AA447 - Feedback Control - Spring 2021

Homework 2

Due Date: Thursday, Apr 15^{th}, 2021 at 11:59 pm

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Cruise Control Model

Consider a car that is moving up a hill while on cruise control. Denoting the force produced by the engine as the input, $u(t)$, and the vehicle's speed as the output, $y(t)$, we can model the vehicle as follows:

$$\dot{v}(t) = -\frac{1}{\tau}v(t) + \frac{1}{m}\big[u(t) + d(t)\big] \tag{1}$$

$$y(t) = v(t) + n(t) \tag{2}$$

Assume:

- The mass of the car is $m = 1000kg$
- The time constant of the system is $\tau = 10s$
- The gravitational constant is $g = 9.81m/s^2$
- The system is subject to a disturbance $d(t)$
- The speed measurement $y(t)$ is corrupted by sensor noise $n(t)$

1. **Simulation**

   For this problem, assume the following:

   - The disturbance is given by $d(t) = -mg\sin(\theta) - \alpha v^2(t)$
   - The noise is given by $n(t) = 0$
   - The initial condition is $v(0) = 0m/s$

   Do the following:

   (a) **(PTS: 0-2)** Let $u(t) = 100$. For $\theta = 10°$ and $\alpha = 0.5kg/m$, use MATLAB's *ode45* function (or a Python equivalent) to integrate Eq. 1 over $t \in [0, 30]s$. Provide your code.

   (b) **(PTS: 0-2)** Provide plots for $y(t)$ and $d(t)$ for the conditions given in part (a).

   (c) **(PTS: 0-2)** Provide a qualitative explanation of the role τ has on the output $y(t)$. For example, set $\alpha = 0kg/m$ and $\theta = 0°$, and experiment with different (positive) values of τ.

   (d) **(PTS: 0-2)** Set $\alpha = 750kg/m$ and $\theta = 0°$. Simulate and plot $y_1(t)$, $y_2(t)$, and $y_3(t)$ for $u_1(t) = 100\sin(\frac{2\pi}{6}t)$, $u_2(t) = 100\sin(\frac{2\pi}{10}t)$, and $u_3(t) = u_1(t) + u_2(t)$. Additionally, plot $y_1(t) + y_2(t)$ on the same plot. Does $y_3(t)$ match $y_1(t) + y_2(t)$?

   (e) **(PTS: 0-2)** Repeat part (d) with $\alpha = 0kg/m$. Does $y_3(t)$ match $y_1(t) + y_2(t)$ in this case? Explain why this result differs from that in part (d).

2. **Open-Loop Control**

Now, let's consider an open-loop control system with the control law:

$$u(t) = k_{OL} v_{ref} \tag{3}$$

where k_{OL} is the open-loop gain, and v_{ref} is the reference velocity. For this problem, assume that $n(t) = 0$, $\theta = 0°$, and $\alpha = 0 kg/m$. Do the following:

(a) **(PTS: 0-2)** What value should k_{OL} have such that $v(t)$ converges to v_{ref}? Give your answer symbolically (i.e. in terms of variables m, τ, etc.).

HINT: Plug Eq. 3 into Eq. 1, and select k_{OL} such that the resulting equation exponentially decays to v_{ref}. That is, select k_{OL} such that you can express Eq. 1 in the following form:

$$\dot{v}(t) = -\beta\big[v(t) - v_{ref}(t)\big] \quad \text{with} \quad \beta > 0$$

(b) **(PTS: 0-2)** Using the value of k_{OL} obtained in part (a), assume $v(0) = 10m/s$ and $v_{ref} = 30m/s$. Turn in a plot of $y(t)$ for $t \in [0, 30]s$. Include a dotted horizontal line that represents the value of v_{ref}.

(c) **(PTS: 0-2)** Now, keep the same k_{OL} as in part (a). Increase m by 25%. What do you notice in the response of $y(t)$? Provide a plot for this case (as you did in part (b)).

(d) **(PTS: 0-2)** Repeat part (c), but this time keep the original m, and set $\theta = 15°$.

3. **Closed-Loop PI Control**

Notice that in the last problem, the control law did not use the measurement $y(t)$, and hence was termed an *open-loop* control law. Now, consider a *closed-loop* system, where the control law is a function of $y(t)$ and is given by:

$$u(t) = k_{CL}\left(v_{ref} - y(t)\right) \tag{4}$$

where k_{CL} is the closed-loop control gain. Unless otherwise specified, assume $n(t) = 0$, $\theta = 0°$, and $\alpha = 0kg/m$. Assume $v(0)$ and v_{ref} are the same as in Problem 2. Do the following:

(a) **(PTS: 0-2)** Through experimentation, find a value of k_{CL} that results $v(t)$ converging to a value (ideally the value $v(t)$ converges to is close to v_{ref}). What are the effects of increasing and decreasing k_{CL} by 50%? Report the numbers of k_{CL} you used, and provide a plot of $y(t)$ vs. t for all three cases. Plot all three cases on the same set of axes.

(b) **(PTS: 0-2)** Are there values of k_{CL} that cause the system to diverge (i.e. where $v(t)$ grows indefinitely)? If so, what seems to be the boundary of stability?

HINT: You can follow a procedure similar to the one you followed in Problem 2a to obtain a differential equation. From there, you can deduce the value of k_{CL} that will result in divergent behavior.

(c) **(PTS: 0-2)** Repeat Problem 2c for the closed-loop case. Compare your results to the open-loop case.

(d) **(PTS: 0-2)** Repeat Problem 2d for the closed-loop case. Compare your results to the open-loop case.

(e) **(PTS: 0-2)** Which control law seems more robust to variations in θ and m? Given your results, what does closed-loop control seem to provide that open-loop control lacks?

(f) **(PTS: 0-2)** Assume that $n(t) = 10\sin(2\pi t)$, $\theta = 0°$, and use the original mass m. Re-run the simulation using the three values of k_{CL} you used in part (a). What seems to be the disadvantage of increasing k_{CL} when noise is corrupting the feedback signal $y(t)$?

(g) **(PTS: 0-2)** Lastly, assume that $n(t) = 0$. Notice that the proportional control law proposed above produces a non-zero steady-state error. That is, $v(t)$ does not match v_{ref} as $t \to \infty$. Modify the control law and your simulation to incorporate an integral term, as follows:

$$u(t) = k_{CL}\left(v_{ref} - y(t)\right) + k_I \int_0^t [v_{ref} - y(\tau)]d\tau \tag{5}$$

Through experimentation select a positive value for k_I. You can use any of the previous values you used for k_{CL}. Can you make the steady-state error disappear? Provide your code, a plot of $y(t)$ versus t, and a plot of $u(t)$ versus time.

4. **Double Integrator PID Model**

Suppose we are given double integrator dynamics,

$$m\ddot{y}(t) = u(t) + d(t)$$

where disturbance is given by $d(t)$. We consider using PID control for the system, whose input is $\xi(t) = y_{ref}(t) - y(t)$,

$$u(t) = k_p\xi(t) + k_d\dot{\xi}(t) + k_I \int_0^t \xi(\tau)d\tau +$$

(a) **(PTS:0-2)** Draw the block diagram of the closed loop system, label each signals

(b) **(PTS:0-2)** Write a state space model of the double integrator dynamics in the form

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

with $x \in \mathbb{R}^3$, ie. add an augmented state so that the PID controller can be written as $u = Kx$. What is K (symbollically) for the PID controller?

(c) **(PTS:0-2)** Derive the transfer functions
- Plant: from U to Y, $G(s) = \frac{Y(s)}{U(s)}$.
- Control: from Ξ (uppercase ξ) to U, $C(s) = \frac{U(s)}{\Xi(s)}$.
- Loop TF: from Ξ to Y, $L(s) = \frac{Y(s)}{\Xi(s)} = G(s)C(s)$.

(d) **(PTS:0-2)** Derive the transfer functions
- from disturbance D to output Y: $T(s) = \frac{Y(s)}{D(s)}$.
- from reference Y_{ref} to output Y: $R(s) = \frac{Y(s)}{Y_{ref}(s)}$.

(e) **(PTS:0-2)** Let $m = 1$ and $k_p = k_d = k_I = 2$. Create bode magnitude plots for the transfer functions $L(s)$, $T(s)$, and $R(s)$. What are the poles of each transfer function? (They should be the same for all three transfer functions.) Is the system stable?