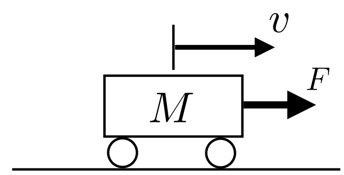
AA447 - Feedback Control - Spring 2021

Homework 7

Due Date: Friday, June 4^{th}, 2021 at 11:59 pm

1. **Loop-Shaping: Velocity Control**

   Consider the velocity control problem illustrated below.



   with $M = 1$.

   (a) **(PTS:0-2)** Create a bode plot of the plant $P(s)$. What is the gain crossover frequency ω_c?

   (b) **(PTS:0-2)** Design a controller to stabilize the closed loop system and reject ramp disturbances.

   (c) **(PTS:0-2)** Create a bode plot of the open-loop transfer function $L(s) = C(s)P(s)$ and plot the step response of the system. Compute the gain crossover frequency and the gain and phase margins.

   (d) **(PTS:0-2)** Adjust the DC gain and add a lead and/or lag compensator to satisfy the following requirements while maintaining system stability.

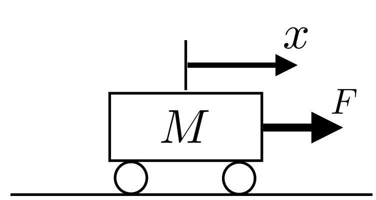
   - Zero steady state error
   - Bandwidth $>$ 1000 rad/sec
   - Max overshoot: 2%
   - Max rise time: 0.1 sec
   - Max settling time: 0.3 sec
   - Upper gain margin $>$ 10 dBs or lower gain margin $<$ 0.1 dBs
   - Phase margin $>$ 60 degs

   (e) **(PTS:0-2)** Write the final form of the controller $C(s)$ and the open loop transfer function $L(s)$. Create bode plots of $L(s)$ with the gain and phase margin labeled. Plot the final closed loop step response.

   (f) **(PTS:0-2)** Show that the closed loop system is still stable and use the final value theorem to show that the controller can reject ramp disturbances.

2. **Loop-Shaping: Position Control**

   Consider the position control problem illustrated below.



with $M = 1$.

(a) **(PTS:0-2)** Create a bode plot of the plant $P(s)$. What is the gain crossover frequency ω_c?

(b) **(PTS:0-2)** Design a controller to stabilize the closed loop system and reject ramp disturbances.

(c) **(PTS:0-2)** Create a bode plot of the open-loop transfer function $L(s) = C(s)P(s)$ and plot the step response of the system. Compute the gain crossover frequency and the gain and phase margins.

(d) **(PTS:0-2)** Adjust the DC gain and add a lead and/or lag compensator to satisfy the following requirements while maintaining system stability.

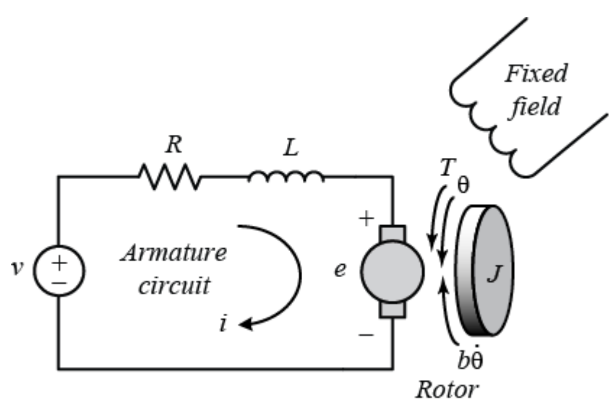
- Zero steady state error
- Bandwidth > 1000 rad/sec
- Max overshoot: 2%
- Max rise time: 0.1 sec
- Max settling time: 0.3 sec
- Upper gain margin > 10 dBs or lower gain margin < 0.1 dBs
- Phase margin > 60 degs

(e) **(PTS:0-2)** Write the final form of the controller $C(s)$ and the open loop transfer function $L(s)$. Create bode plots of $L(s)$ with the gain and phase margin labeled. Plot the final closed loop step response.

(f) **(PTS:0-2)** Show that the closed loop system is still stable and use the final value theorem to show that the controller can reject ramp disturbances.

3. **Loop-Shaping: DC Motor Control**

Consider the car and trailer illustrated below.



The equations of motion are given by

$$J\ddot{\theta} + b\dot{\theta} = Ki$$

$$L\frac{di}{dt} + Ri = V - K\dot{\theta}$$

where

- Motor angle: θ
- Current: i

- Voltage (input): v
- Moment of inertia of rotor: $J = 0.01$ kgm^2
- Motor viscous friction constant: $b = 0.1$ Nms
- Electromotive force constant: $K_e = 0.01$ V/rad/sec
- Motor torque constant: $K_t = 0.01$ Nm/Amp
- Resistance: $R = 1$ Ohm
- Inductance: $L = 0.5$ H

(a) **(PTS:0-2)** Write a state-space model for the system with state vector $x = [\dot{\theta}\ i]^T$ with the output being rotor speed $\dot{\theta}$.

(b) **(PTS:0-2)** Write the transfer function from voltage v to rotor speed $\dot{\theta}$.

$$P(s) = \frac{\dot{\Theta}(s)}{V(s)}$$

(c) **(PTS:0-2)** Create a bode plot of the plant $P(s)$. What is the gain crossover frequency ω_c?

(d) **(PTS:0-2)** Design a controller to stabilize the closed loop system and reject ramp disturbances.

(e) **(PTS:0-2)** Create a bode plot of the open-loop transfer function $L(s) = C(s)P(s)$ and plot the step response of the system. Compute the gain crossover frequency and the gain and phase margins.

(f) **(PTS:0-2)** Adjust the DC gain and add a lead and/or lag compensator to satisfy the following requirements while maintaining system stability.

- Zero steady state error
- Bandwidth > 1000 rad/sec
- Max overshoot: 2%
- Max rise time: 0.1 sec
- Max settling time: 0.3 sec
- Upper gain margin > 10 dBs or lower gain margin < 0.1 dBs
- Phase margin > 60 degs

(g) **(PTS:0-2)** Write the final form of the controller $C(s)$ and the open loop transfer function $L(s)$. Create bode plots of $L(s)$ with the gain and phase margin labeled. Plot the final closed loop step response.

(h) **(PTS:0-2)** Show that the closed loop system is still stable and use the final value theorem to show that the controller can reject ramp disturbances.