AA447 - Feedback Control - Spring 2021

Homework 3

Due Date: Wednesday, Apr 28th, 2021 at 11:59 pm

1. BIBO Stability Analysis

Consider a feedback control system with plant and control transfer functions

$$G(s) = \frac{1}{ms^2}, \qquad C(s) = k_p + k_d s + \frac{k_I}{s}$$

(a) **(PTS:0-2)** Determine the BIBO stability requirements on k_p, k_d, k_I for stability of the system, ie. use Routh-Hurwitz to find conditions on k_p, k_d, k_I that guarantee that the roots of $1 + L(s) = 0$ are in the open left half plane (OLHP). Here $L(s) = G(s)C(s)$.

(b) **(PTS:0-2)** Suppose for this part that $m = 1$ and you have the additional constraint that $k_p + k_d + k_I = 1, k_p, k_d, k_I > 0$. Find the values for k_p, k_d that result in BIBO stability and plot the region in $k_p - k_d$ space (draw a plot with k_p on the horizontal axis and k_d on the vertical axis and indicate the region of the space where the closed loop system is stable. Note that given the constraint determining k_p and k_d fixes $k_I = 1 - k_d - k_p$. Also, be sure to take into account the conditions that $k_p, k_d, k_I > 0$.)

(c) **(PTS:0-2)** Plot the Bode plot of loop transfer function $L(s)$ with BIBO stable coefficients. Describe the effects of each component of the PID controller on the plot.

(d) **(PTS:0-2)** Simulate y(t) with BIBO stable coefficients for $t \in [0, 30]$ given the following initial conditions:

$$y_{ref} = 30, \ y(0) = 5, \ \dot{y}(0) = 1, \ \nu(0) = 0, \ d(t) = 1$$

See below part (e) for tips

(e) **(PTS:0-2)** Simulate y(t) with BIBO stable coefficients for $t \in [0, 30]$ given the following initial conditions:

$$y_{ref} = 30, \ y(0) = 5, \ \dot{y}(0) = 1, \ \nu(0) = 0, \ d(t) = \cos(2t)$$

The Matlab command `lsim` may be helpful. What do you notice when the disturbance is varying with time compared to when it is constant?

Note: You can ignore the initial conditions $y(0)$ and $\dot{y}(0)$, ie. set $y(0) = 0$ and $\dot{y}(0) = 0$. To simulate a system with both a reference and a disturbance, you can compute the transfer functions from the reference to the output, $T(s) = \frac{C(s)G(s)}{1+C(s)G(s)}$, and from the disturbance to the output, $\frac{G(s)}{1+C(s)G(s)}$. You can then combine these functions into a Matlab `sys` model with two inputs (one for the reference and one for the disturbance) and one output and pass it

in to `lsim`. Another option would be to use the `impulse` command. This can be done by computing the Laplace transform of the desired reference $Y_{\text{ref}}(s)$ and disturbance $D(s)$ and then computing the impulse response of $Y(s)$

$$Y(s) = \frac{C(s)G(s)}{1 + C(s)G(s)} Y_{\text{ref}}(s) + \frac{G(s)}{1 + C(s)G(s)} D(s)$$

(This gives you the appropriate output because the Laplace transform of an impulse is 1. $\mathcal{L}\big[\delta(t)\big] = 1$. Conceptually, this is like thinking of the reference and disturbance as part of the internal workings of the system and the impulse as simply turning the system on, but the results would are the same)

(f) **(PTS:0-2)** Why can we reject a constant disturbance? Show this analytically. (Hint: consider the disturbance transfer function $T_d(s)$).
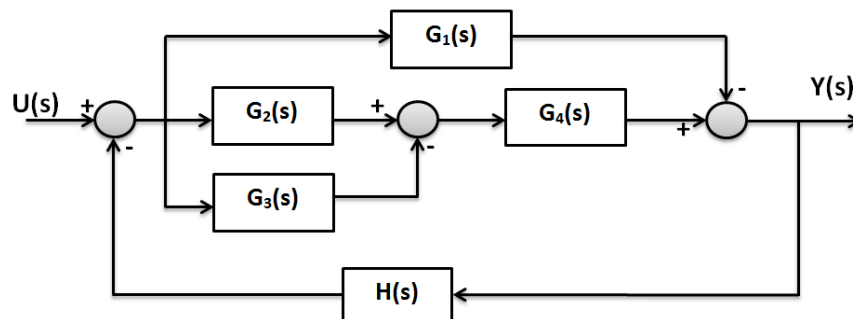
2. **Block Diagrams**

Find the transfer functions from $U(s)$ to $Y(s)$ for the following closed-loop systems:

(a) **(PTS:0-2)** System 1: Assume $H_1(s) = 0$.
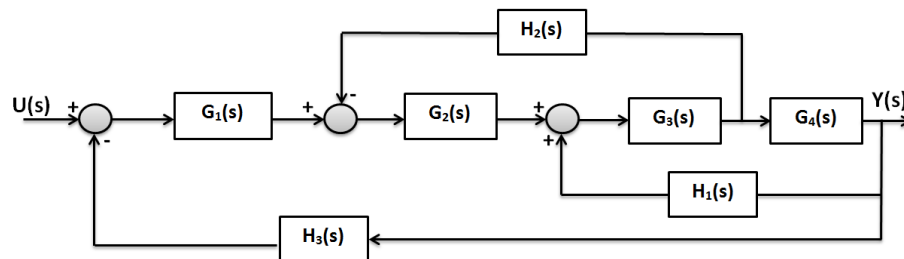
(b) **(PTS:0-2)** System 2:

3. **Problem 3**

Consider the spacecraft system model given in class:

Where

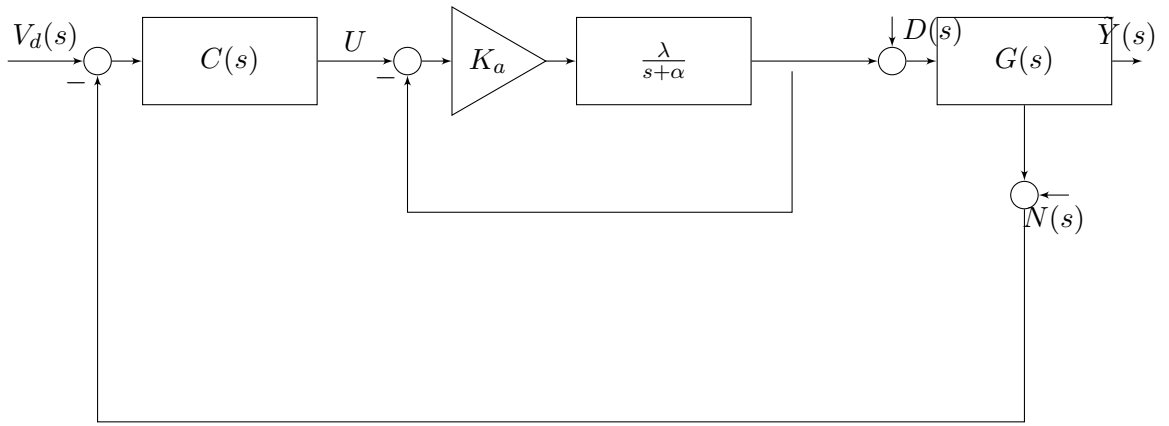$$G(s) = \frac{cs + k}{s[Mms^2 + (M + m)(cs + k)]}$$

Figure 1: Full Spacecraft Model Block Diagram

and the internal transfer function for the thruster is given by

$$\Gamma(s) = \frac{\lambda k_a}{s + (\alpha + \lambda k_a)}$$

If we were to consider an ideal thruster and treat the system as a rigid body we would have:

$$\Gamma(s) = 1$$

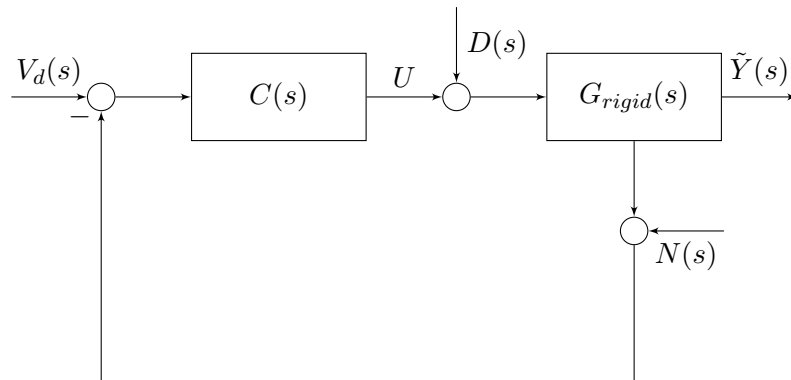$$G_{rigid}(s) = G_r = \frac{1}{(M + m)s}$$

Figure 2: Simplified Spacecraft Model Block Diagram

We will consider using 3 different controllers.

i $C(s) = k_p$

ii $C(s) = k_p + \frac{k_I}{s}$

iii $C(s) = k_p + \frac{k_I}{s} + k_d \frac{s}{\beta s + 1}$

We will compare the performance of the controllers on the simplified vs actual systems when they are designed using the simplified system. You may use MATLAB throughout, to guide your process. The closed loop transfer functions for the actual system and the approximated system are respectively:

$$Y(s) = T(s)[V_d(s) - N(s)] + R(s)D(s)$$
$$Y_r(s) = T_r(s)[V_d(s) - N(s)] + R_r(s)D(s)$$

(Subscript r refers to the rigid body dynamics.) Unless specific values are mentioned, you may use a set of default constants: $m = 1, M = 10, k = 1, c = 1, k_a = 1, \beta = 1, \alpha = 1, \lambda = 5, v_d = 50$

(a) **(PTS:0-2)** Determine the closed loop transfer function components: $T(s), T_r(s), R(s), R_r(s)$ (you may keep these in terms of the symbolic transfer functions $C(s), G(s), G_r(s)$,etc. until you have values to plug in in MATLAB).

(b) **(PTS:0-2)** Using the simplified model, find the necessary and sufficient conditions for stability for the 3 controllers. (It may be helpful to save these conditions as expressions in Matlab so they can be checked if you change the values). Can you drive this system unstable with positive gains for any of the controllers?

(c) **(PTS:0-2)** Select values for the controller gains that satisfy these conditions and check them by finding the roots of $L_r(s) + 1$. Compare with the roots for $L(s) + 1$.

(d) **(PTS:0-2)** With $n(t) = d(t) = 0$, and your chosen gains, simulate the simplified model using the matlab frequency domain commands (i.e. lsim, impulse, etc.)

(e) **(PTS:0-2)** With $n(t) = d(t) = 0$, simulate the full model. Describe what you observer.

(f) **(PTS:0-2)** Suppose a large disturbance begins influencing the system. Let $d(t) = 50 * (1 + cos(2t))$, simulate both the simplified system as well as the full system, compare the 3 controllers. Describe what you observe.