AA447 - Feedback Control - Spring 2021

Homework 4

Due Date: Wednesday, May 5^{th}, 2021 at 11:59 pm

1. **Final Value Theorem**

   (PTS:0-2) Consider the function

   $$f(t) = t^n e^{-at}$$

   with $n \geq 1$ and $a > 0$. Show using the final value theorem that:

   $$\lim_{t \to \infty} t^n e^{-at} = 0$$

2. **Space-Craft Model Design for Disturbance Rejection**

   Consider again the spacecraft system model given in class:

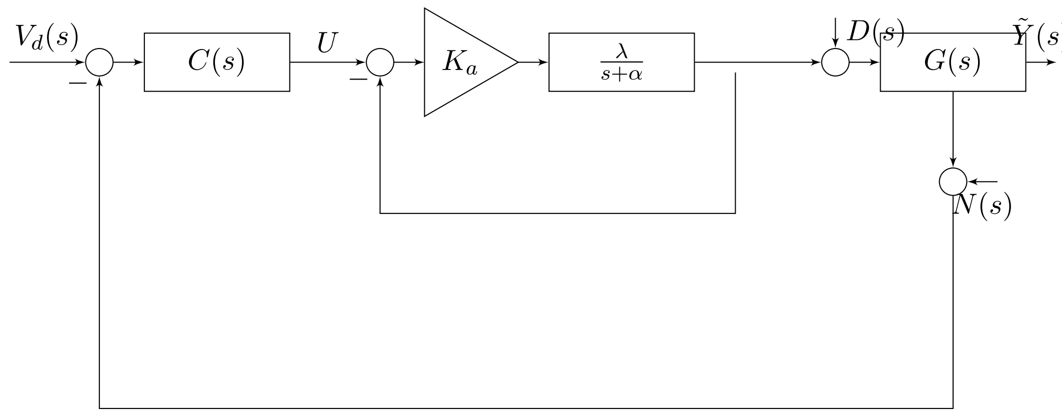

Figure 1: Full Spacecraft Model Block Diagram

where

$$G(s) = \frac{cs + k}{s[Mms^2 + (M + m)(cs + k)]}$$

and the transfer function for the thruster is given by

$$\Gamma(s) = \frac{\lambda k_a}{s + (\alpha + \lambda k_a)}$$

If we were to consider an ideal thruster and treat the system as a rigid body we would have the following approximations:

$$\Gamma(s) = 1$$

$$G_{rigid}(s) = G_r = \frac{1}{(M + m)s}$$

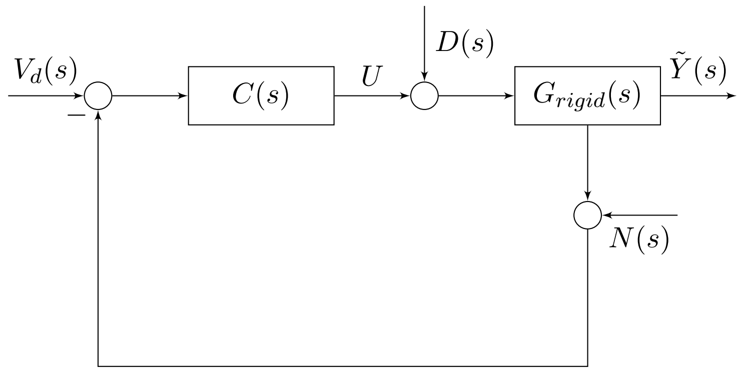

Figure 2: Simplified Spacecraft Model Block Diagram

The closed loop transfer functions for the actual system and the approximated system are respectively:

$$Y(s) = T(s)[V_d(s) - N(s)] + R(s)D(s)$$
$$Y_r(s) = T_r(s)[V_d(s) - N(s)] + R_r(s)D(s)$$

Unless specific values are mentioned, you may use a set of default constants: $m = 1, M = 3, k = 1, c = 1, k_a = 1, \beta = 1, \alpha = 1, \lambda = 5, v_d = 50, n(t) = 0$

(a) **(PTS:0-2)** In the previous homework we saw that the controllers we used would not remove the sinusoidal disturbance $d(t) = 50(1 + \cos(2t))$. What kind of controller will you need to achieve this and why? Write out your equation for the form of $C(s)$.

(b) **(PTS:0-2)** Let us assume instead that we have a disturbance $d(t) = t + t^2 + cos(5t)$ what form of controller is needed? (You do not need to solve for any parameters here)

(c) **(PTS:0-2)** With the original disturbance $d(t) = 50(1 + \cos(2t))$. Find values for your controller parameters that result in BIBO stability in the simplified system. Hint: Fix values in your inequalities and try to find solutions to the resulting lower order inequalities. Make sure you are using the correct inequalities.

(d) **(PTS:0-2)** Simulate the simplified system with the disturbance $d(t) = 50(1+\cos(2t))$ using your controller.

(e) **(PTS:0-2)** Simulate the full system with the disturbance $d(t) = 50(1 + \cos(2t))$ using your controller. What difference do you see? Why do you think that behavior is occurring? How does this compare to the controllers in the previous homework?

(f) **(PTS:0-2)** What happens if you increase the frequency of the disturbance such that $d(t) = 50(1 + cos(5t))$?

(g) **(PTS:0-4)** Extra credit: Suppose you knew the disturbance was in a range of frequencies (w_{low}, w_{high}) but not exactly the frequency, how might you want to design your controller to reject it? Can you guarantee perfect rejection?

3. **Disturbance Rejection**

For the plant $G(s)$ given, design a controller that stabilizes the system and rejects the disturbance $D(s)$ of the form shown. In each case, show that the closed loop system is stable and that the steady state error from the disturbance is 0.

(a) **(PTS:0-6)**

$$G(s) = \frac{1}{ms}, \qquad D(s) = \frac{1}{s} + \frac{1}{s^2}$$

(b) **(PTS:0-6)**

$$G(s) = \frac{1}{ms^2}, \qquad D(s) = \frac{1}{s} + \frac{1}{s^2}$$

(c) **(PTS:0-6)**

$$G(s) = \frac{1}{ms^2 + cs + k}, \qquad D(s) = \frac{1}{s} + \frac{1}{s^2}$$

with $m = 1$, $c = 1$, $k = 1$.