AA447 - Feedback Control - Spring 2021

Homework 5

Due Date: Wednesday, May 12th, 2021 at 11:59 pm

1. **Time Delay**

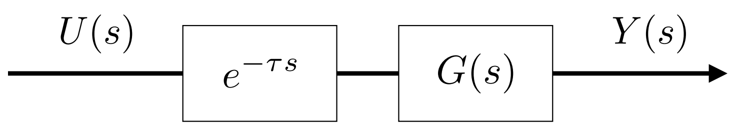
   (a) **(PTS:0-2)** Prove the time delay property of the Laplace transform. I.e. let $f : \mathbb{R} \to \mathbb{R}$ be a generic time domain function, if

   $$y(t) = f(t-\tau)u_{step}(t-\tau)$$

   Show that

   $$Y(s) = e^{-\tau s}F(s)$$

   (b) **(PTS:0-2)** Consider a system transfer function $G(s)$ and with a time delay of τ seconds added in series as shown below.



   How do the magnitude and phase of the frequency response change for a given frequency ω?

2. **Nyquist Plots**

   Sketch Nyquist plots for the following systems by hand by computing the following points

   - $\omega = 0,\ \omega = \infty$
   - Re-axis intercept, Im-axis intercept
   - Min/max of real component
   - Min/max of imaginary component

   (You can check your results using Matlab or an online visualizer.)

   `https://mathlets.org/mathlets/bode-and-nyquist-plots/`

   Decide from the Nyquist plot if the closed-loop system is stable.

   (a) **(PTS: 0-2)**

   $$L(s) = \frac{1}{(s-2)}$$

   (b) **(PTS: 0-4)**

   $$L(s) = \frac{1}{(s-1)(s-2)}$$

(c) **(PTS: 0-4)**

$$L(s) = \left(k_p + \tfrac{k_I}{s}\right)\frac{1}{(s-1)}$$

Choose two sets of values for k_p and k_I: one set that makes the system stable and one set that makes it unstable and note how the Nyquist plot changes between the two.

(d) **(PTS: 0-6)**

$$L(s) = \left(k_p + k_d s + \tfrac{k_I}{s}\right)\frac{1}{(s-1)(s-2)}$$

Choose two sets of values for k_p, k_d, and k_I,: one set that makes the system stable and one set that makes it unstable and note how the Nyquist plot changes between the two.

3. **Nyquist Stability**

Let $G : \mathbb{C} \to \mathbb{C}$ be a rational function from the complex plane to itself. Let $\Gamma \subset \mathbb{C}$ be a closed clockwise contour in the complex plane and let $G(\Gamma)$ be the image of that contour under the map $G(s)$.

The Cauchy Argument principle says that

$$\begin{array}{c}\text{Num. of times}\\ G(\Gamma)\text{ circles }0\\ \text{(clockwise)}\end{array} = \begin{array}{c}\text{Zeros}\\ \text{inside }\Gamma\end{array} \quad - \quad \begin{array}{c}\text{Poles}\\ \text{inside }\Gamma\end{array}$$

Let $L(s) = C(s)G(s)$ be the open-loop transfer function for a system with plant $G(s)$ and controller $C(s)$ and let Γ_{N} be the Nyquist contour that encircles the right-half of the complex plane.

(PTS:0-6) Prove the Nyquist stability criteria that states that the closed-loop system is stable if

$$\begin{array}{c}\text{Num. of times}\\ L(\Gamma_{\text{N}})\text{ circles -1}\\ \text{counter-clockwise}\end{array} = \begin{array}{c}\text{Poles}\\ \text{of }L(s)\\ \text{inside }\Gamma_{\text{N}}\end{array}$$

4. **State-Space Realizations**

Consider a plant $G(s)$, PID controller $C(s)$, and open-loop transfer function $L(s)$.

$$G(s) = \frac{1}{ms^2}, \qquad C(s) = k_d s + k_p + \tfrac{k_I}{s}, \qquad L(s) = G(s)C(s)$$

Write a separate state-space minimal realization for the following transfer functions

(a) **(PTS:0-2)**

$$L(s)$$

(b) **(PTS:0-2)**

$$\frac{1}{1+L(s)}$$

(c) **(PTS:0-2)**

$$\frac{L(s)}{1+L(s)}$$

(d) **(PTS:0-2)**

$$\frac{G(s)}{1+L(s)}$$