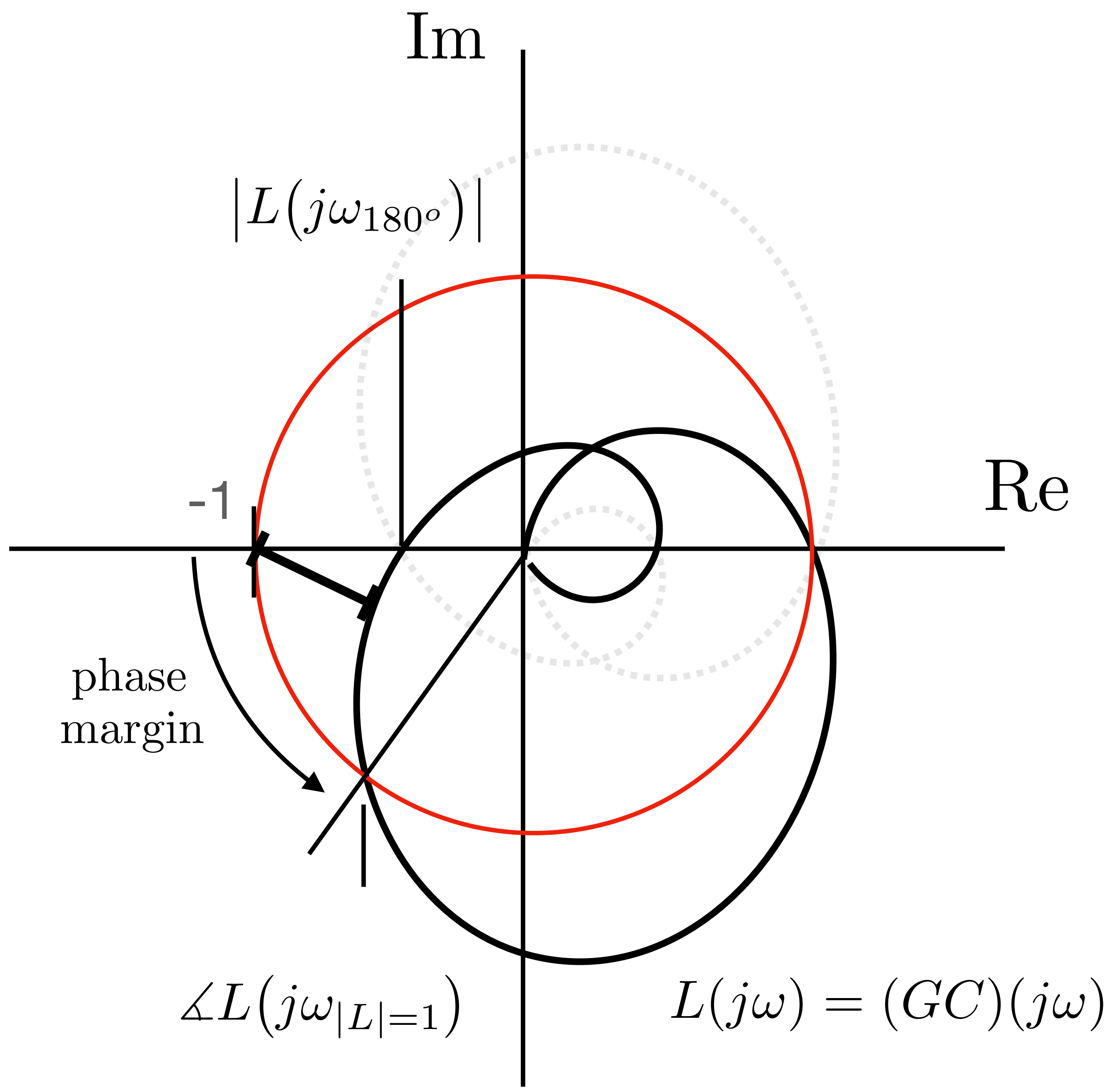
Complementary Sensitivity $T = (I + GC)^{-1}GC$

… fundamental limitation $S + T = I$

Gain Margin $\dfrac{1}{|L(j\omega_{180^o})|}$

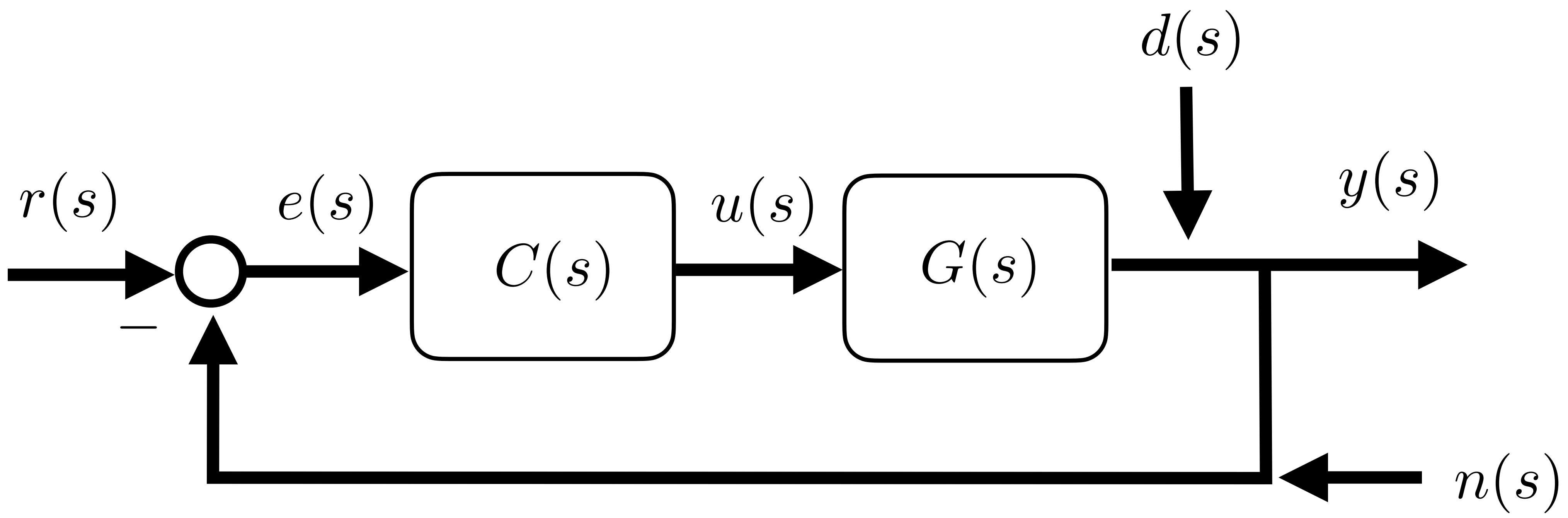
Phase Margin $180 - \measuredangle L(j\omega_{|L|=1})$

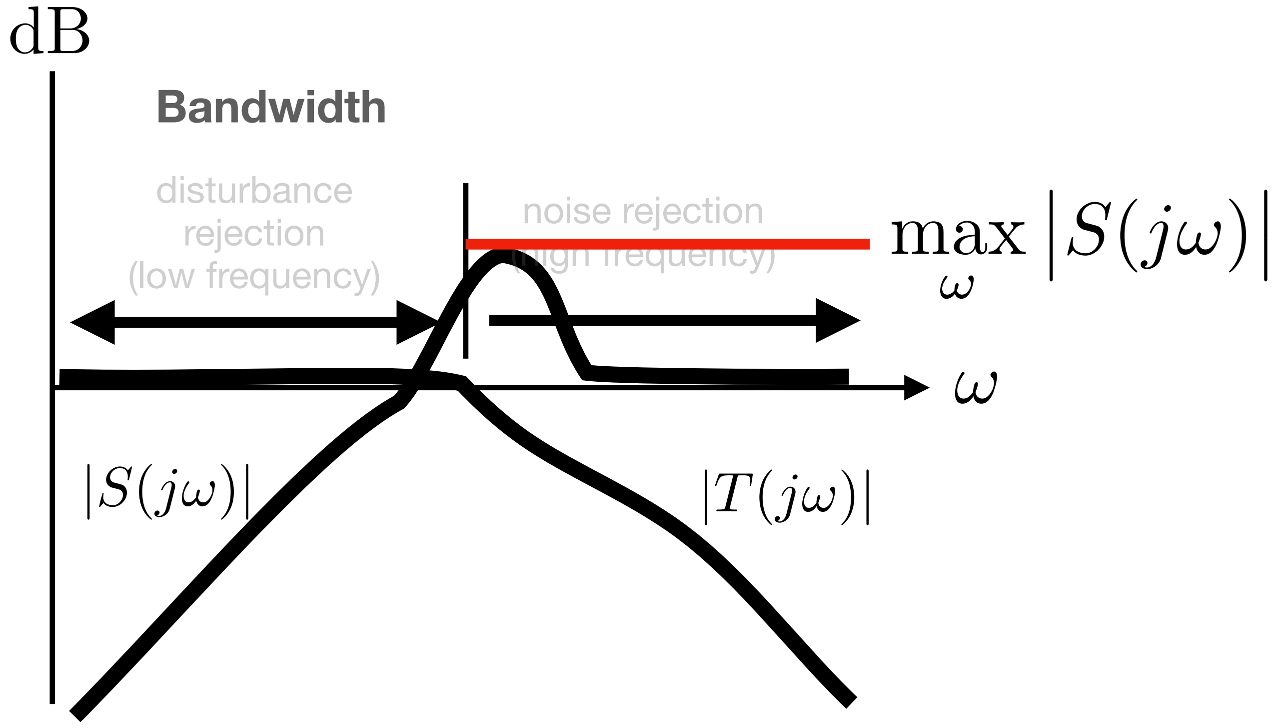
Stability Margin $|1 + L| = |1 + GC|$

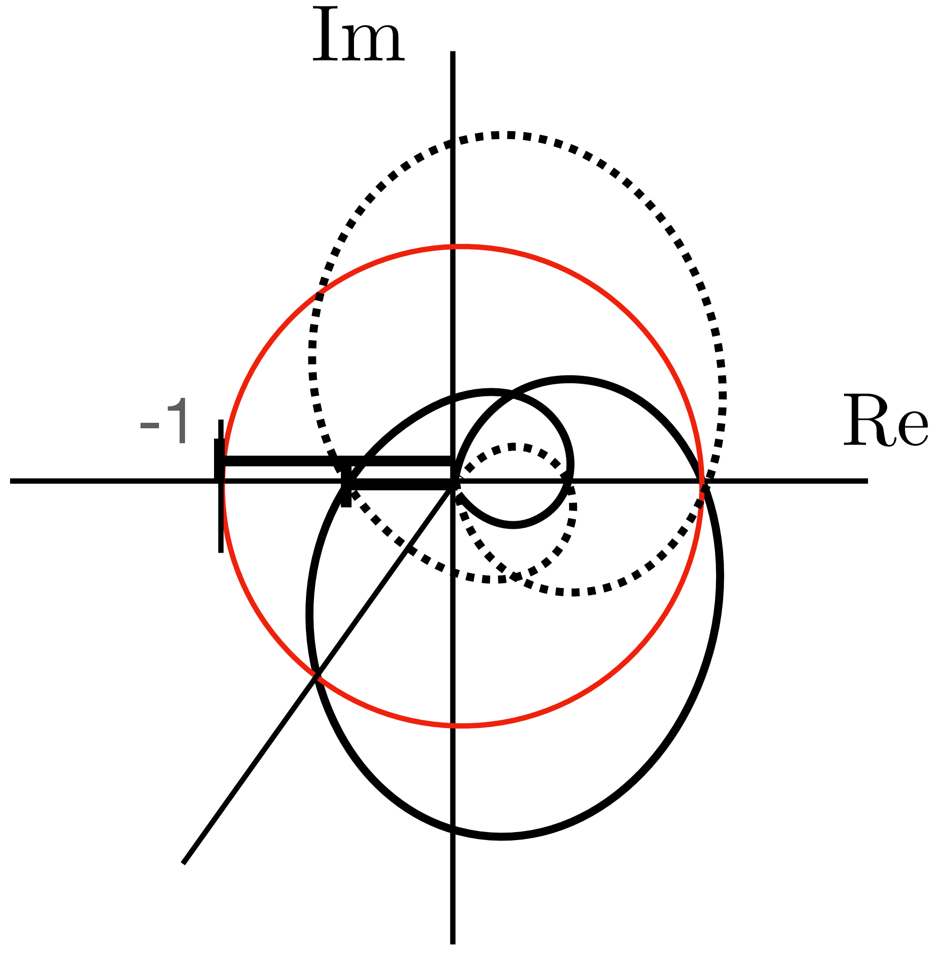


$L(j\omega) = (GC)(j\omega)$

SISO: Sensitivity and Robustness







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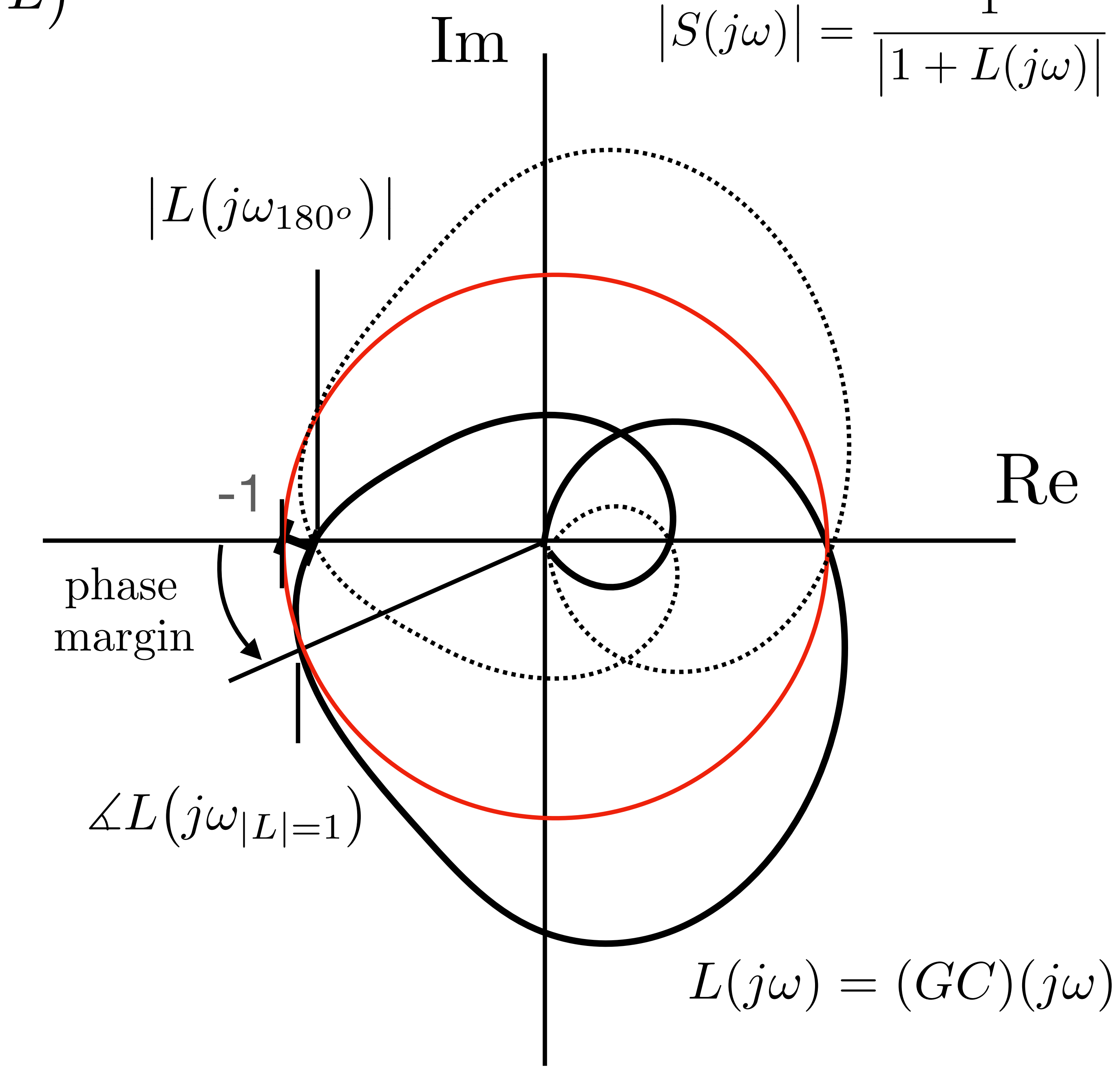
$$y = GC(r - y - n) + d$$

$$\Rightarrow \; y = \underbrace{(I + GC)^{-1}GC}_{T}(r - n) + \underbrace{(I + GC)^{-1}}_{S} d$$

$$S = (I + L)^{-1} = \underbrace{\frac{1}{\det(I + L)}}_{\text{char poly}} \text{Adj}(I + L)$$

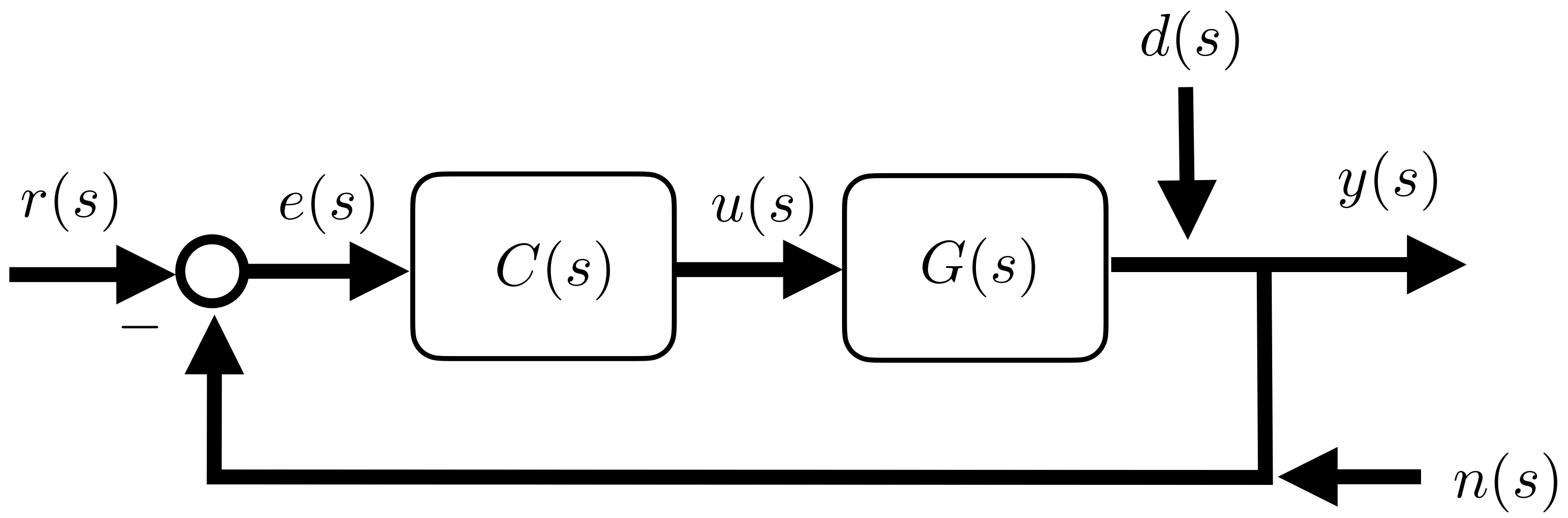
$$e = r - y = \underbrace{(I + GC)^{-1}}_{S} r + \underbrace{(I + GC)^{-1}GC}_{T} n - \underbrace{(I + GC)^{-1}}_{S} d$$

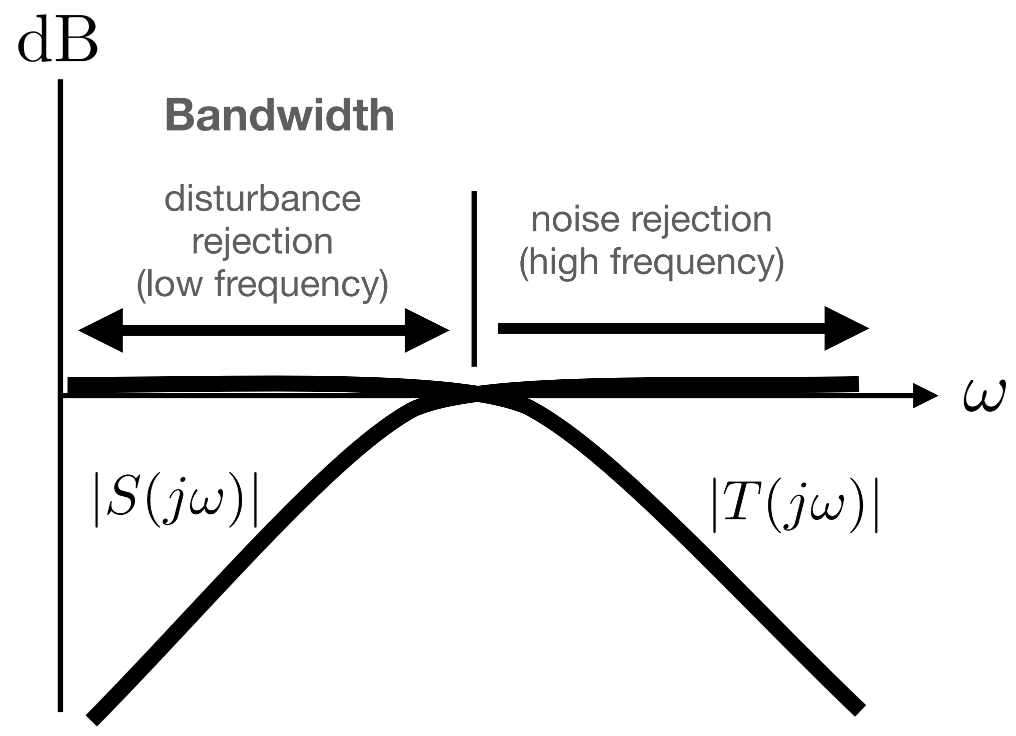
$$|S(j\omega)| = \frac{1}{|1 + L(j\omega)|}$$

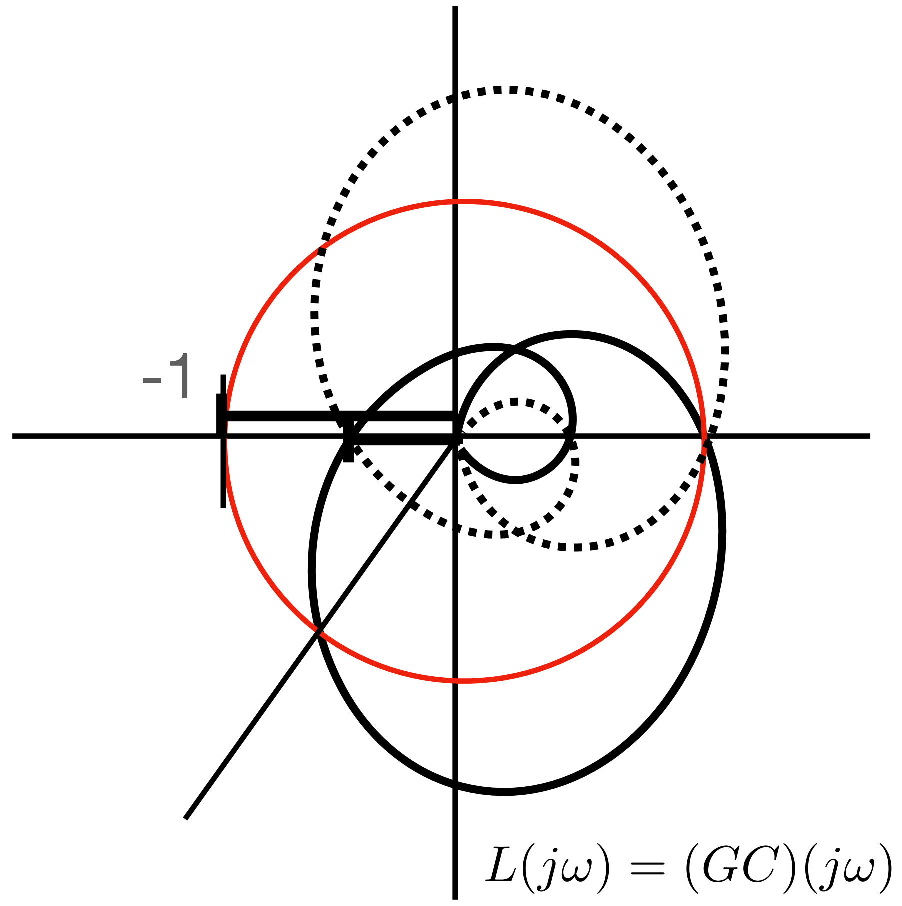


Sensitivity	$S = (I + GC)^{-1}$	**Gain Margin**	$\dfrac{1}{\lvert L(j\omega_{180^o})\rvert}$
Complementary Sensitivity	$T = (I + GC)^{-1}GC$	**Phase Margin**	$180 - \measuredangle L(j\omega_{\lvert L\rvert=1})$
… fundamental limitation	$S + T = I$	**Stability Margin**	$\lvert 1 + L\rvert = \lvert 1 + GC\rvert$

SISO Design - Final Value Theorem







Loop Transfer $L = GC = \dfrac{\mathrm{n}_G}{\mathrm{d}_G}\dfrac{\mathrm{n}_C}{\mathrm{d}_C}$ $\quad G = \dfrac{\mathrm{n}_G}{\mathrm{d}_G} \quad C = \dfrac{\mathrm{n}_C}{\mathrm{d}_C}$

…causal $\mathrm{d}_G, \mathrm{d}_C$ higher order than… $\mathrm{n}_G, \mathrm{n}_C$

Output $y = GC(r - y - n) + d$

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Error $e = r - y$

$$= \underbrace{(I + GC)^{-1}}_{S}r + \underbrace{(I + GC)^{-1}GC}_{T}n - \underbrace{(I + GC)^{-1}}_{S}d$$

Sensitivity $S = (I + GC)^{-1}$

Complementary Sensitivity $T = (I + GC)^{-1}GC$

… fundamental limitation $S + T = I$

SISO:

given

$$\frac{1}{1 + GC} = \frac{1}{1 + \frac{\mathrm{n}_G\mathrm{n}_C}{\mathrm{d}_G\mathrm{d}_C}} = \frac{\mathrm{d}_G\mathrm{d}_C}{\underbrace{\mathrm{d}_G\mathrm{d}_C + \mathrm{n}_G\mathrm{n}_C}_{\text{char poly}}}$$

1. Design for disturbance rejection

2. Design for stability

FVT:

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \lim_{s\to 0}\frac{s^{n_\mathrm{n}} + \cdots + \alpha_k s^k}{s^{n_\mathrm{d}} + \cdots + \alpha_{k'} s^{k'}}$$

$k > k' \quad \rightarrow \quad 0$

$k = k' \quad \rightarrow \quad \frac{\alpha_k}{\alpha_{k'}}$

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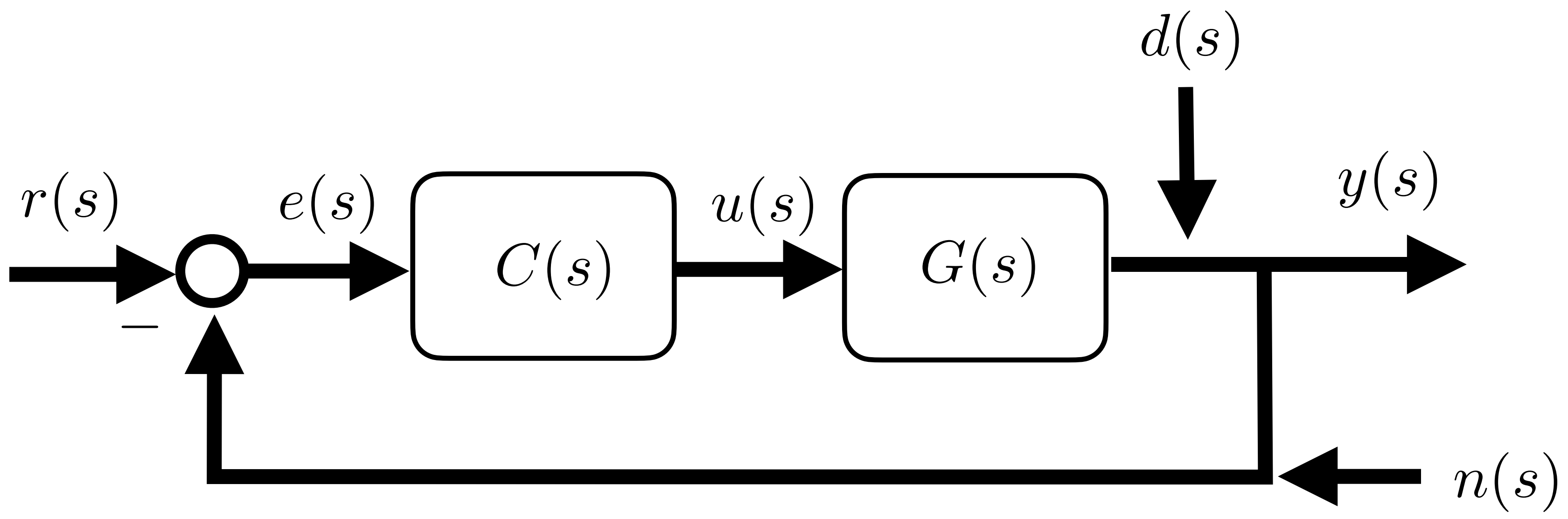
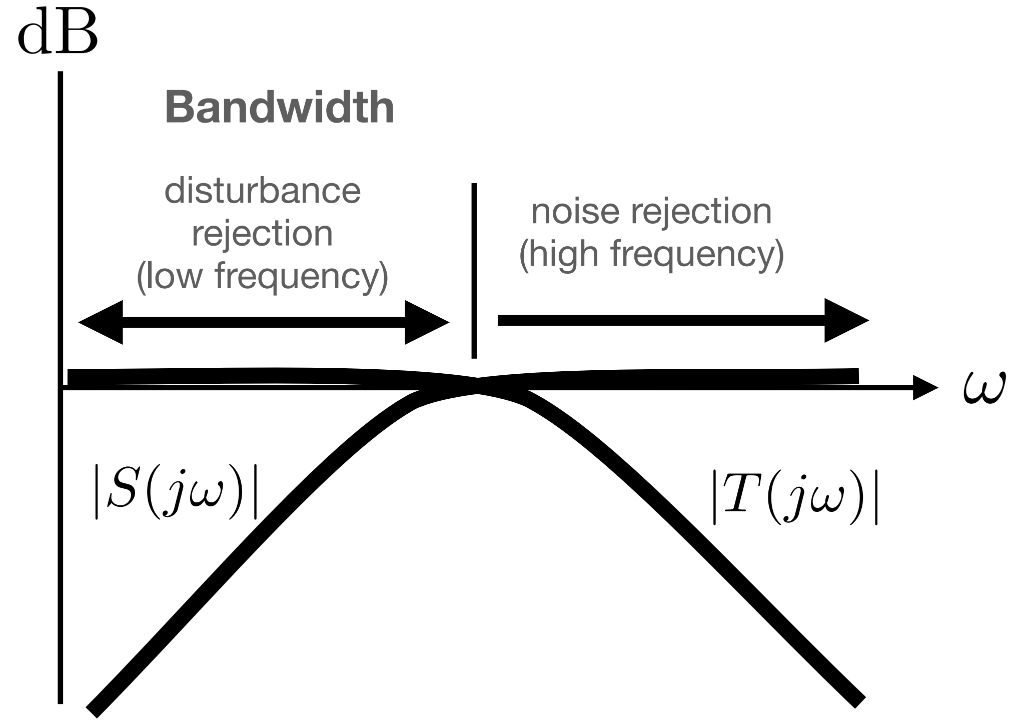
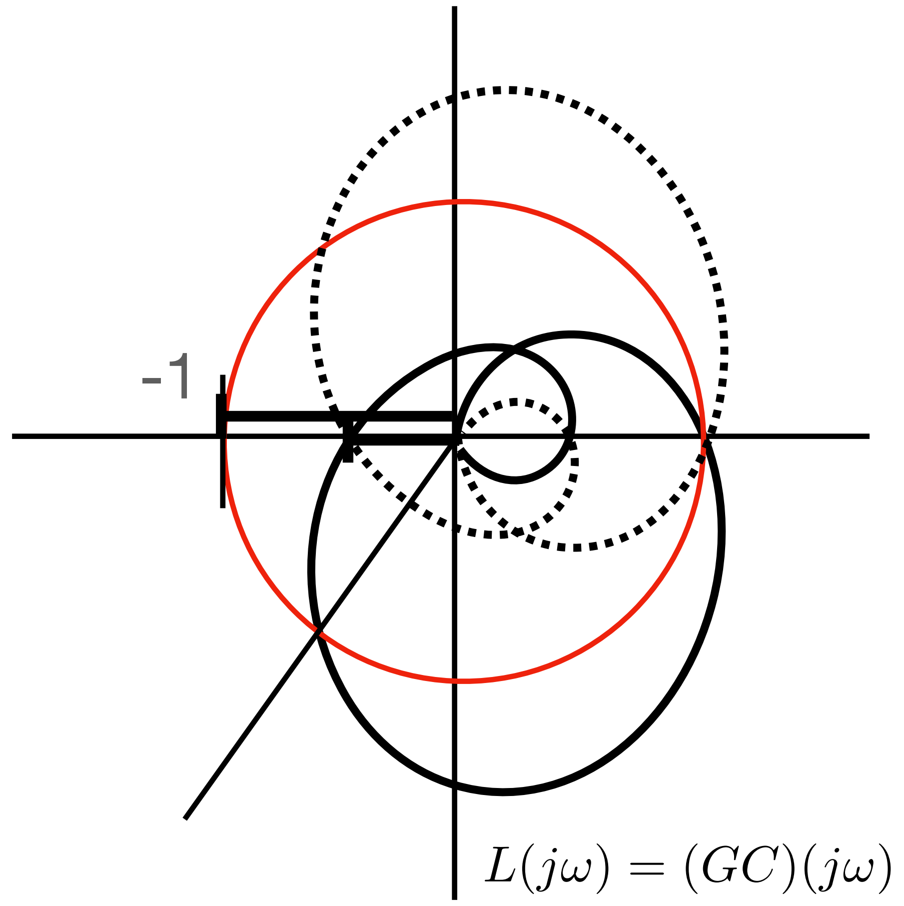
SISO Design - 1. Disturbance Rejection







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given

1. Design for disturbance rejection

2. Design for stability

FVT:

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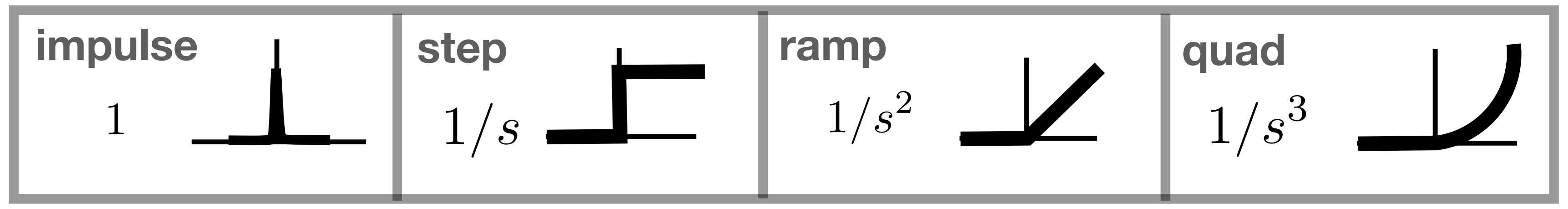
Disturbance types $d(s) = \dfrac{n_d}{d_d}$

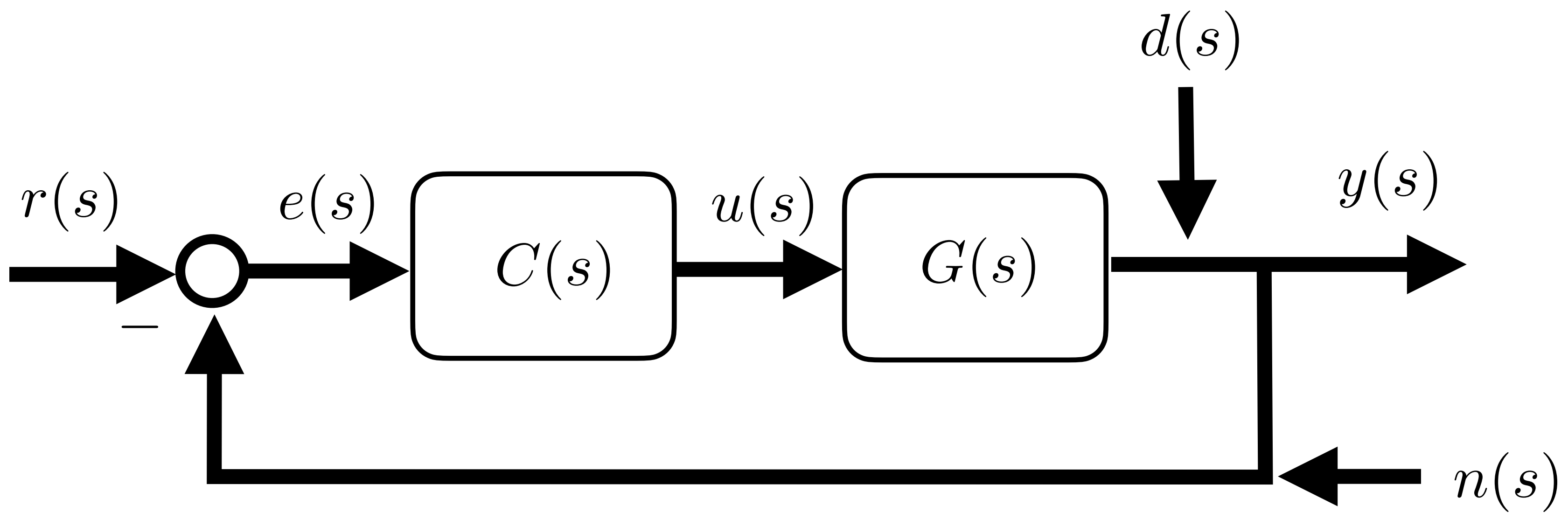


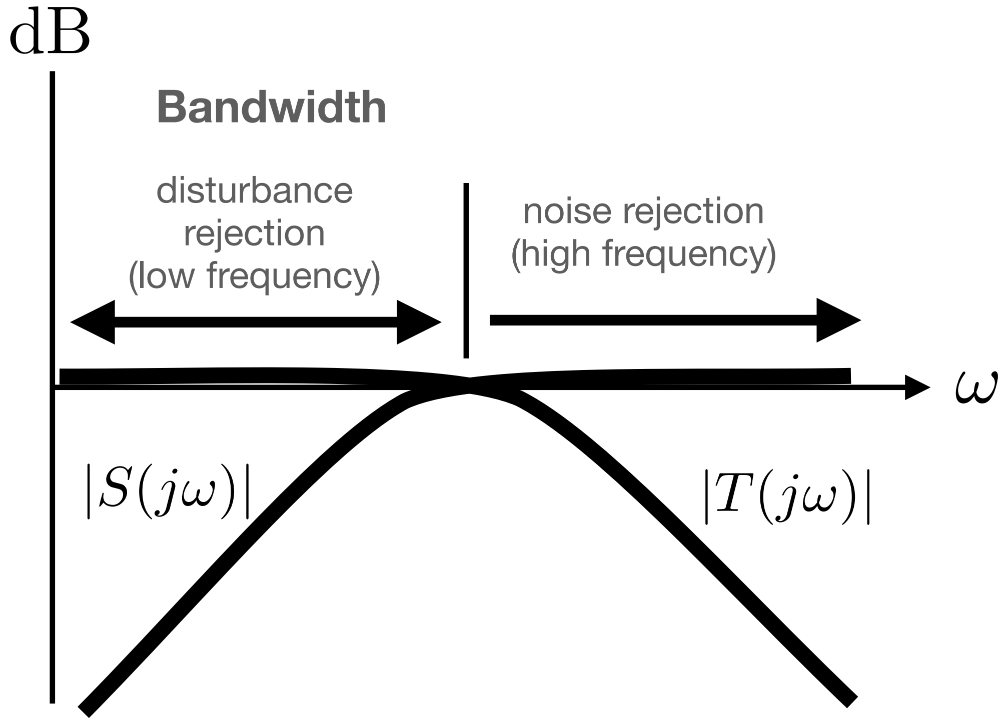
1. Design for disturbance rejection

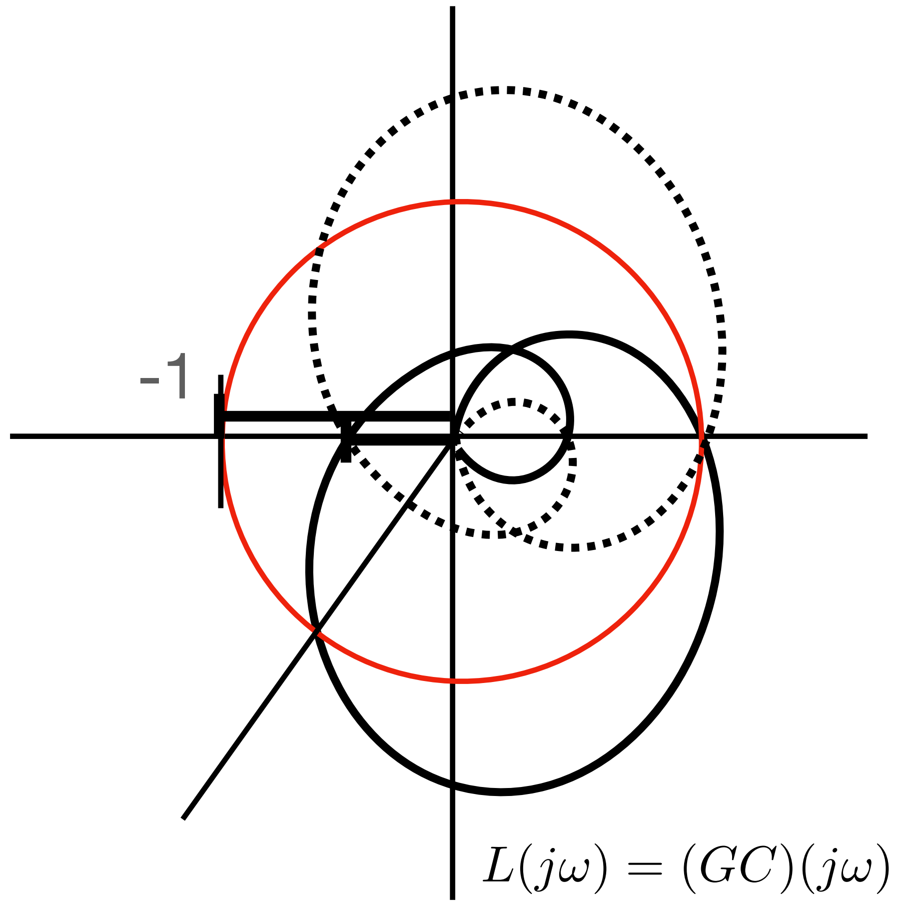
$$\lim_{s\to 0} sS(s)d(s) = \lim_{s\to 0}\frac{s\; d_G d_C \; n_d}{(d_G d_C + n_G n_C)\; d_d}$$

2. Stability

SISO Design - 1. Disturbance Rejection







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…causal $\mathrm{d}_G, \mathrm{d}_C$ higher order than… $\mathrm{n}_G, \mathrm{n}_C$

Output $y = GC(r - y - n) + d$

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given

1. Design for disturbance rejection

2. Design for stability

FVT:

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{s^{n_n} + \cdots + \alpha_k s^k}{s^{n_d} + \cdots + \alpha_{k'} s^{k'}}$$

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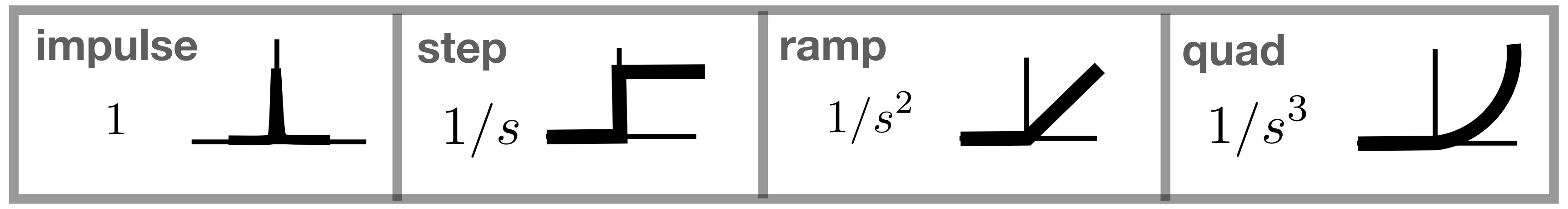


1. Design for disturbance rejection

2. Stability

…probably has constant

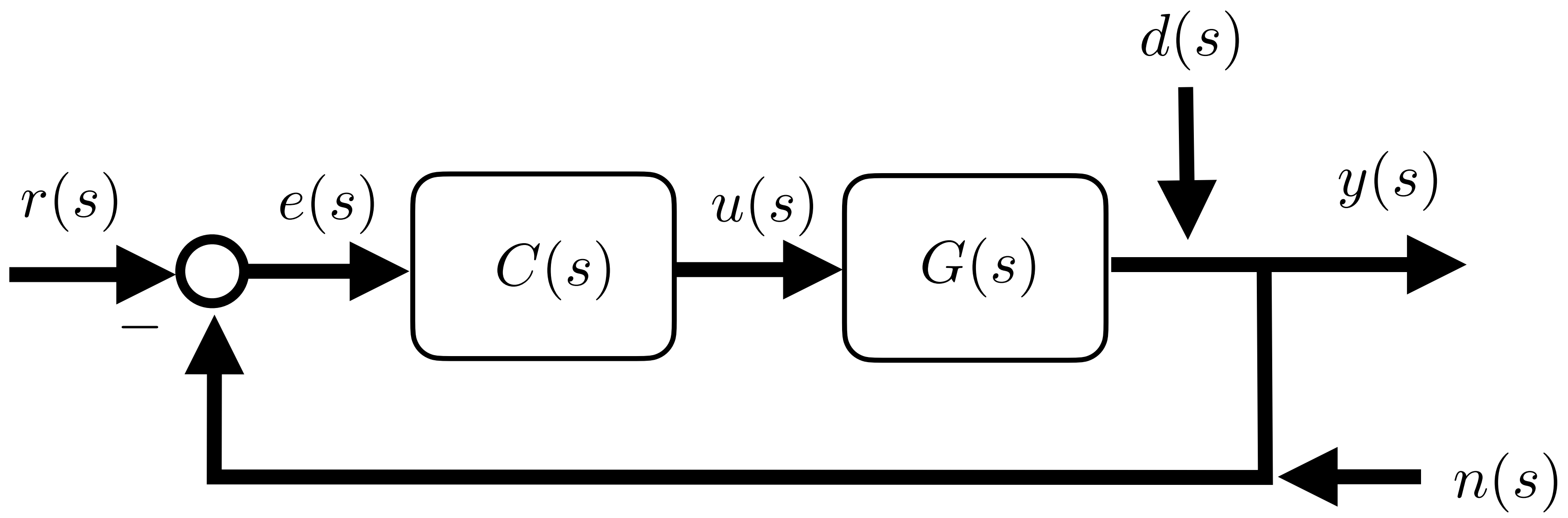
$$\lim_{s\to 0} sS(s)d(s) = \lim_{s\to 0} \frac{s\,\mathrm{d}_G\mathrm{d}_C}{\underbrace{\mathrm{d}_G\mathrm{d}_C}_{\text{higher order}} + \underbrace{\mathrm{n}_G\mathrm{n}_C}_{\text{lower order}}} \frac{\mathrm{n}_d}{\mathrm{d}_d}$$

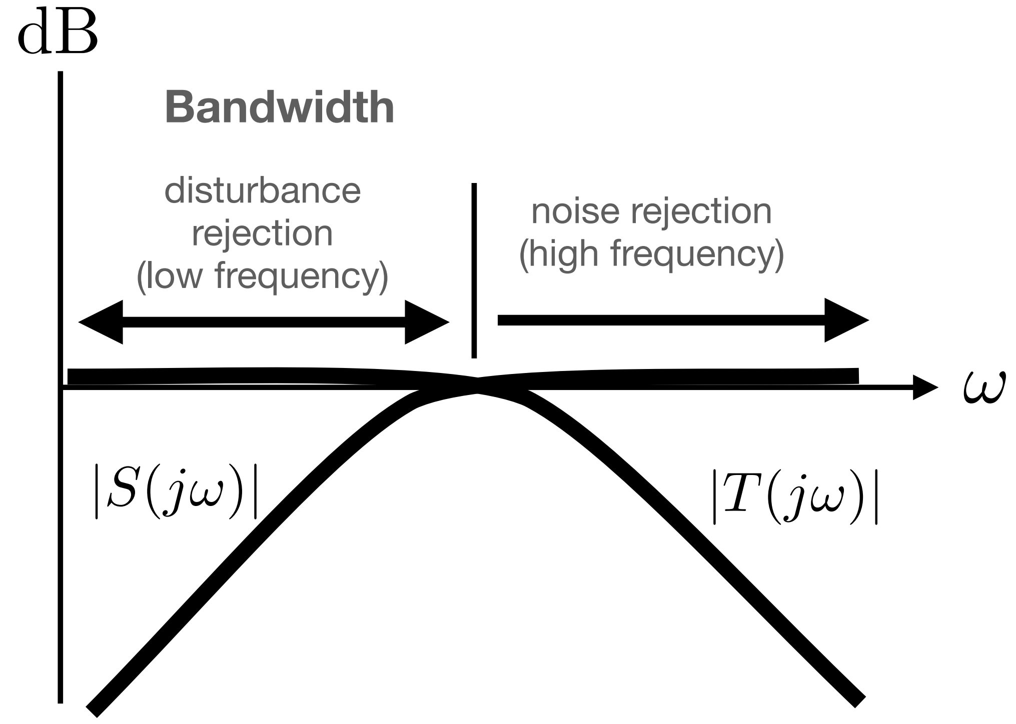
…low order or constant

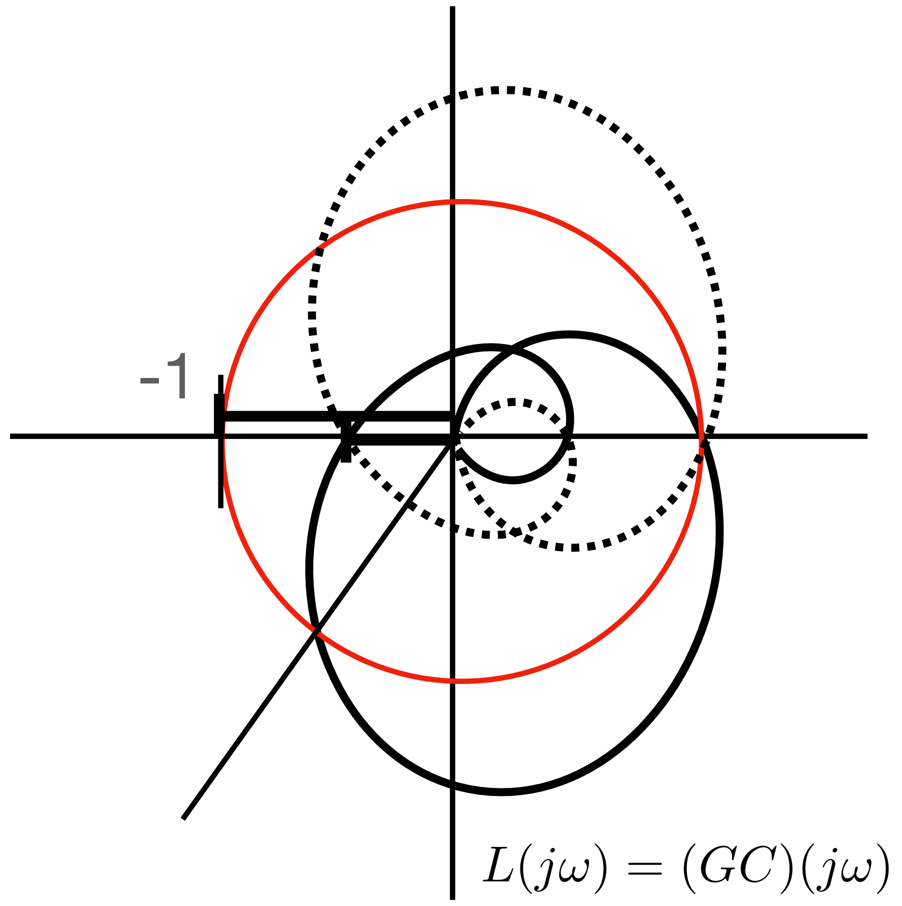
…higher order

…must have constant for stability

SISO Design - Internal Model Principle







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given (d_G)

1. Design for disturbance rejection (numerator)

2. Design for stability (denominator)

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Disturbance types $d(s) = \frac{n_d}{d_d}$

impulse	step	ramp	quad
1	$1/s$	$1/s^2$	$1/s^3$

1. Design for disturbance rejection

2. Stability

…probably has constant

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n_d — …low order or constant

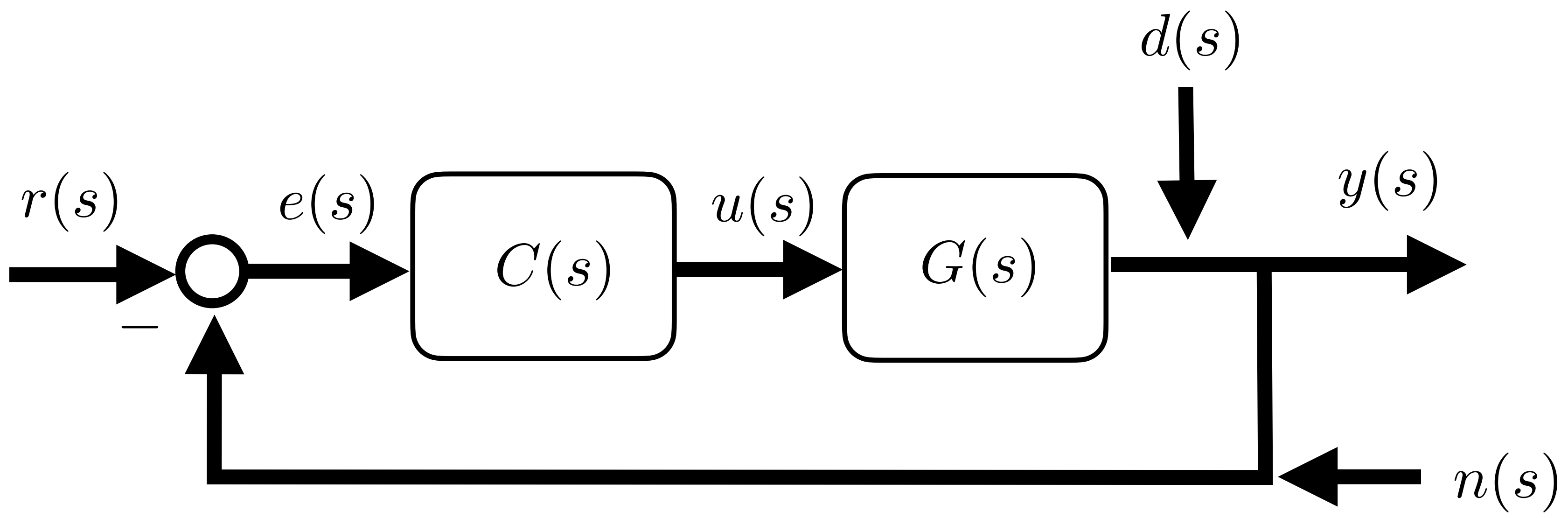
d_d — …higher order

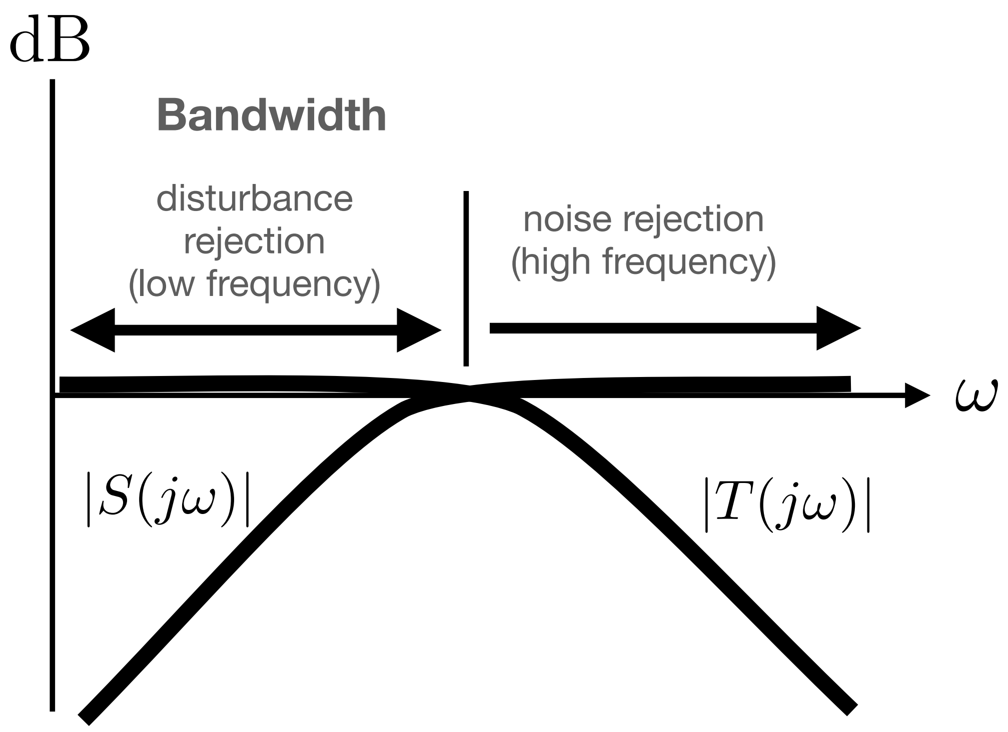
…must have constant for stability

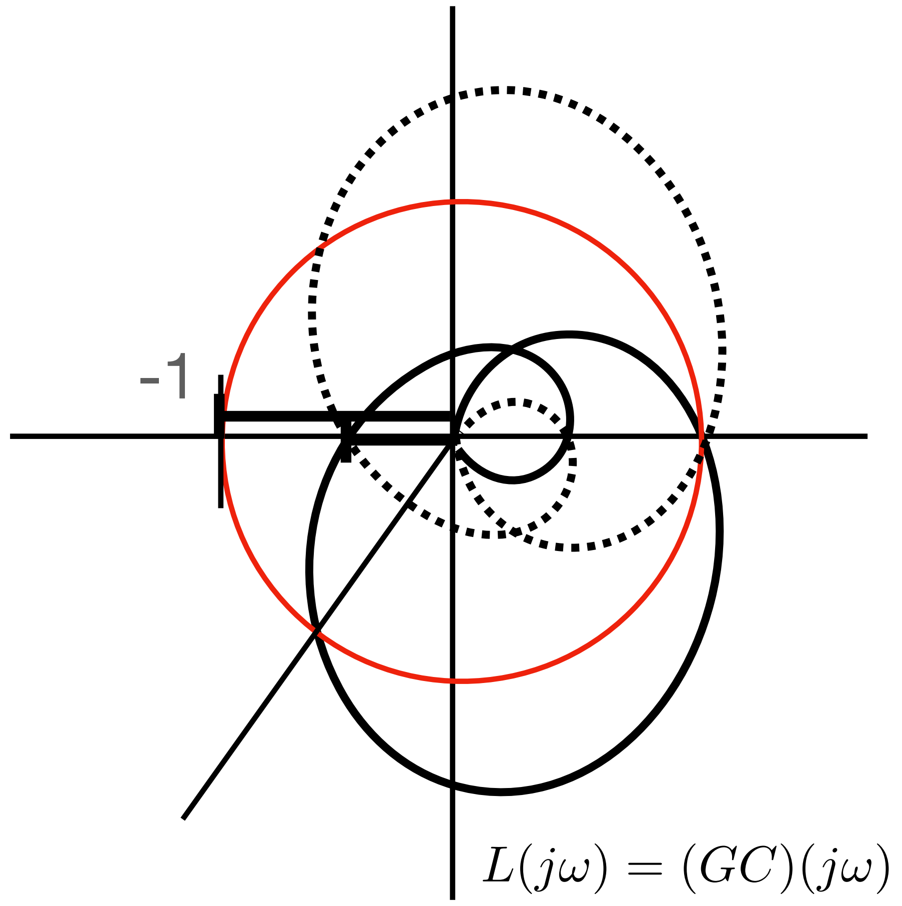
CHOOSE

degree $d_C \geq$ degree d_d

SISO Design - 2. Stability







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given

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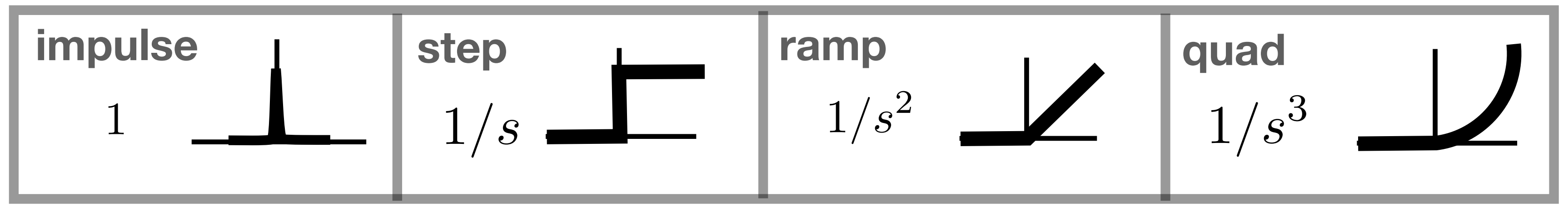


1. Design for disturbance rejection

CONDITION 1:
degree $d_C \geq$ degree d_d

$$\lim_{s\to 0} \frac{s\; d_G d_C}{\underbrace{\underbrace{d_G d_C}_{\text{higher order}} + \underbrace{n_G n_C}_{\text{lower order}}}_{\text{...must be stable}}} \frac{n_d}{d_d}$$

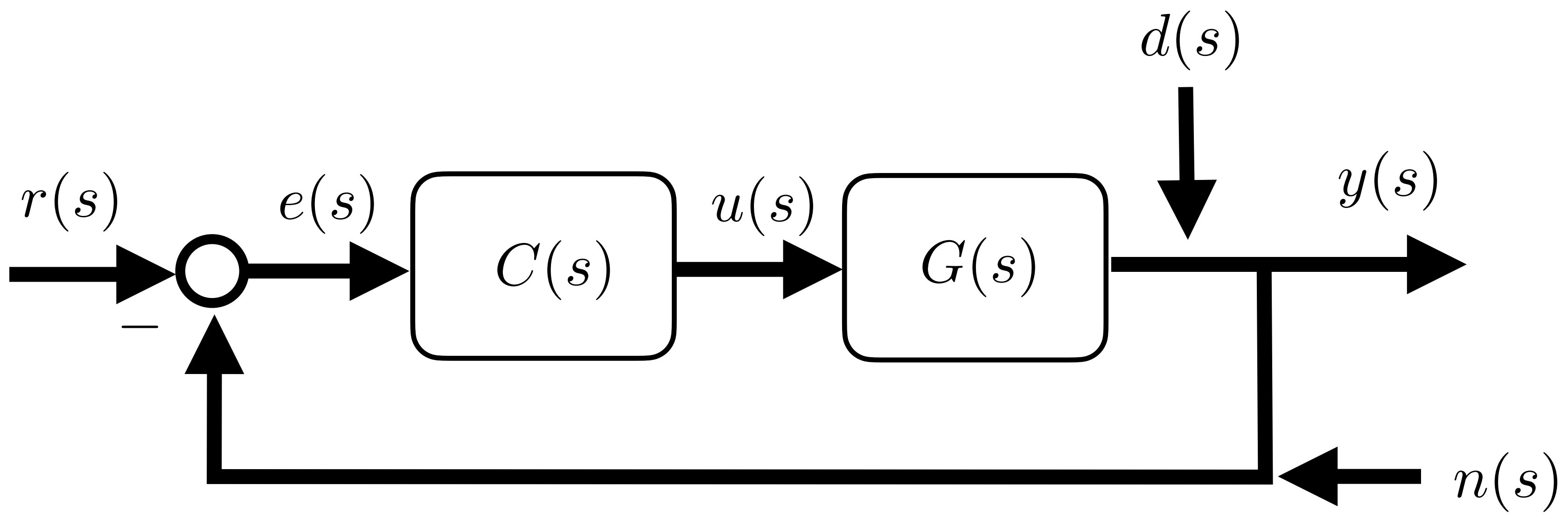
2. Stability

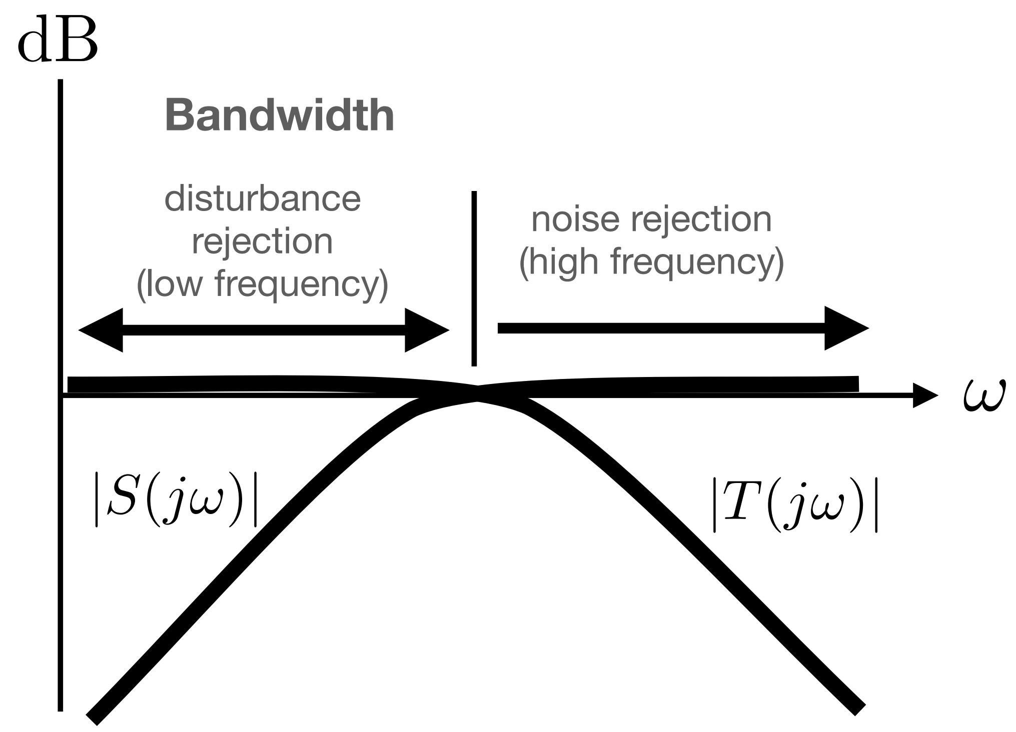
CHOOSE n_C

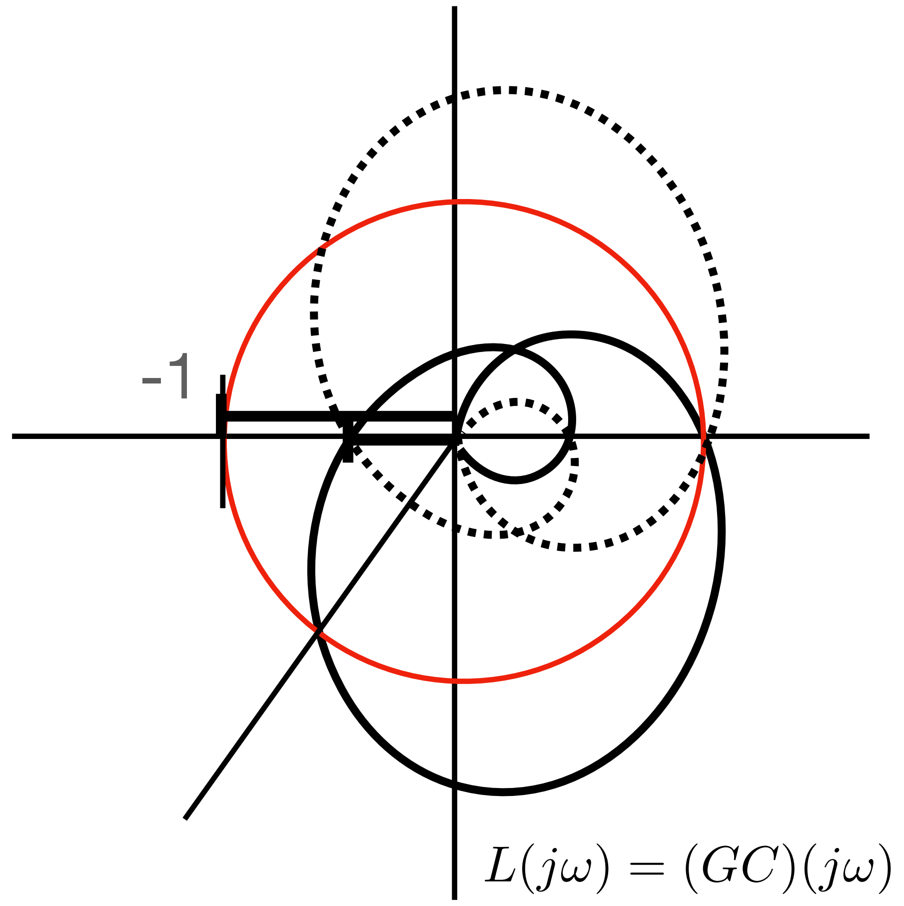
$d_G d_C + n_G n_C$ **stable**

- **work backwards from desired roots**
- **Routh-Hurwitz**
- **Root-locus**

SISO Design - 2. Stability







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given

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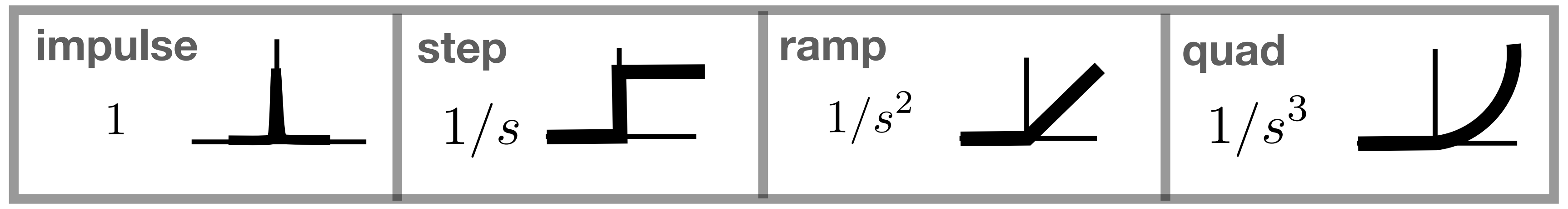

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degree $d_C \geq$ degree d_d

$$\lim_{s \to 0} \frac{s\, d_G d_C}{d_G d_C + n_G n_C} \frac{n_d}{d_d}$$

higher order — lower order

…must be stable

2. Stability

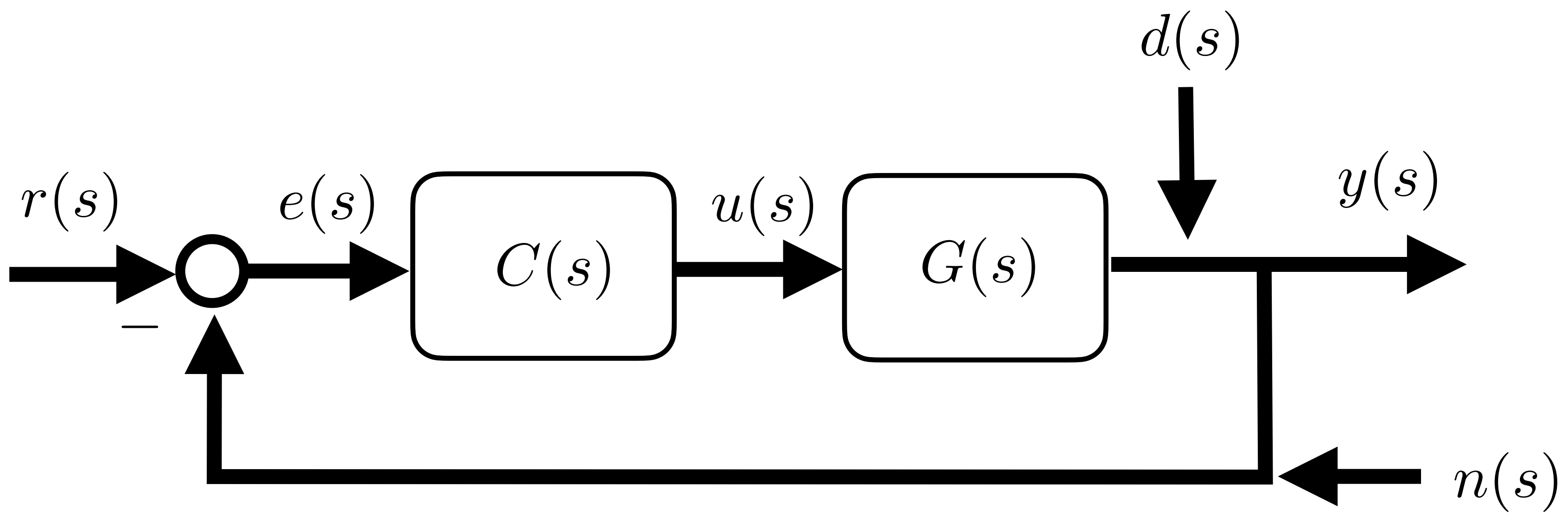
CONDITION 2:
$d_G d_C + n_G n_C$ **stable**

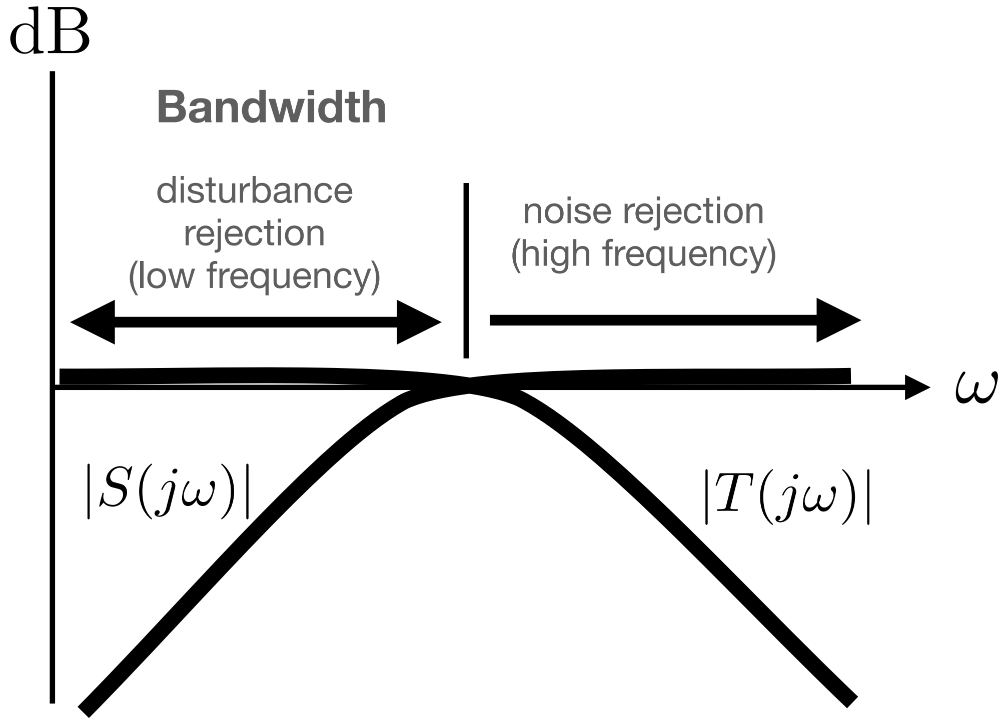
CHOOSE n_C

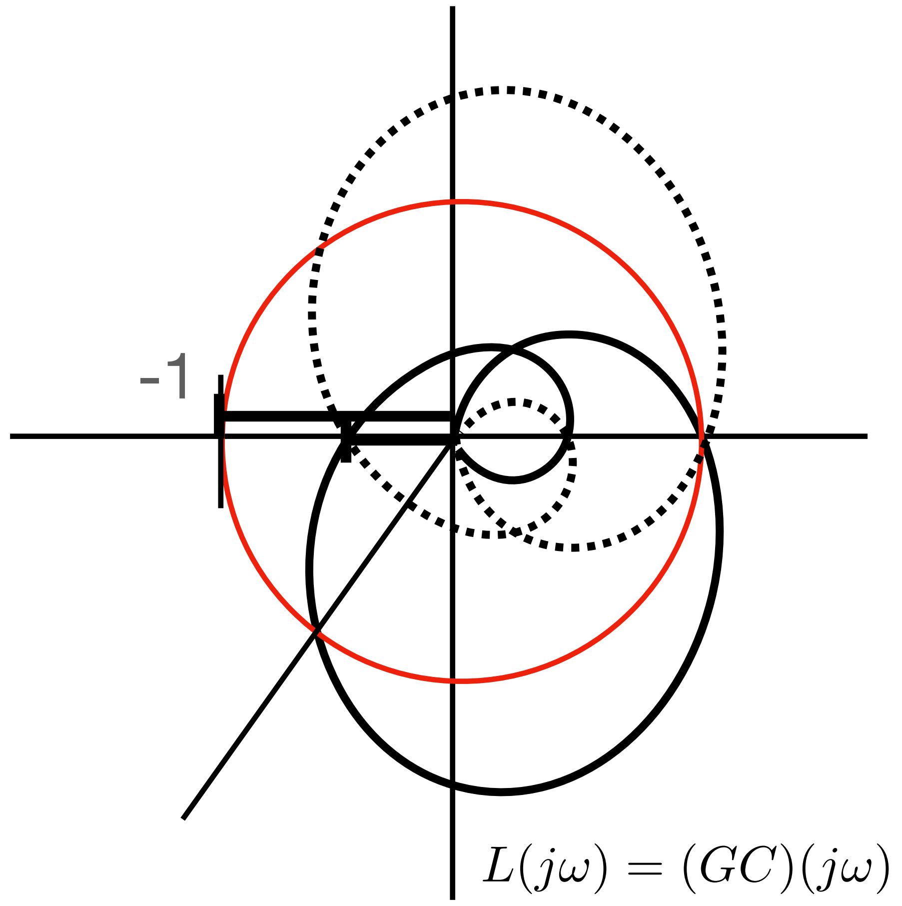
$d_G d_C + n_G n_C$ **stable**

- **work backwards from desired roots**
- **Routh-Hurwitz**
- **Root-locus**

SISO Design - Example







Loop Transfer $L = GC = \frac{n_G}{d_G}\frac{n_C}{d_C}$ $\quad G = \frac{n_G}{d_G} \quad C = \frac{n_C}{d_C}$

…causal d_G, d_C higher order than… n_G, n_C

FVT: $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{s^{n_n} + \cdots + \alpha_k s^k}{s^{n_d} + \cdots + \alpha_{k'} s^{k'}}$

$k > k' \rightarrow 0$

$k = k' \rightarrow \frac{\alpha_k}{\alpha_{k'}}$

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Output $y = \underbrace{(I+GC)^{-1}GC}_{T}(r-n) + \underbrace{(I+GC)^{-1}}_{S} d$

Error $e = \underbrace{(I+GC)^{-1}}_{S} r + \underbrace{(I+GC)^{-1}GC}_{T} n - \underbrace{(I+GC)^{-1}}_{S} d$

Disturbance types $d(s) = \frac{n_d}{d_d}$

impulse	step	ramp	quad
1	$1/s$	$1/s^2$	$1/s^3$

$\lim_{s\to 0} \frac{s\, d_G\, d_C}{d_G d_C + n_G n_C} \frac{n_d}{d_d}$

disturbance…

1. Disturbance rejection

CONDITION 1:
degree $d_C \geq$ degree d_d

2. Stability

CONDITION 2:
$d_G d_C + n_G n_C$ stable

Plant:

$G(s) = \frac{n_G}{d_G} =$

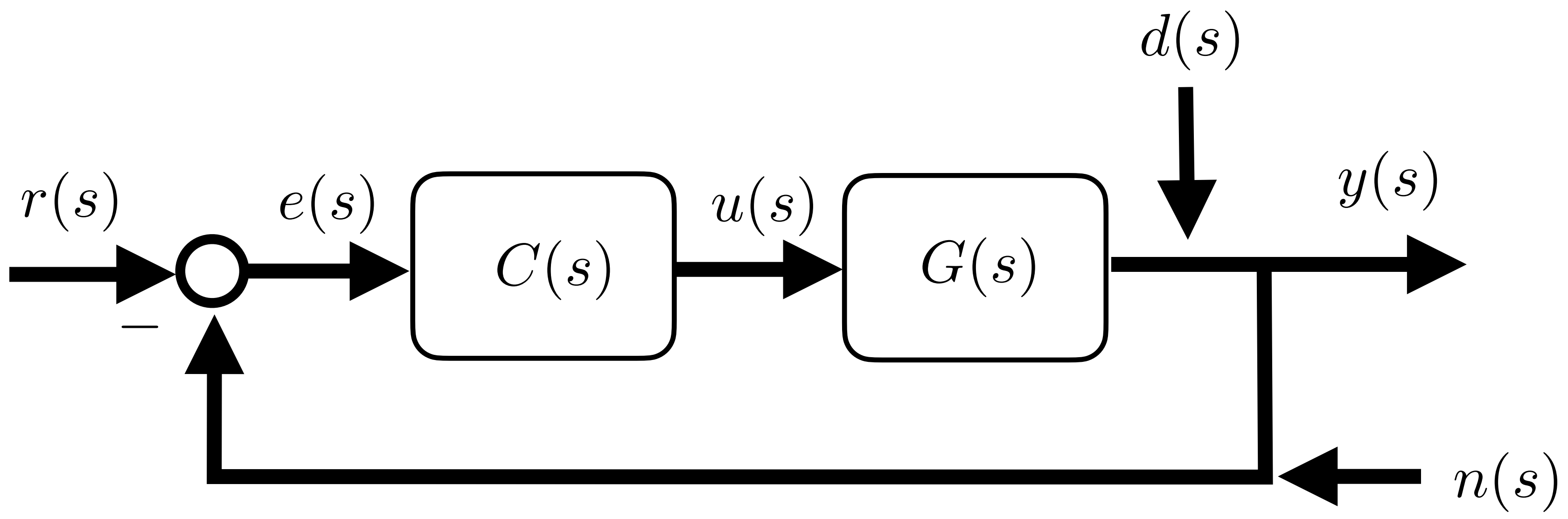
Controller:

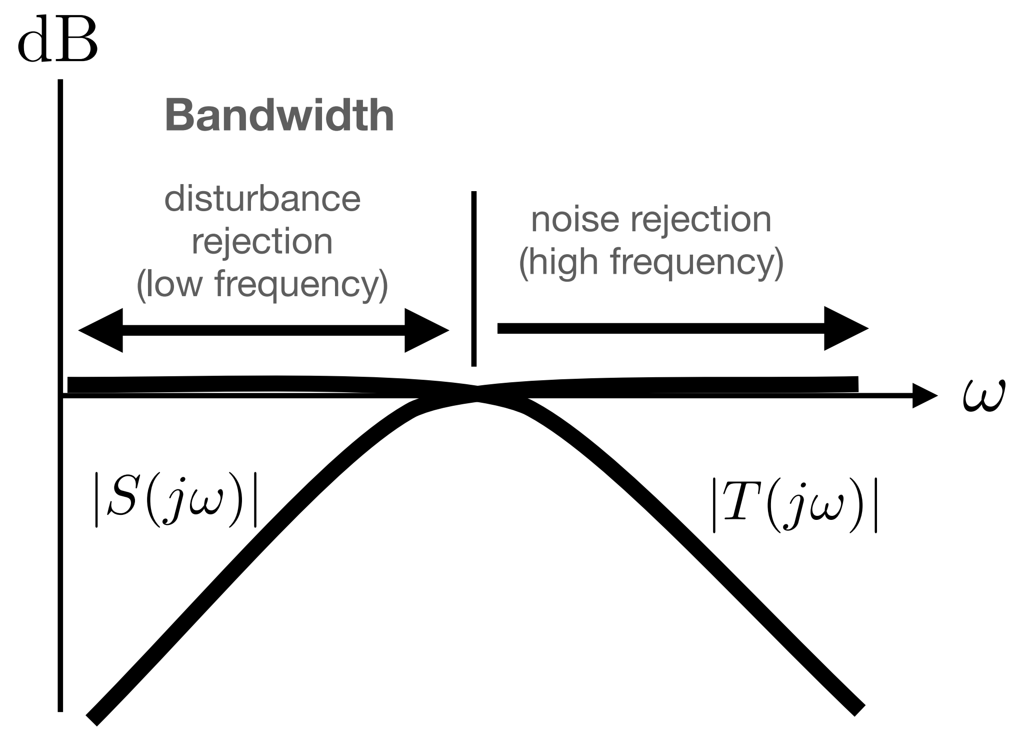
$C(s) = \frac{n_C}{d_C} =$

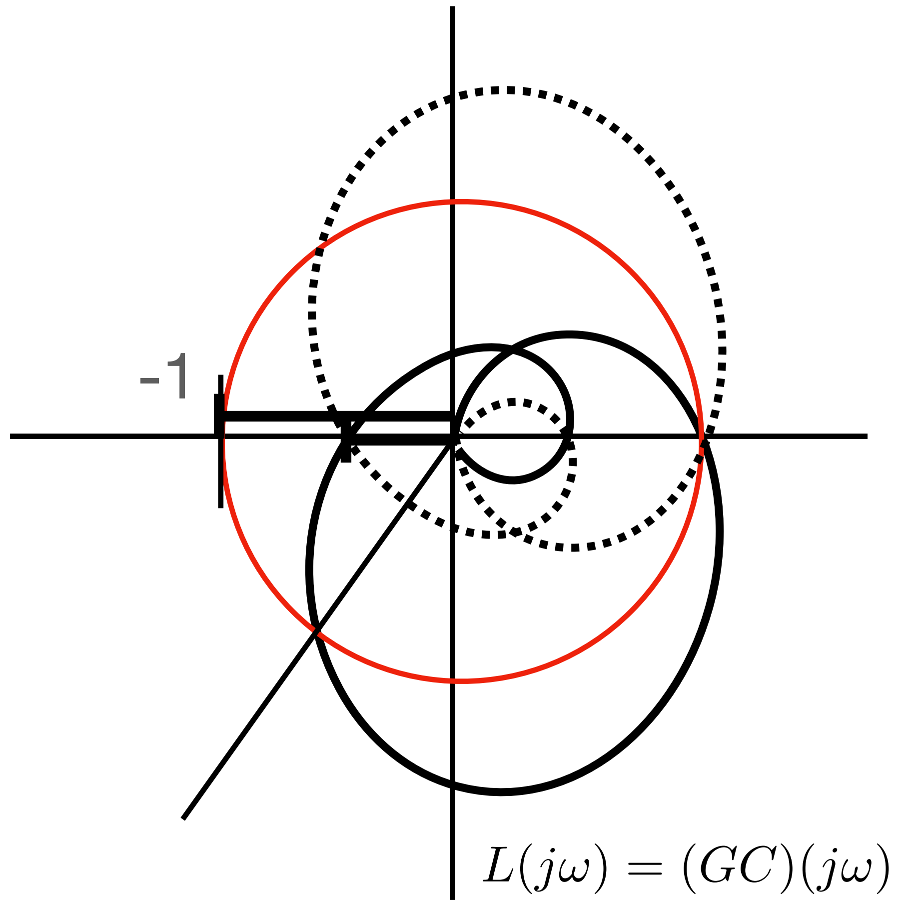
disturbance rejection… $d_C =$

stability… $n_C =$

SISO Design - Example







Loop Transfer $L = GC = \frac{\mathrm{n}_G}{\mathrm{d}_G}\frac{\mathrm{n}_C}{\mathrm{d}_C}$ $\quad G = \frac{\mathrm{n}_G}{\mathrm{d}_G} \quad C = \frac{\mathrm{n}_C}{\mathrm{d}_C}$

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impulse	step	ramp	quad
1	$1/s$	$1/s^2$	$1/s^3$

$$\lim_{s\to 0} \frac{s\,(s^2 + 2\zeta\omega_n s + \omega_n^2)\,\mathrm{d}_C}{(s^2 + 2\zeta\omega_n s + \omega_n^2)\,\mathrm{d}_C + 1\,\mathrm{n}_C} \frac{\mathrm{n}_d}{\mathrm{d}_d}$$

disturbance…

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CONDITION 1:
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2. Stability

CONDITION 2:
$\mathrm{d}_G\mathrm{d}_C + \mathrm{n}_G\mathrm{n}_C$ stable

Plant: **oscillator…**

$G(s) = \frac{\mathrm{n}_G}{\mathrm{d}_G} = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

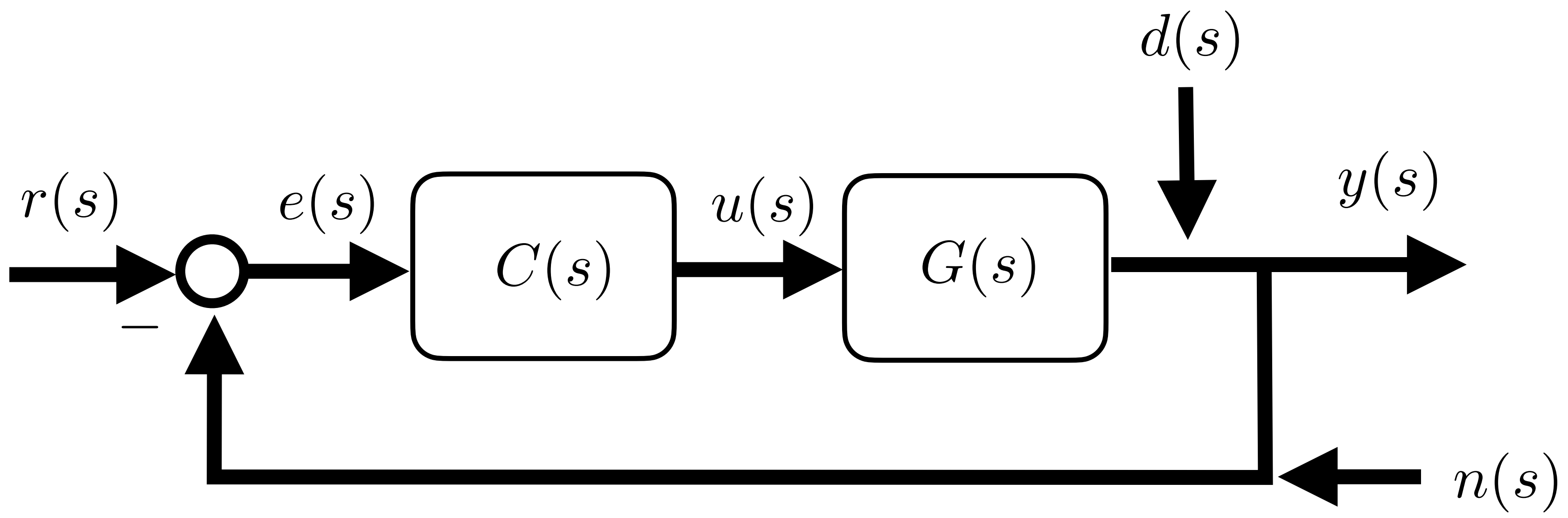
Controller:

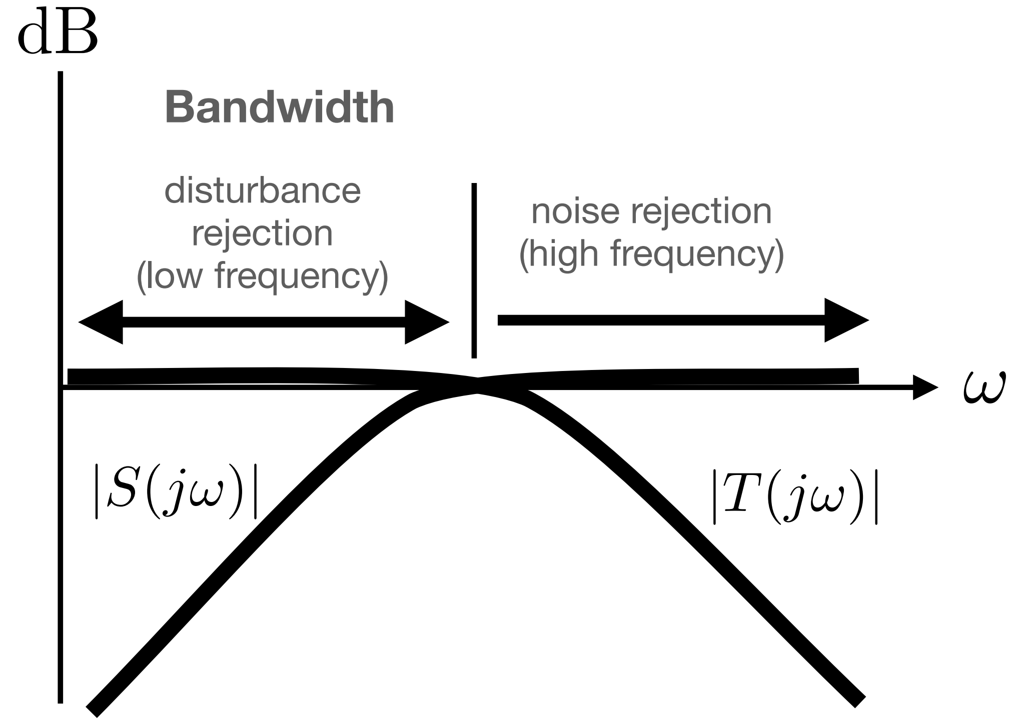
$C(s) = \frac{\mathrm{n}_C}{\mathrm{d}_C} =$

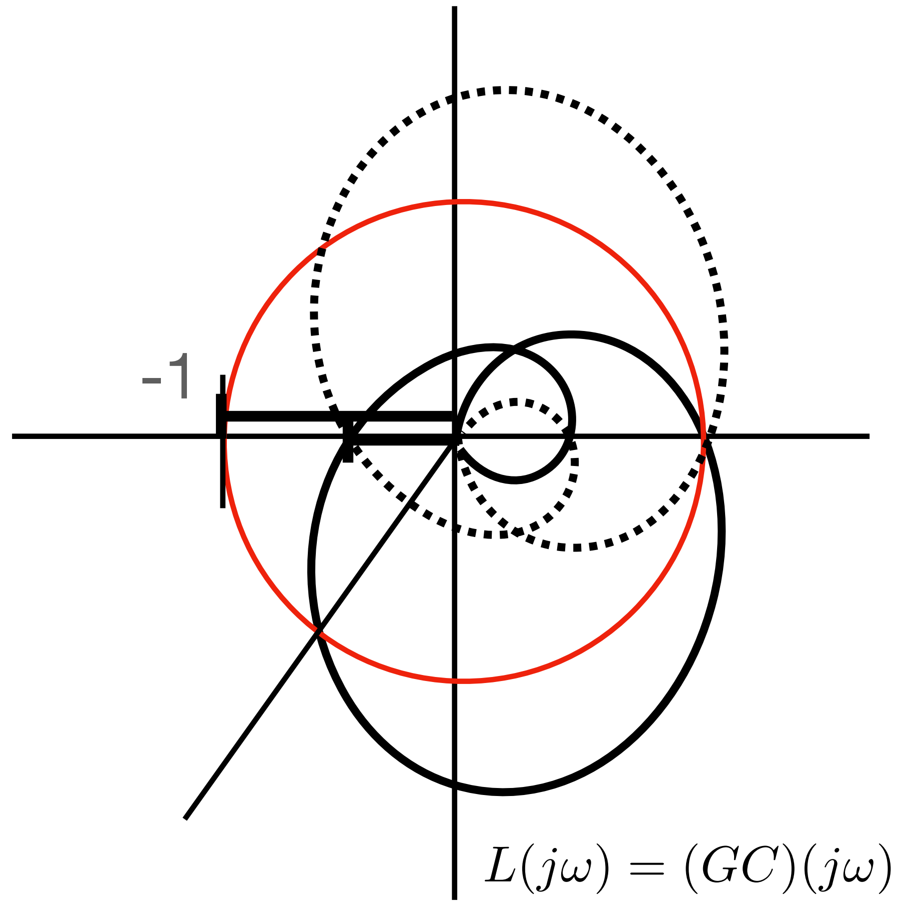
disturbance rejection… $\mathrm{d}_C =$

stability… $\mathrm{n}_C =$

SISO Design - Example







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Disturbance types $\quad d(s) = \dfrac{\mathrm{n}_d}{\mathrm{d}_d}$

impulse	step	ramp	quad
1	$1/s$	$1/s^2$	$1/s^3$

$$\lim_{s\to 0} \frac{s\,(s^2 + 2\zeta\omega_n s + \omega_n^2)\,\mathrm{d}_C}{(s^2 + 2\zeta\omega_n s + \omega_n^2)\,\mathrm{d}_C + 1\,\mathrm{n}_C}\;\frac{1}{1}$$

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Plant: **oscillator…**

$$G(s) = \frac{\mathrm{n}_G}{\mathrm{d}_G} = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Controller:

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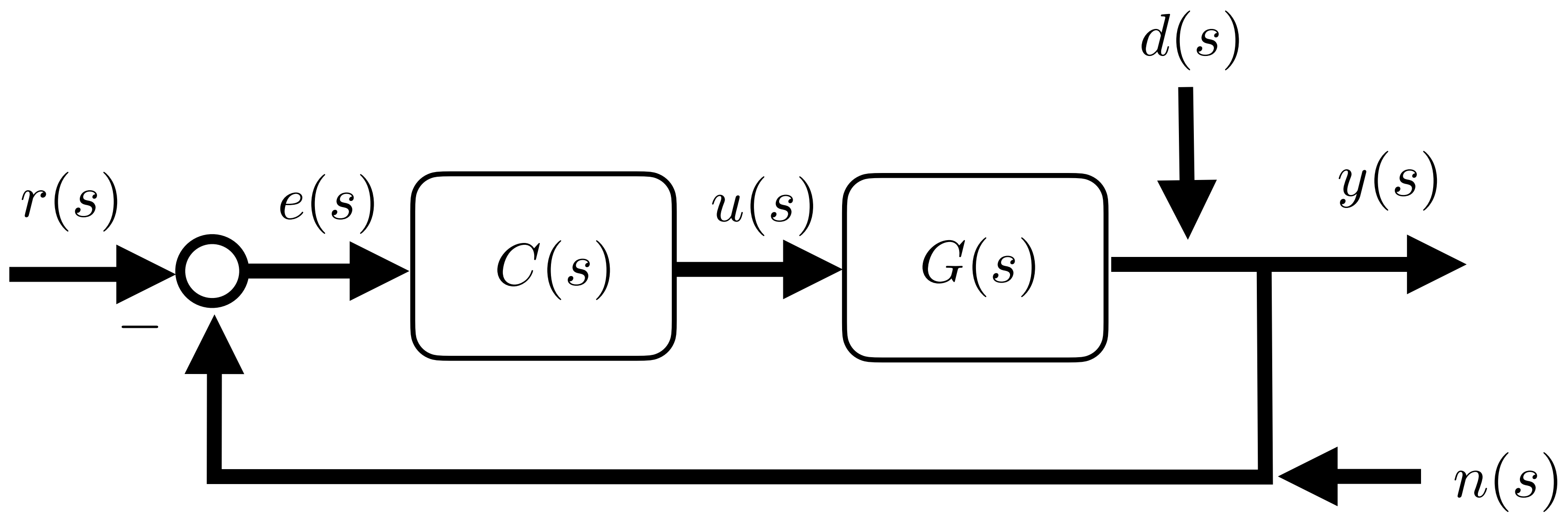
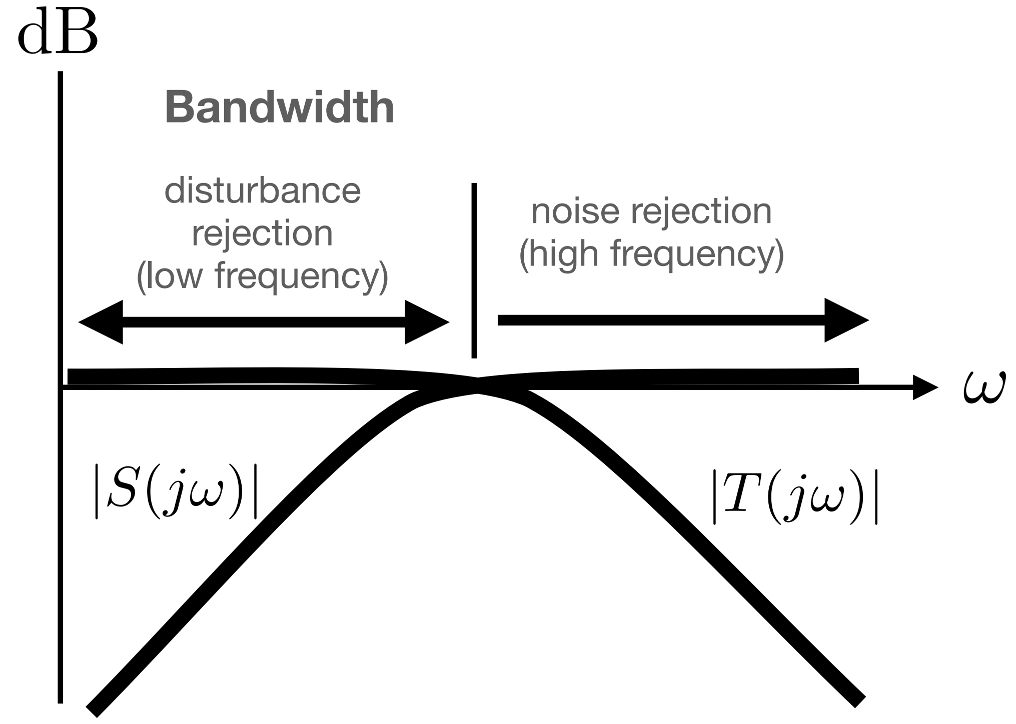
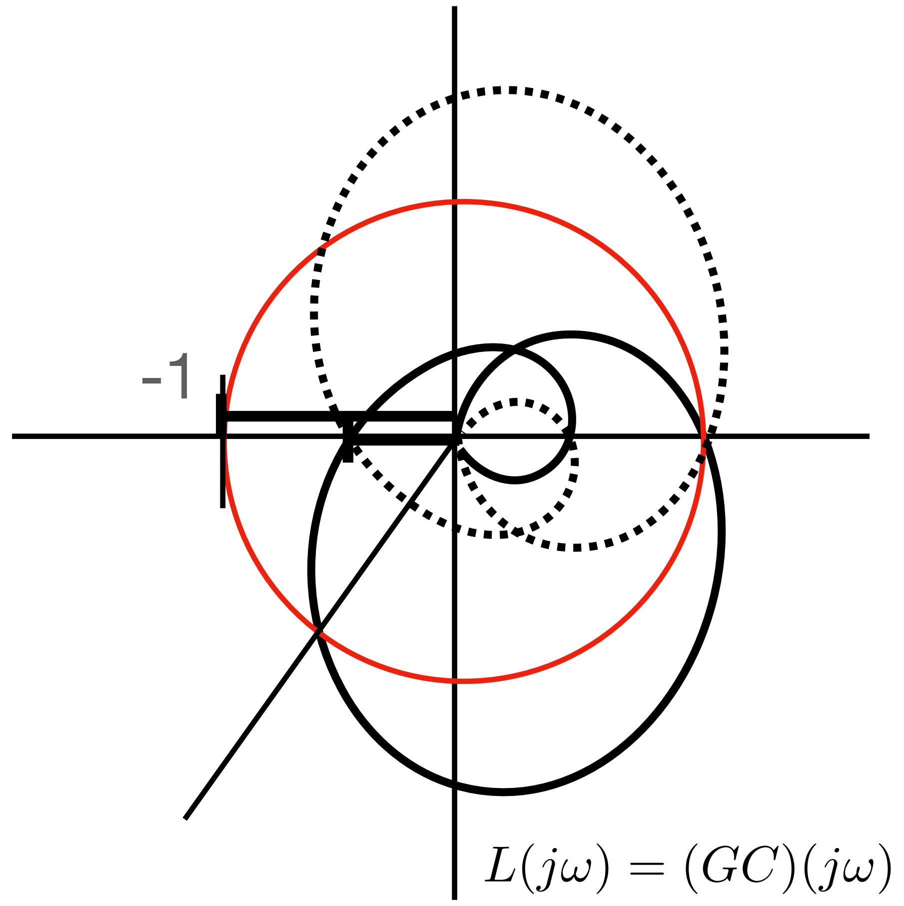
SISO Design - Example







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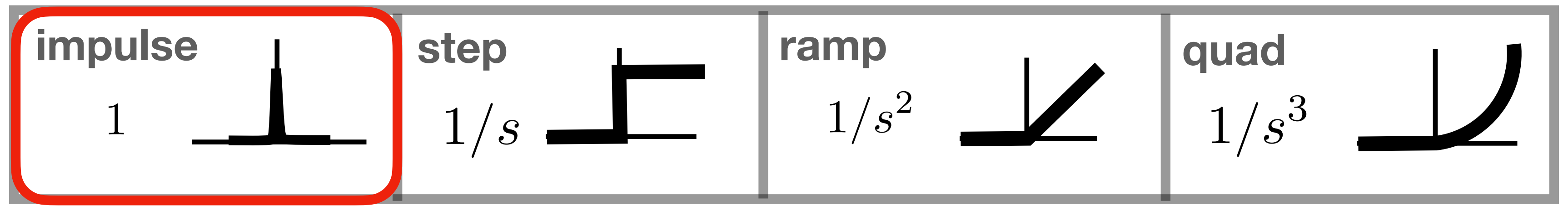

$$\lim_{s\to 0} \frac{s\,(s^2 + 2\zeta\omega_n s + \omega_n^2)\,1}{(s^2 + 2\zeta\omega_n s + \omega_n^2)\,1 + 1\,K_p}\;\frac{1}{1}$$

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Controller:

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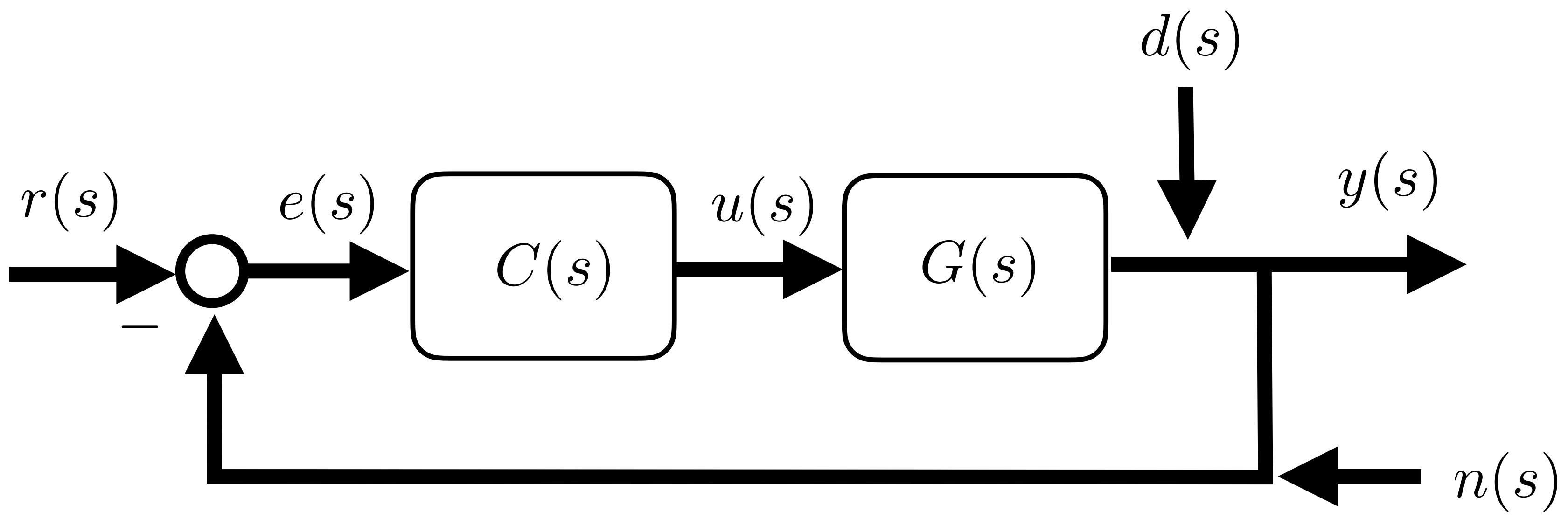
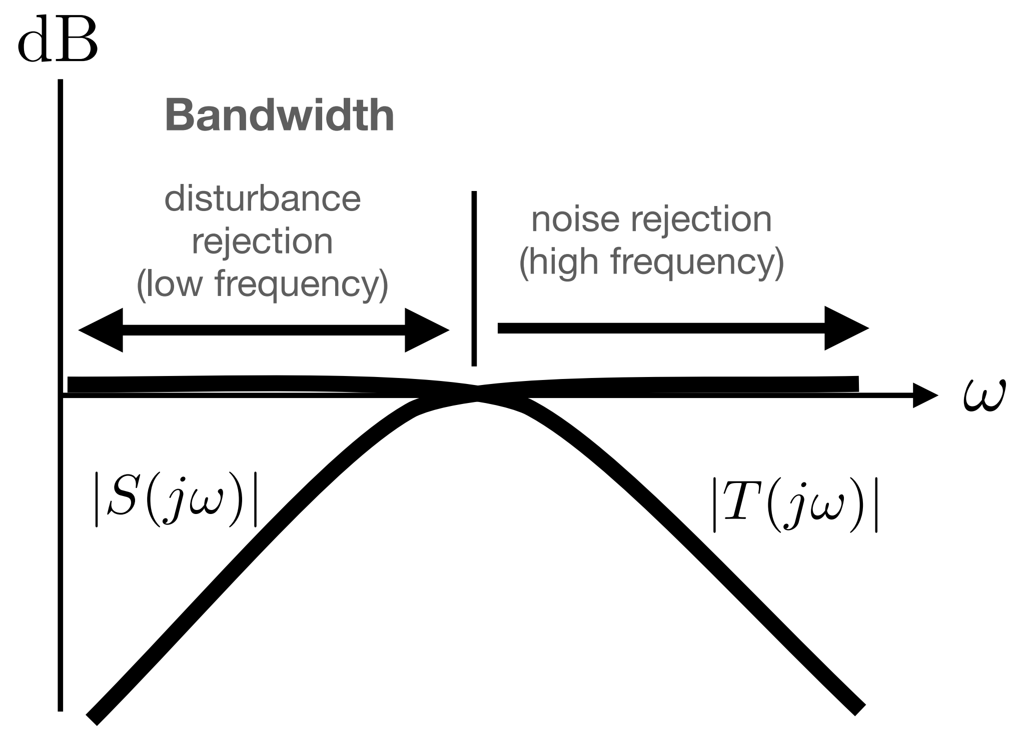
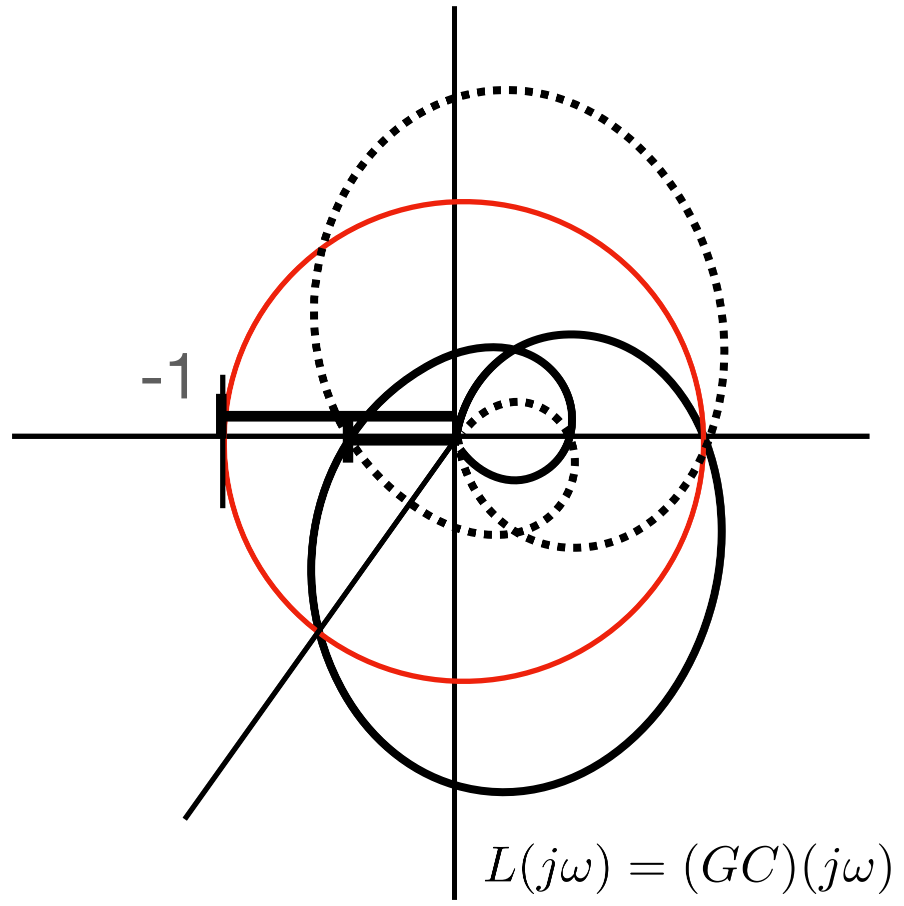
disturbance rejection… $d_C = 1$

stability… $n_C = K_p$

SISO Design - Example







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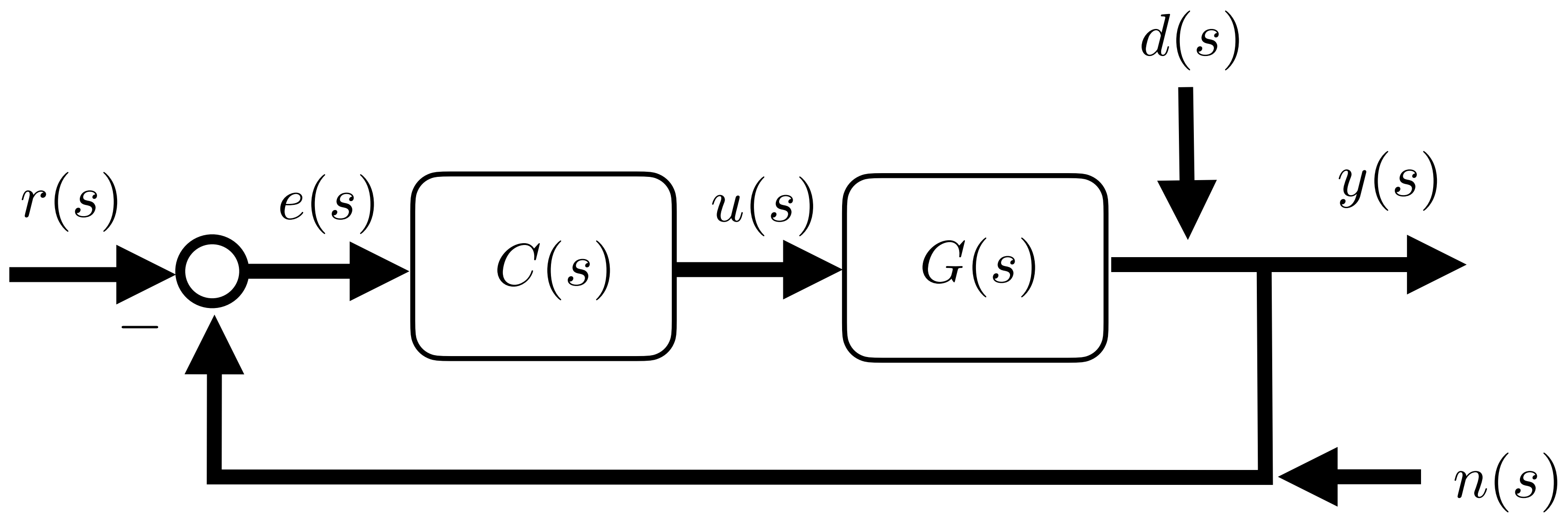
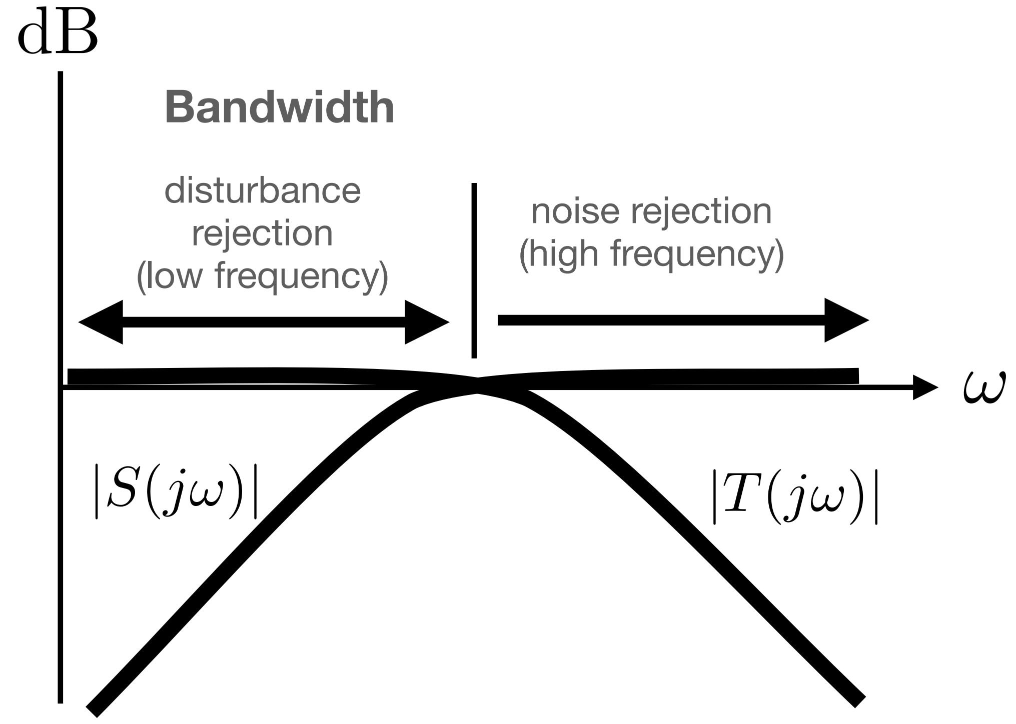
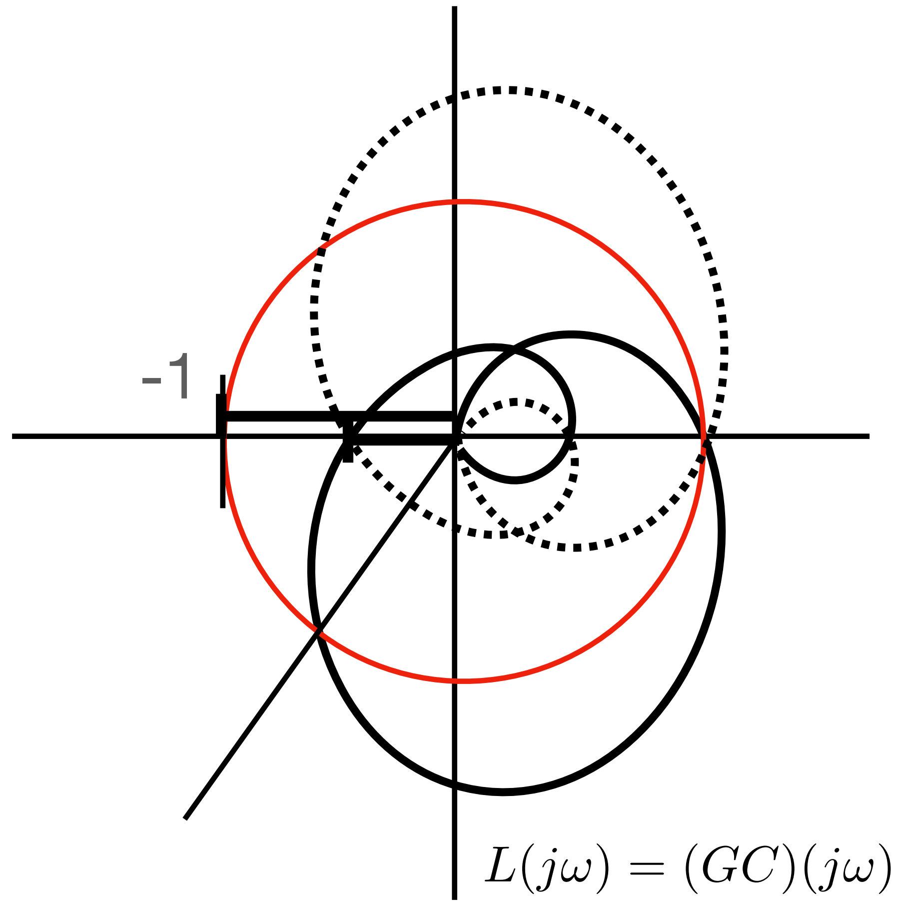
SISO Design - Example







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1	$1/s$	$1/s^2$	$1/s^3$

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Controller:

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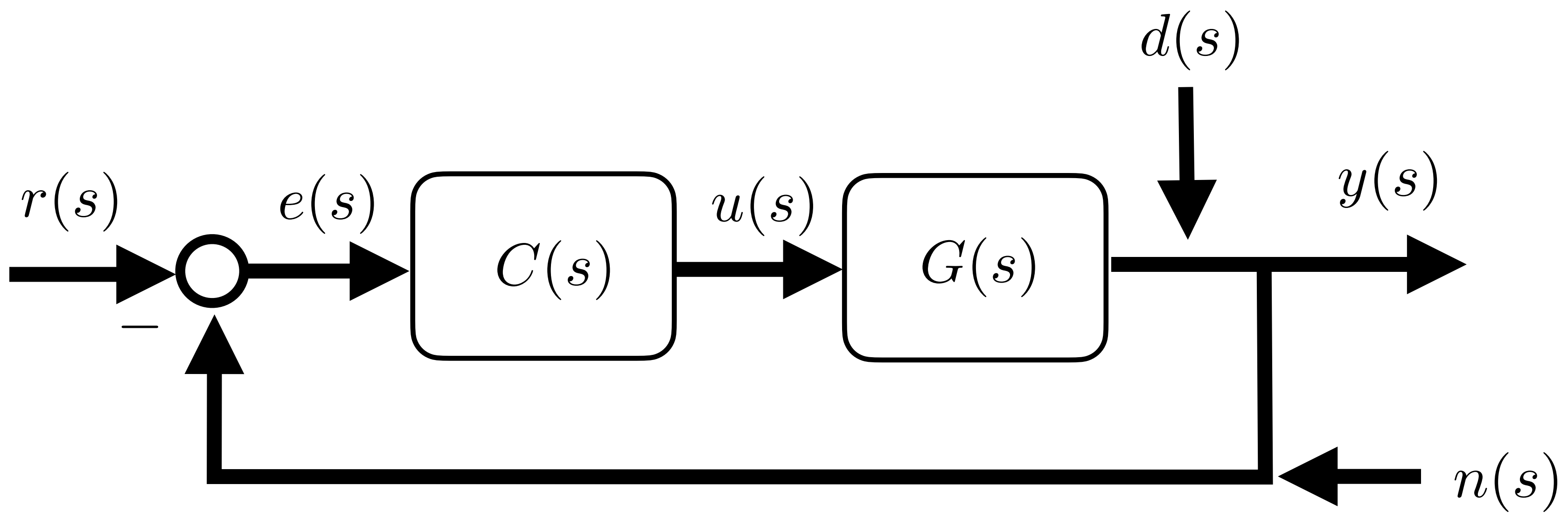
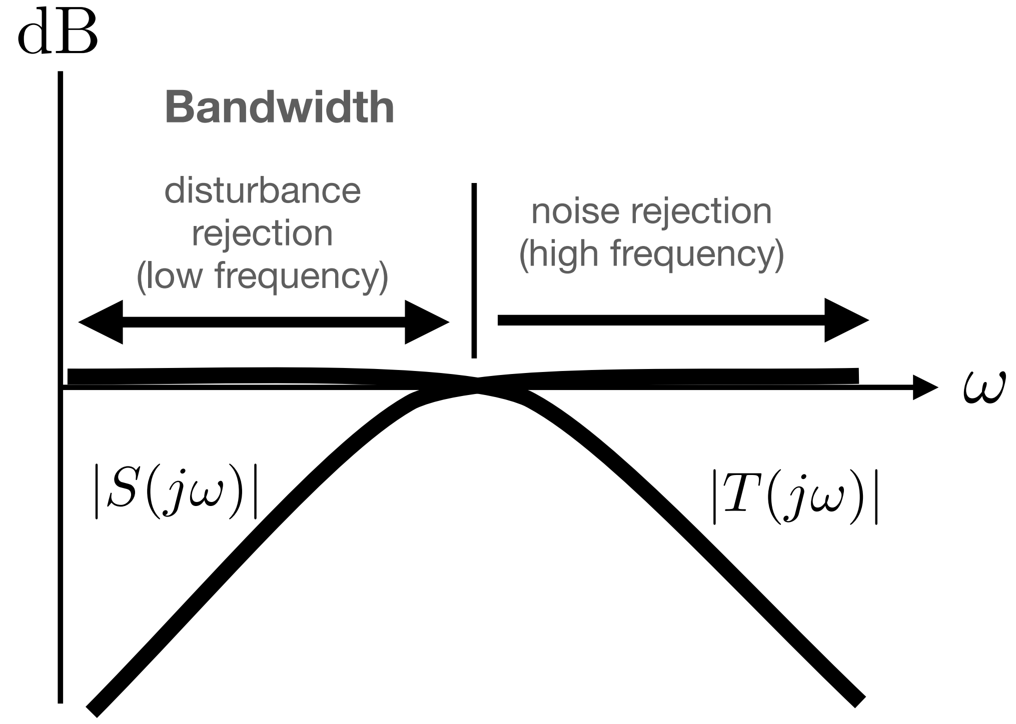
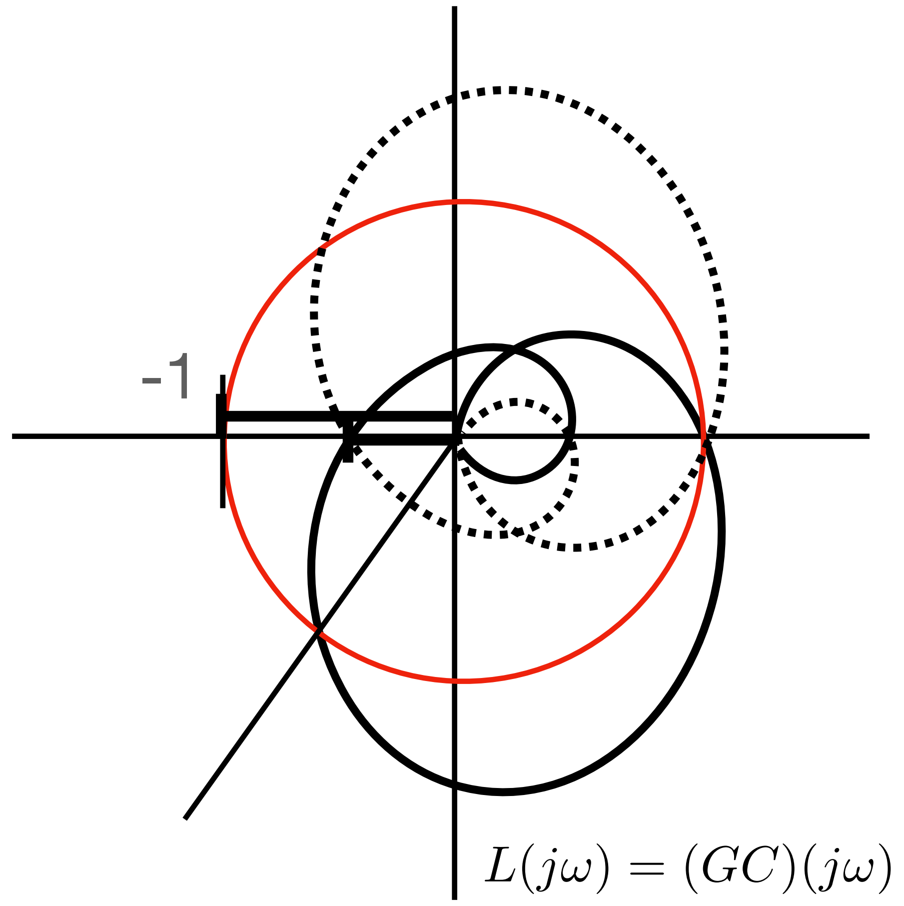
disturbance rejection… $d_C = s$

stability… $n_C = K_p s + K_I$

SISO Design - Example







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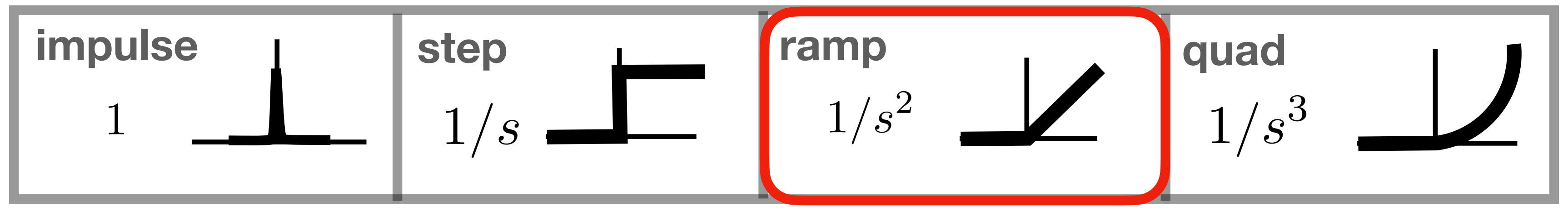

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disturbance rejection… $\quad d_C =$

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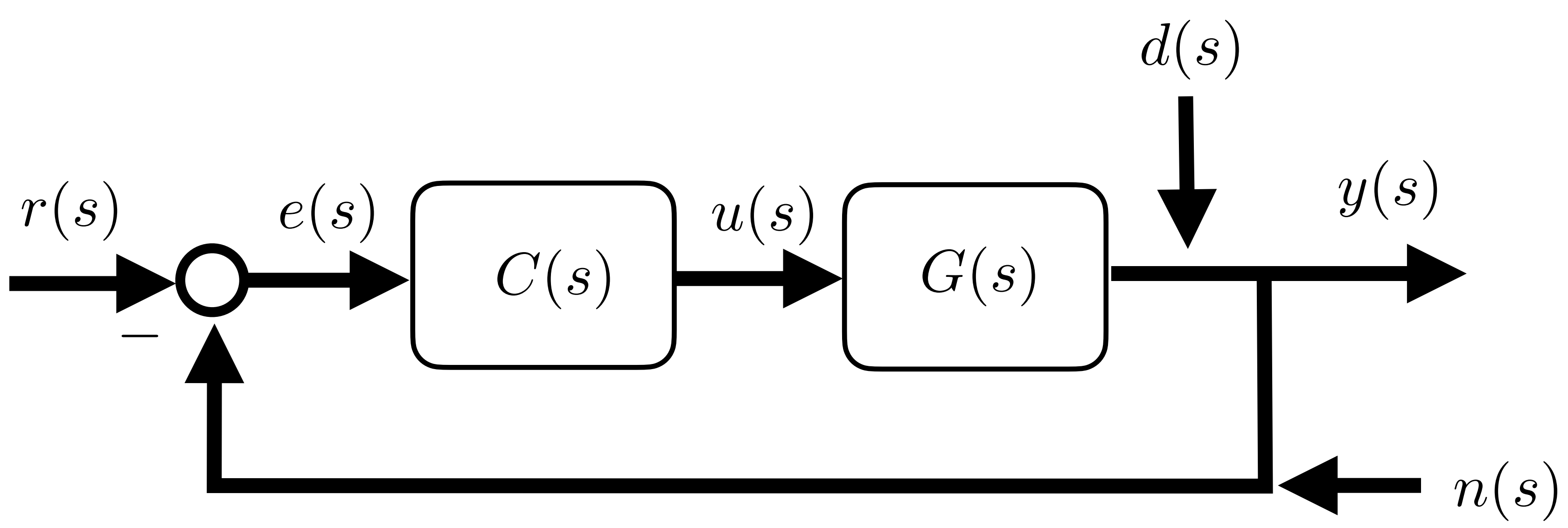
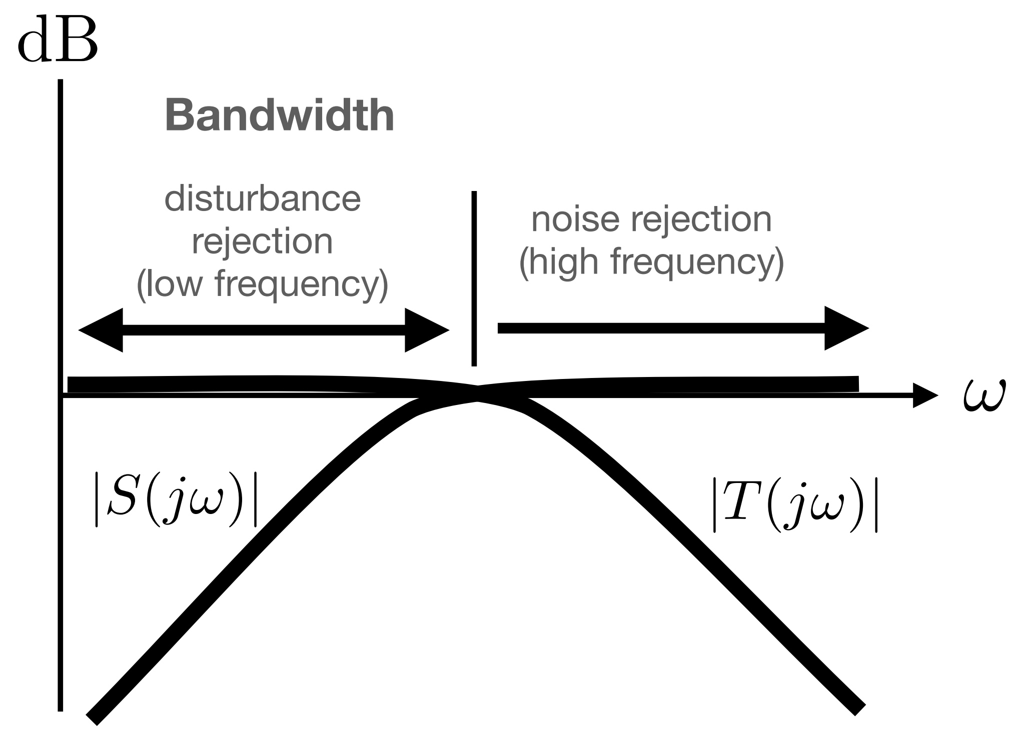
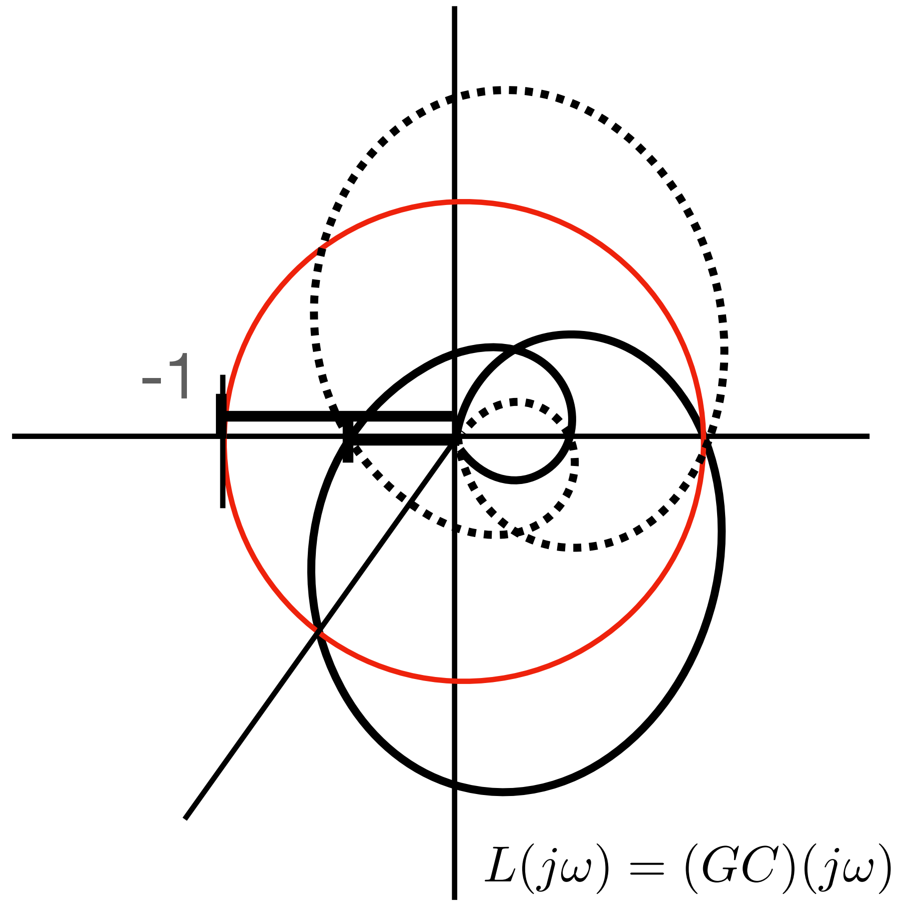
SISO Design - Example







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1. Disturbance rejection

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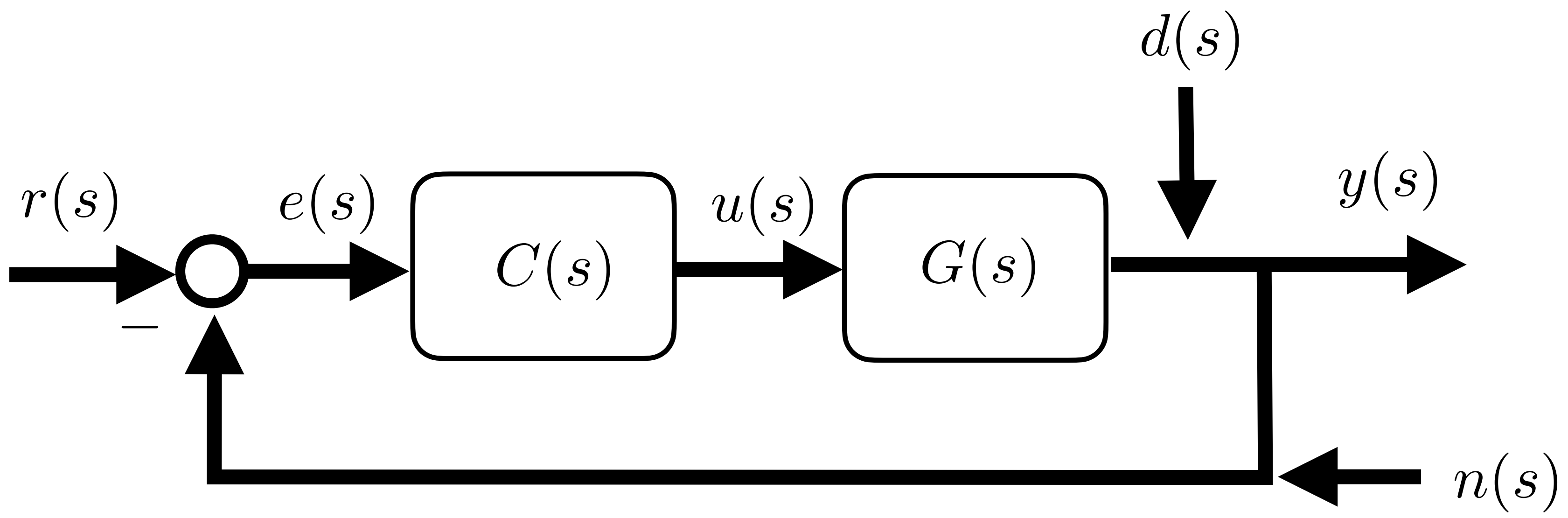
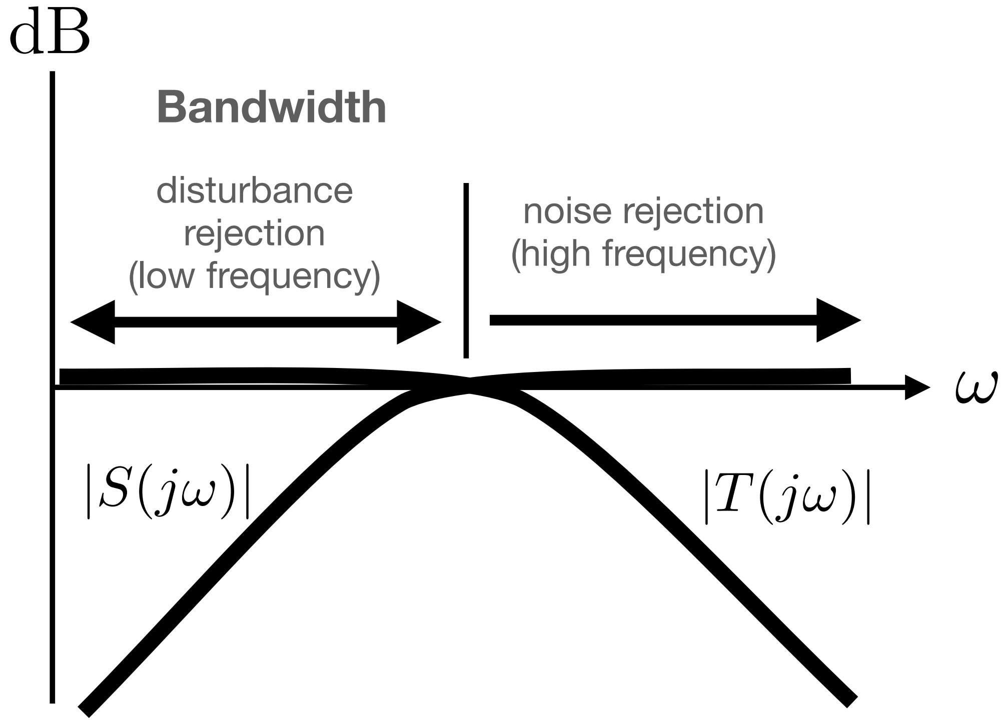
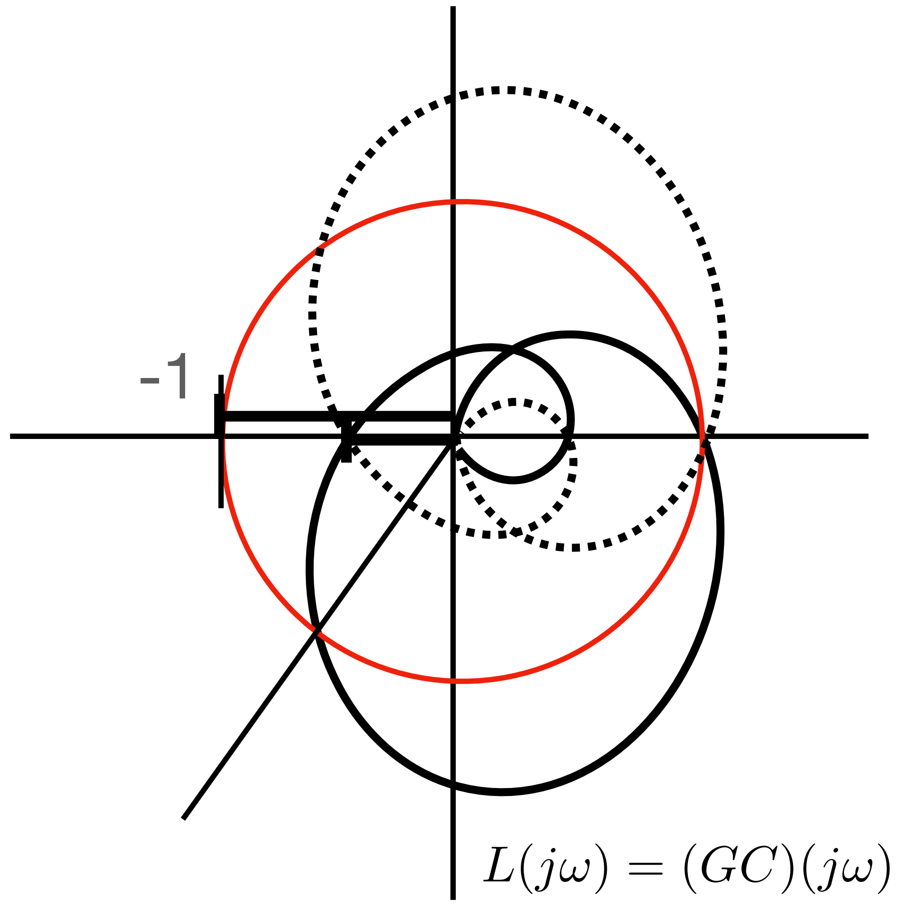
disturbance rejection… $\mathrm{d}_C = s^2$

stability… $\mathrm{n}_C = K_p s^2 + K_I s + K_{II}$

SISO Design - Example







Loop Transfer $L = GC = \dfrac{n_G}{d_G}\dfrac{n_C}{d_C}$ $\quad G = \dfrac{n_G}{d_G} \quad C = \dfrac{n_C}{d_C}$

…causal d_G, d_C higher order than… n_G, n_C

Output $y = \underbrace{(I+GC)^{-1}GC}_{T}(r-n) + \underbrace{(I+GC)^{-1}}_{S} d$

Error $e = \underbrace{(I+GC)^{-1}}_{S} r + \underbrace{(I+GC)^{-1}GC}_{T} n - \underbrace{(I+GC)^{-1}}_{S} d$

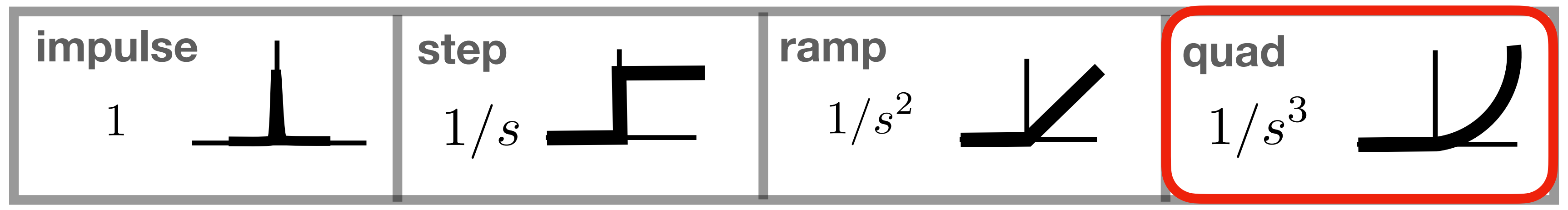
FVT: $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \dfrac{s^{n_n} + \cdots + \alpha_k s^k}{s^{n_d} + \cdots + \alpha_{k'} s^{k'}}$

$k > k' \quad \to \quad 0$

$k = k' \quad \to \quad \dfrac{\alpha_k}{\alpha_{k'}}$

$k < k' \quad \to \quad \infty$

Disturbance types $d(s) = \dfrac{n_d}{d_d}$



$$\lim_{s\to 0} \frac{s\,(s^2 + 2\zeta\omega_n s + \omega_n^2)\, d_C}{(s^2 + 2\zeta\omega_n s + \omega_n^2)\, d_C + 1\, n_C} \frac{1}{s^3}$$

disturbance…

1. Disturbance rejection

CONDITION 1: degree $d_C \geq$ degree d_d

2. Stability

CONDITION 2: $d_G d_C + n_G n_C$ stable

Plant: **oscillator…**

$G(s) = \dfrac{n_G}{d_G} = \dfrac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

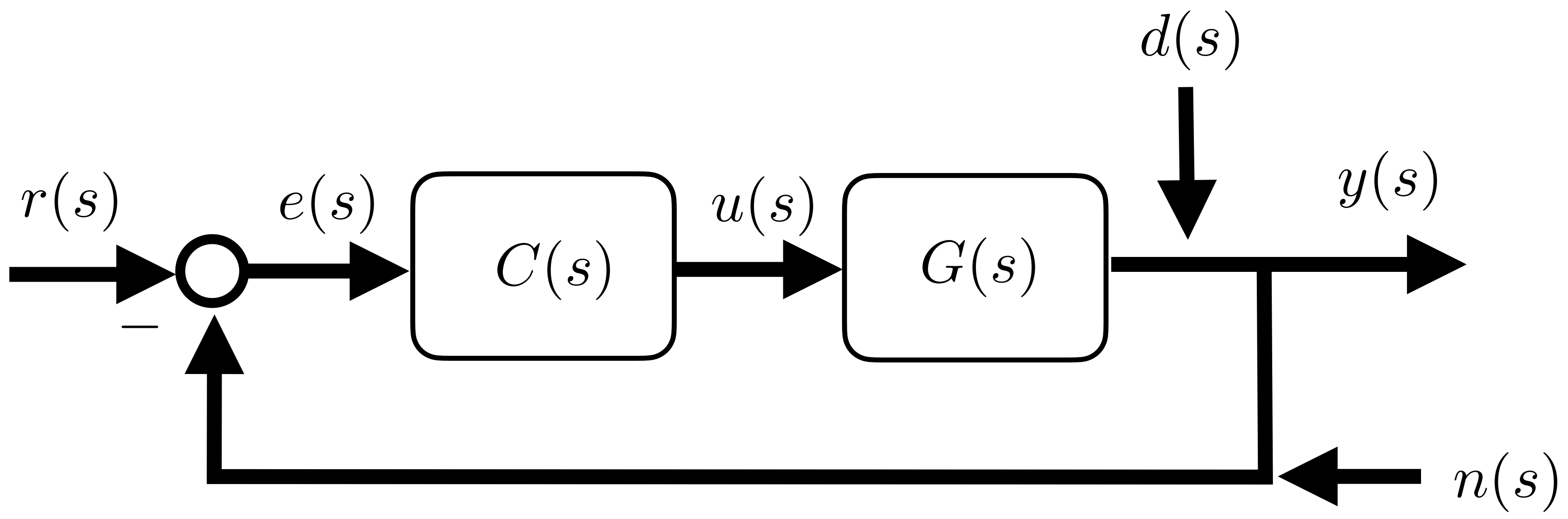
Controller:

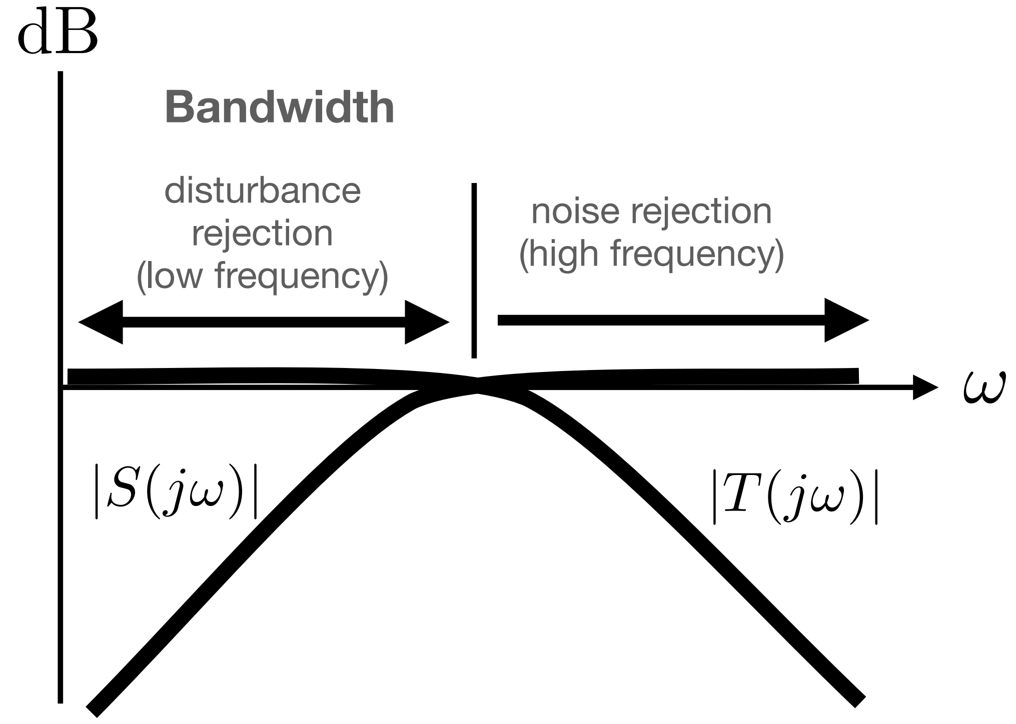
$C(s) = \dfrac{n_C}{d_C} =$

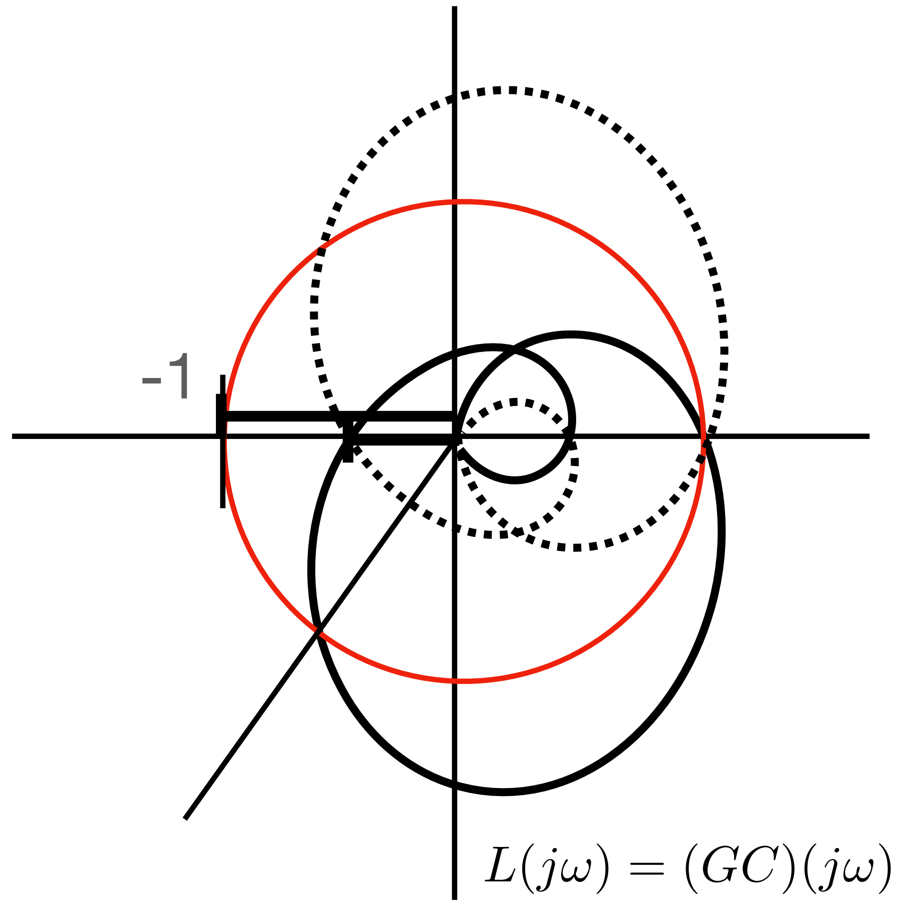
disturbance rejection… $d_C =$

stability… $n_C =$

SISO Design - Example







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Disturbance types $d(s) = \frac{n_d}{d_d}$

impulse	step	ramp	quad
1	$1/s$	$1/s^2$	$1/s^3$

$$\lim_{s\to 0} \frac{s\,(s^2+2\zeta\omega_n s+\omega_n^2)\,s^3}{(s^2+2\zeta\omega_n s+\omega_n^2)\,s^3 + 1\,(K_p s^3 + K_I s^2 + K_{II} s + K_{III})} \frac{1}{s^3}$$

disturbance…

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degree $d_C \geq$ degree d_d

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CONDITION 2:
$d_G d_C + n_G n_C$ stable

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Controller:

$$C(s) = \frac{n_C}{d_C} = K_p + \frac{K_I}{s} + \frac{K_{II}}{s^2} + \frac{K_{III}}{s^3}$$

disturbance rejection… $d_C = s^3$

stability… $n_C = K_p s^3 + K_I s^2 + K_{II} s + K_{III}$