AA447 - Feedback Control - Spring 2021

Homework 6

Due Date: Wednesday, May 26^{th}, 2021 at 11:59 pm

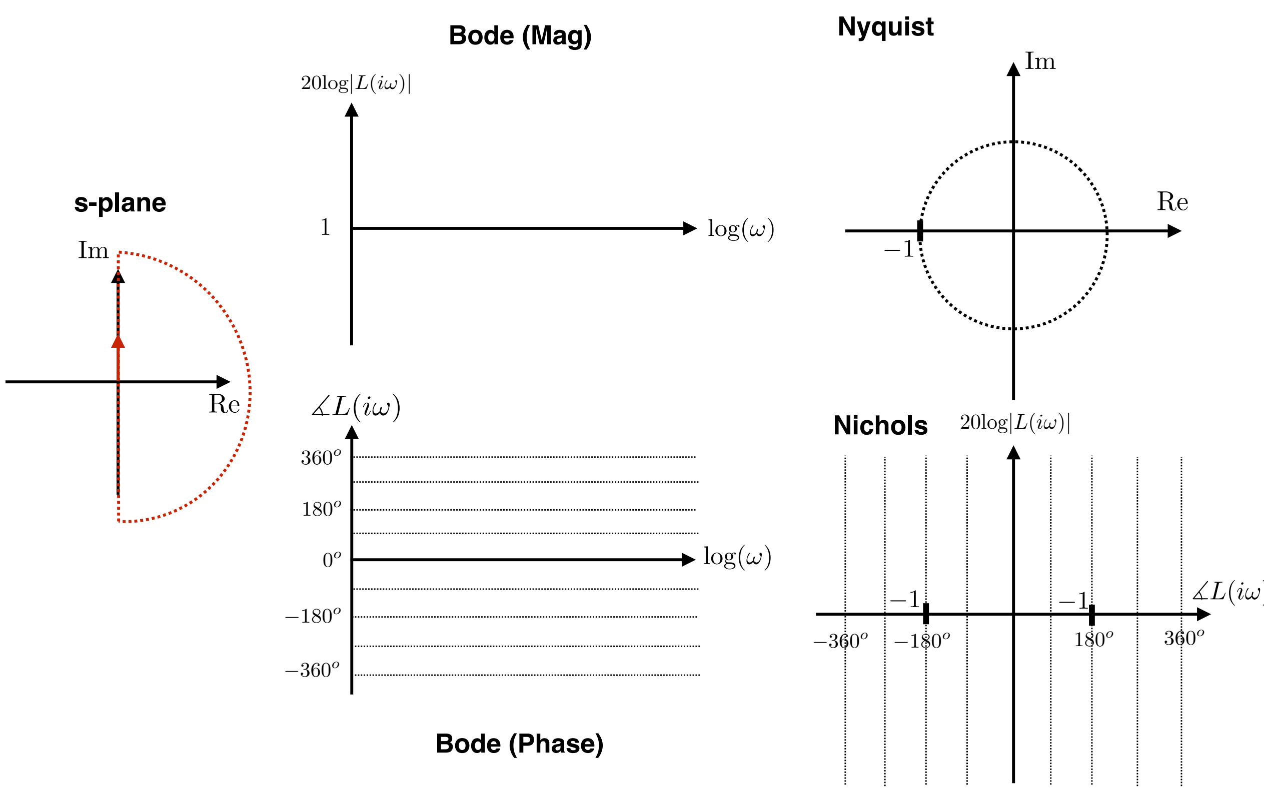
Draw the zero/pole locations, Bode, Nyquist, and Nichols plots for each of the following transfer functions. For each plot consider the following points.

- **s-plane:** Location of poles and zeros, phasors $(i\omega - z_k)$ and $(i\omega - p_k)$
- **Bode plots:** $|L(0)|, \omega_c, \omega_{180}$, log slope of $|L(i\omega)|$.
- **Nyquist plot:** $L(0), L(i\infty), L(i\omega_c), L(i\omega_1 80)$, direction of contour
- **Nichols plot:** $L(0), L(i\infty), L(i\omega_c), L(i\omega_1 80)$, direction of contour

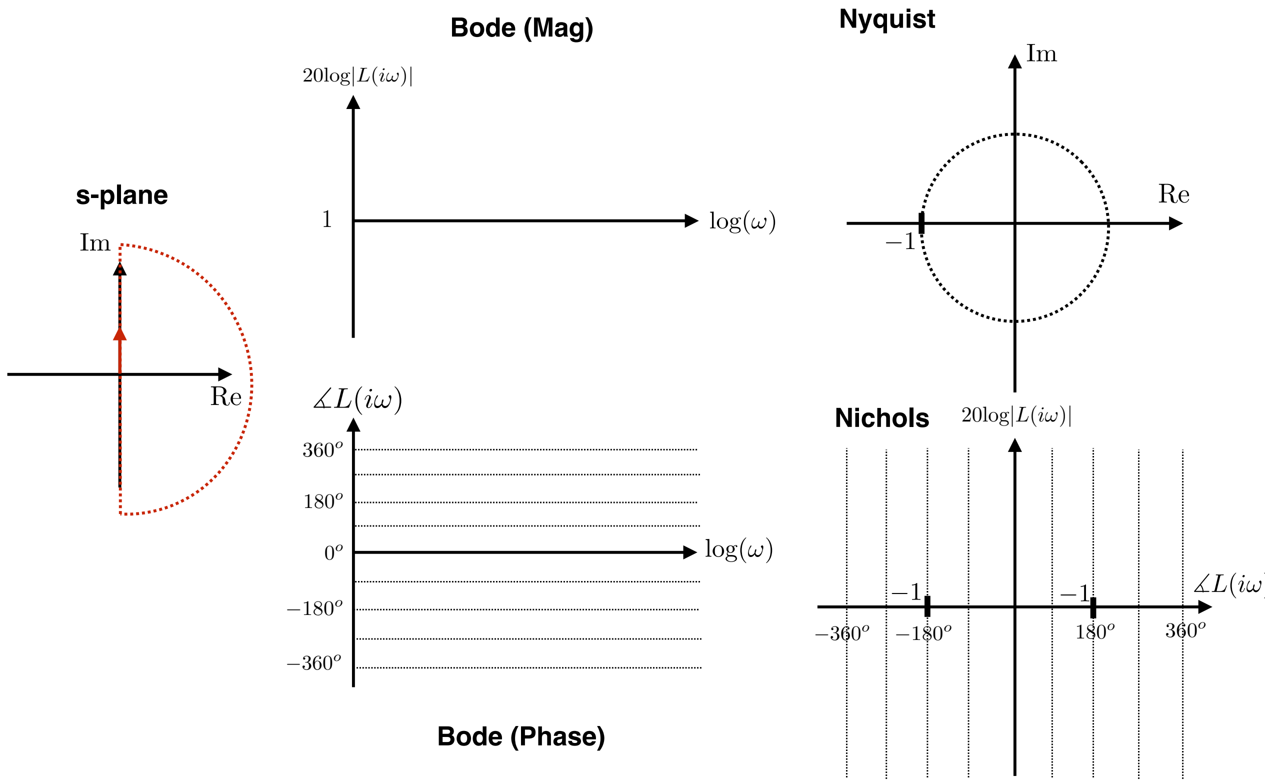
Label these points whenever appropriate. (If you're plotting multiple variations of a family curves on one plot you don't need to label each value on all of them.) You may use whatever software you want. Understanding qualitative behavior is more important than graphical precision.

1. **First order systems**

   - $L(s) = s + z$ for $z = 0, z < 0, z > 0$



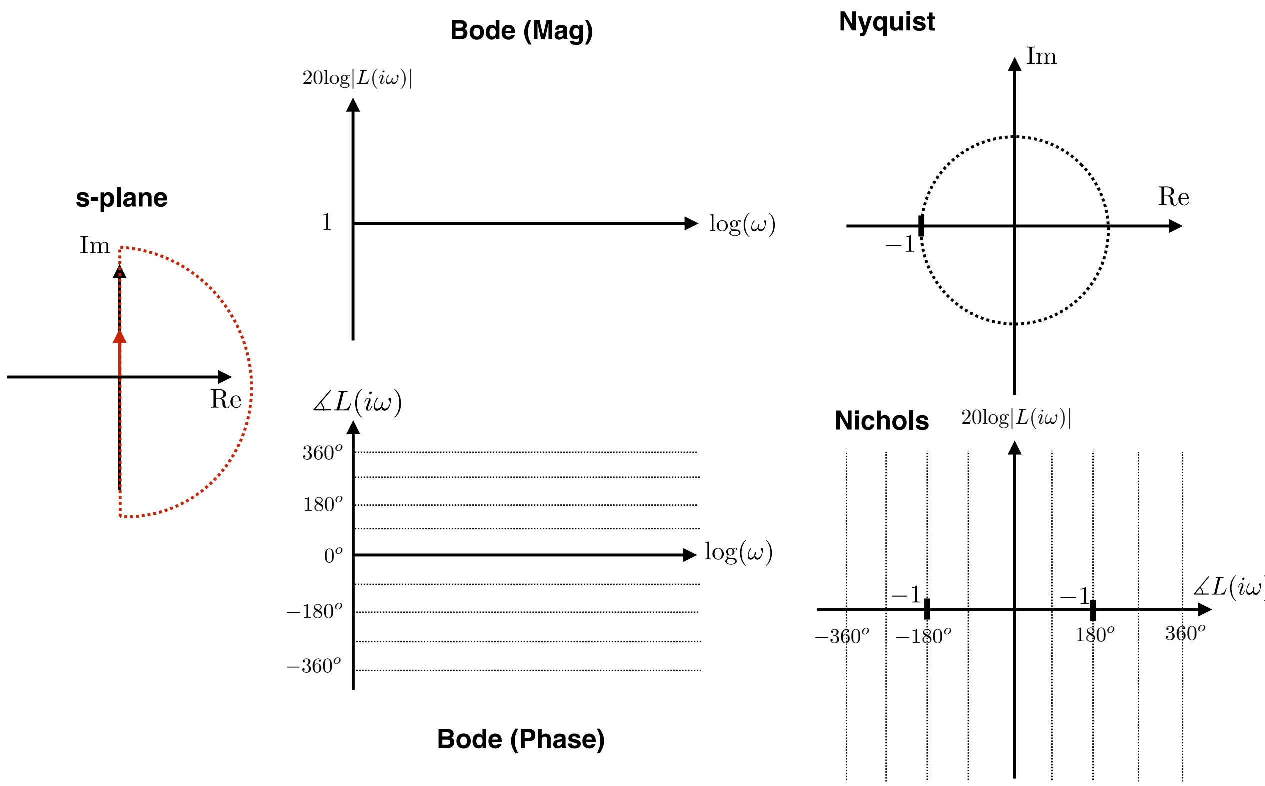
- $L(s) = \frac{1}{s+p}$ for $p = 0, p < 0, p > 0$



2. **Second order systems**

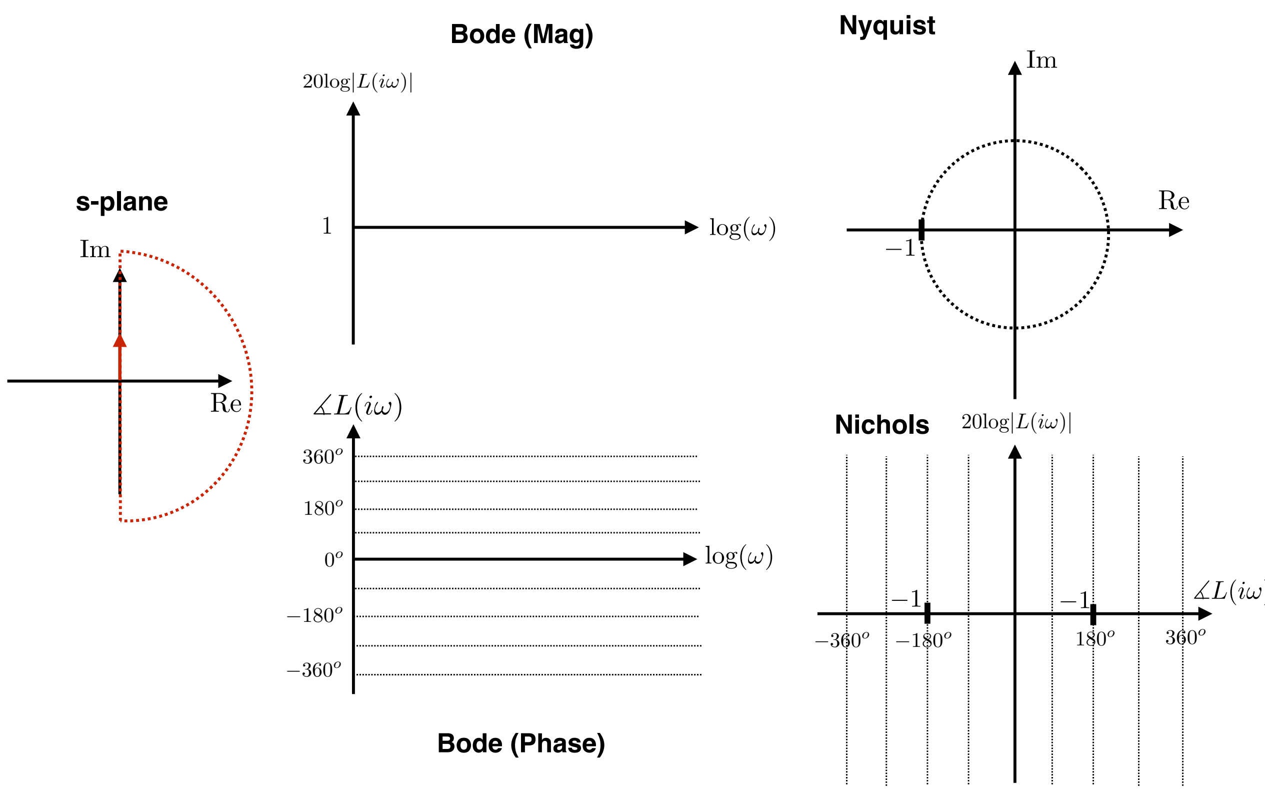
- $L(s) = (s + z_1)(s + z_2)$
  - Real z_1, z_2: z_1 and $z_2 = 0$, z_1 and $z_2 < 0$, z_1 and $z_2 > 0$, $z_1 < 0$ and $z_2 > 0$
  - Complex z_1, z_2: $\mathrm{Re}(z_1)$ and $\mathrm{Re}(z_2) = 0$, $\mathrm{Re}(z_1)$ and $\mathrm{Re}(z_2) < 0$, $\mathrm{Re}(z_1)$ and $\mathrm{Re}(z_2) > 0$, For these, write the polynomial in terms of the natural frequency ω_0 and the damping ratio ξ. Draw plots for $\xi > 1$, $\xi = 1$, $\xi < 1$, $\xi = 0$

$$(s + z_1)(s + z_2) = s^2 + (z_1 + z_2)s + z_1 z_2 = s^2 + 2\xi\omega_0 s + \omega_0^2$$



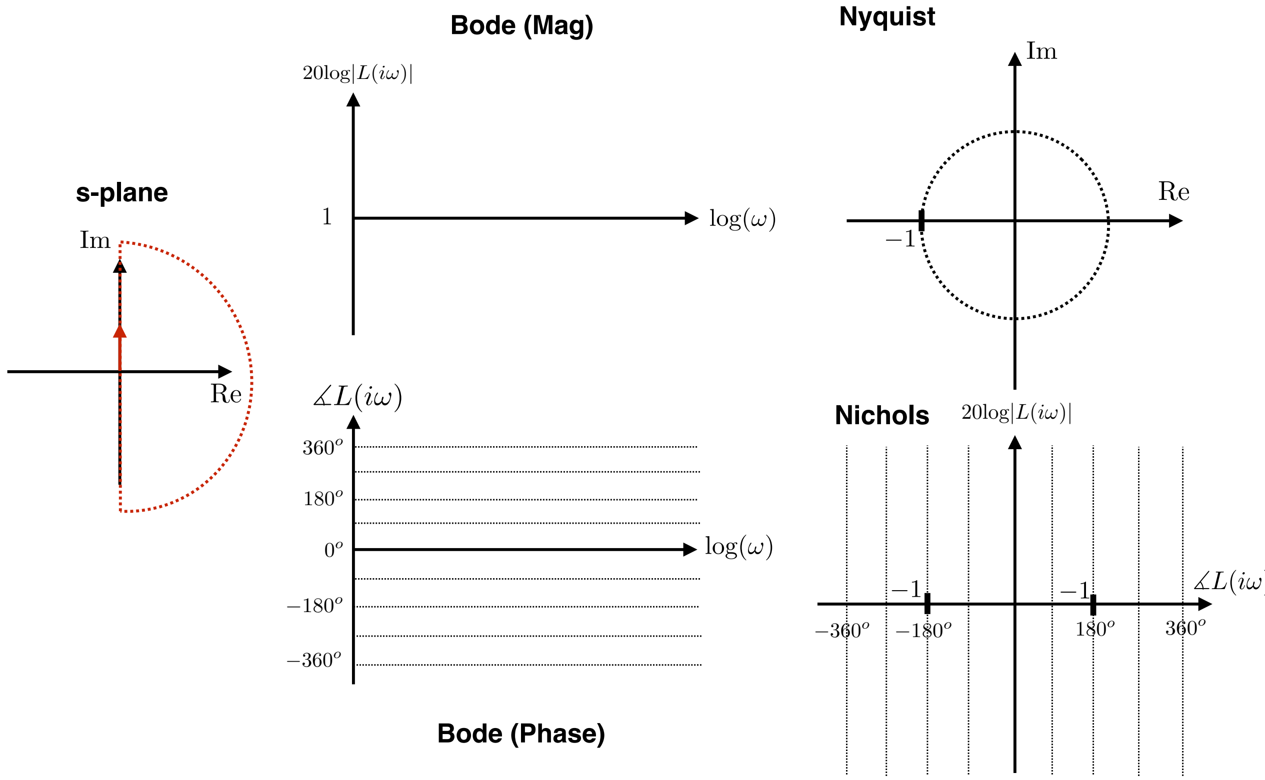
- $L(s) = \frac{1}{(s+p_1)(s+p_2)}$
  - Real p_1, p_2: p_1 and $p_2 = 0$, p_1 and $p_2 < 0$, p_1 and $p_2 > 0$, $p_1 < 0$ and $p_2 > 0$
  - Complex p_1, p_2: $\text{Re}(p_1)$ and $\text{Re}(p_2) = 0$, $\text{Re}(p_1)$ and $\text{Re}(p_2) < 0$, $\text{Re}(p_1)$ and $\text{Re}(p_2) > 0$, For these, write the polynomial in terms of the natural frequency ω_0 and the damping ratio ξ. Draw plots for $\xi > 1$, $\xi = 1$, $\xi < 1$, $\xi = 0$

$$\frac{1}{(s+p_1)(s+p_2)} = \frac{1}{s^2 + (p_1+p_2)s + p_1 p_2} = \frac{1}{s^2 + 2\xi\omega_0 s + \omega_0^2}$$

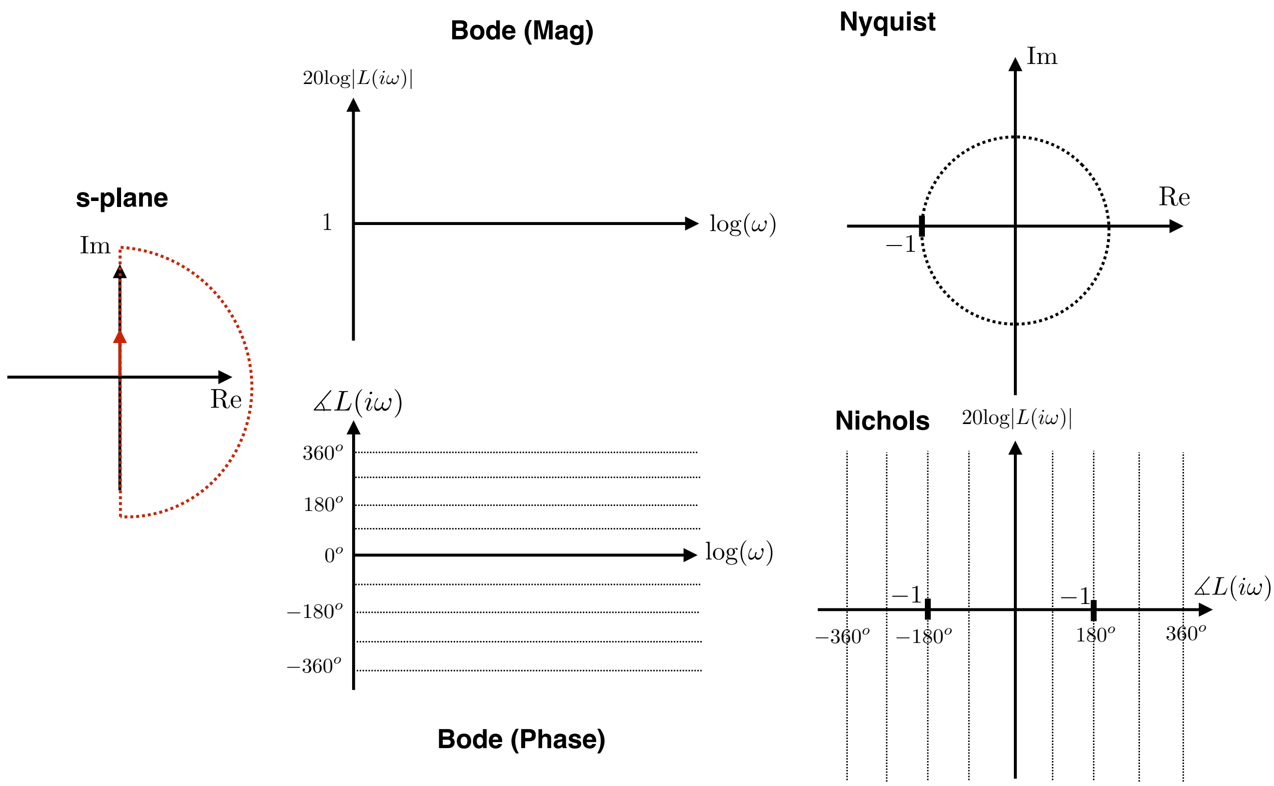


3. **General Transfer Functions**

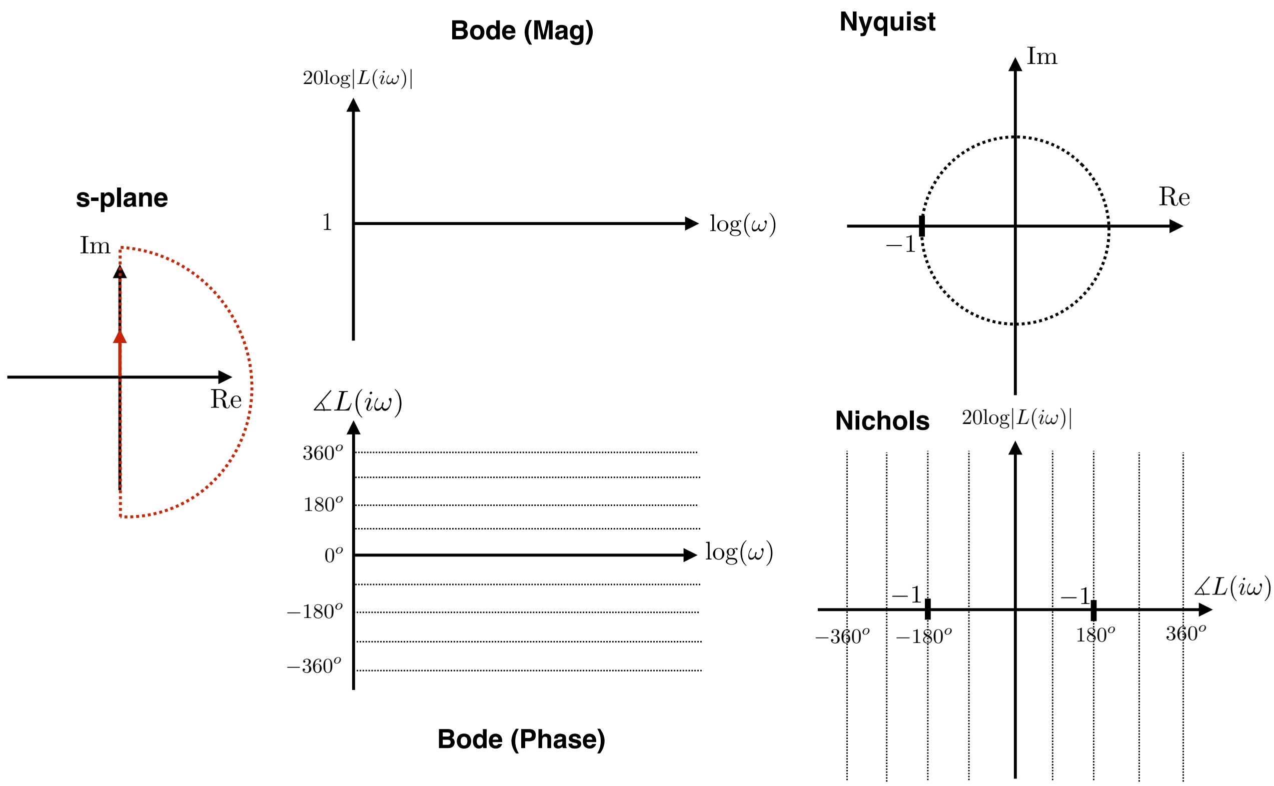
- $L(s) = \frac{s+z}{s+p}$: $z > p$, $z < p$, $z = 0$, $p = 0$.



- $L(s) = \frac{(s+z)}{(s+p_1)(s+p_2)}$
  - Real p_1, p_2: one stable-minimum phase, one not stable-minimum phase.
  - Complex p_1, p_2: one stable-minimum phase, one not stable-minimum phase.



- $L(s) = \frac{(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)}$
  - Real z_1, z_2, p_1, p_2: one stable-minimum phase, one not stable-minimum phase.
  - Complex z_1, z_2, p_1, p_2: one stable-minimum phase, one not stable-minimum phase.



- $L(s) = \frac{(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)(s+p_3)}$
  - Real z_1, z_2, p_1, p_2: one stable-minimum phase, one not stable-minimum phase.
  - Complex z_1, z_2, p_1, p_2: one stable-minimum phase, one not stable-minimum phase.