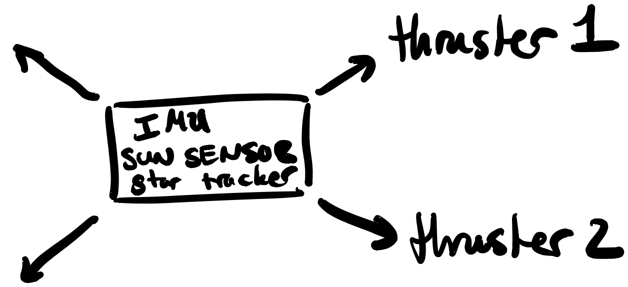
Feedback Control (AA447)

Feedback ctrl brings software and hardware together

CPS - Cyber Physical Systems

- Simple Example

A spacecraft (S/C) deep space



Sensors feed info to the S/C

- position
- velocity
- orientation
- angular velocity

$\underbrace{\hphantom{angular velocity}}$

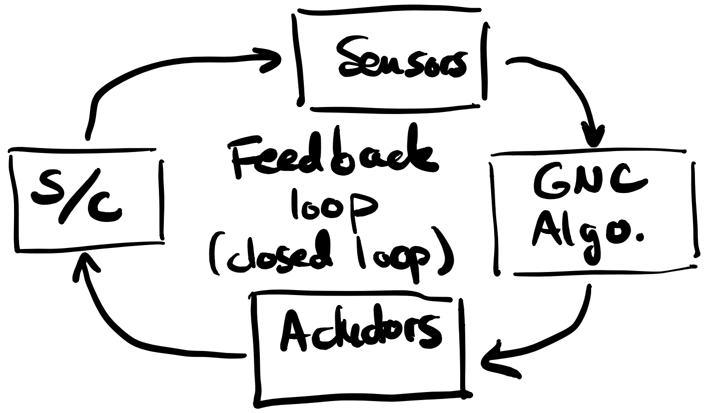
State of the system

This sensed information is used to make control decisions on board in real time.

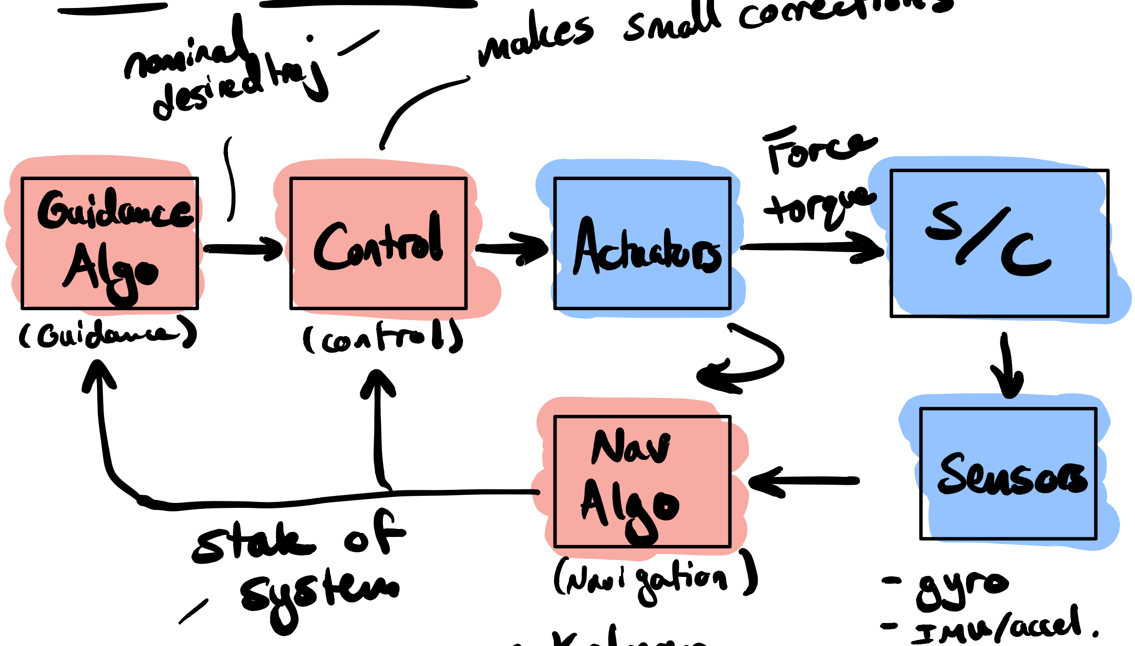
These decisions are made by GNC algorithms on board.

GNC
- Guidance
- Navigation
- Control

These decisions are communicated to actuators as commands



Block Diagram



- Kalman filter
- particle filter
- SLAM

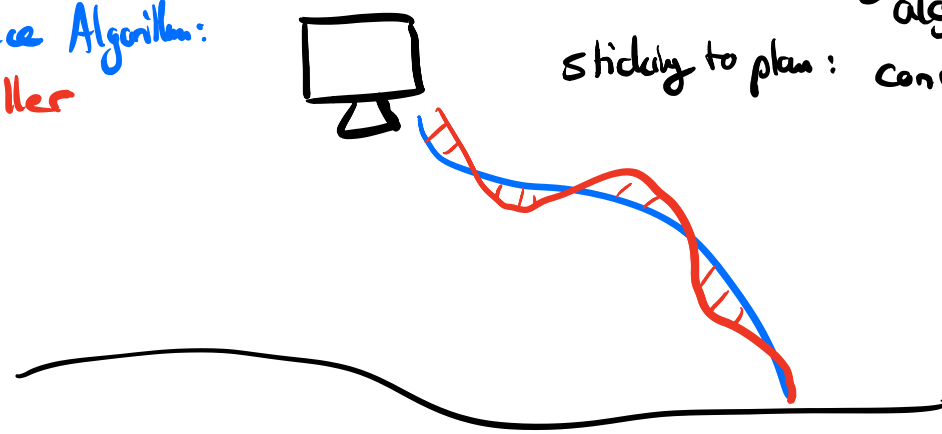
- gyro
- IMU/accel.
- camera
- thermal
- lidar
- radar

- ☐ : Dynamical system hardware & software
- ⟶ : Signal

Guidance vs. Control

Guidance Algorithm:
Controller

overall plan : guidance algorithm
sticking to plan : controller



Decision Heirarchy:

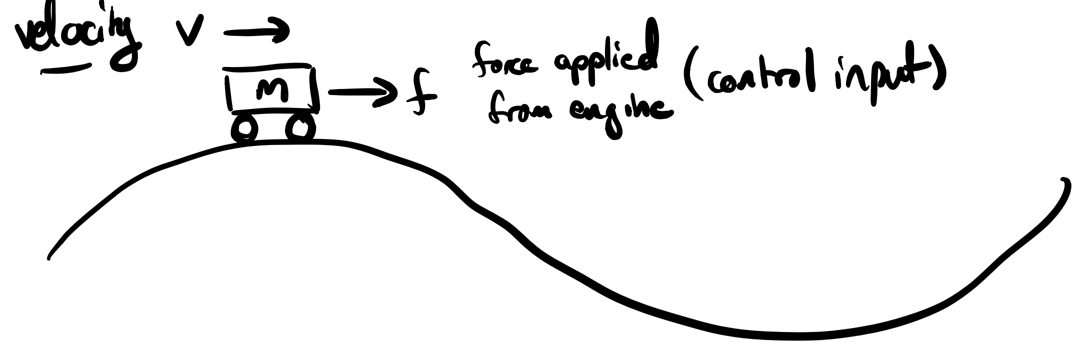
time scale	block	
hrs/days	Mission planner	overall mission plan
hrs/min	Mode Commander	switch between diff. parts of the mission
min/secs	Guidance	plan nominal trajectories ←
micro seconds	Controller	sticks to nominal trajectory
	↓ ←	actuator commands
	Actuators	
	↓ ←	forces & torques
	Vehicle	

focus of this class

each lower block passes performance constraints to block above it.

A SIMPLE FEEDBACK CONTROL SYS:

A car driving on a road ($\bar{w}$ simplifications)



Goal: $v(t) = v_d(t)$ desired velocity

u: (or f) control force

measure velocity. $y = v$

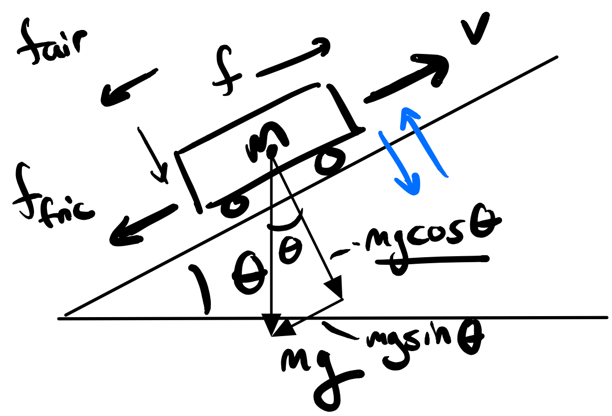
Uncertainty:

- mass (payload, fuel) ← want to design a controller to compensate
- friction (road & tires)
- drag (wind resistance, vehicle shape)
- gradient of terrain
- engine efficiency

System Model:

Newton's 2nd law

Free Body Diagram



$f_{grav} = -mg\sin\theta$

$f_{fric} = -\underbrace{\mu}_{\text{coeff of friction}} \operatorname{sgn}(v)\, mg\cos\theta$

$\operatorname{sgn}(v) = \begin{cases} 1 & \text{if } v > 0 \\ 0 & \text{if } v = 0 \\ -1 & \text{if } v < 0 \end{cases}$

drag coeff.

$f_{air} = -\frac{1}{2}\rho A C_d v|v|$

$|v|$: speed

effective area

air density.

$\beta = \frac{1}{2}\rho A C_d$

$f_{net} = f + f_{grav} + f_{fric} + f_{air}$

Dynamics

$$\boxed{m\dot{v} = f_{net} = f - \beta v|v| - mg\sin\theta - \mu mg \operatorname{sgn}(v)\cos\theta}$$

Dynamic Modeling:

→ - Newton's 2nd Law

- Euler-Lagrange eqns (Lagrangian mechanics)
- Conservation laws
  - energy
  - momentum
  - mass flow
- Kirchoff's laws (Voltage & Current Laws)

Non linear ODE (dynamics of v)
for control input f.

Uncertainties: m, β, θ, μ

- Measurement : $v \longrightarrow v + n$ → measurement noise
- control : $f \longrightarrow f + \underbrace{\gamma f}_{\text{proportional process noise}} + w$ → additive process noise

More accurate model $|\gamma| \leq \bar{\gamma}$ →

$$\boxed{\begin{aligned} m\dot{v} &= f + \gamma f + w - \beta v|v| - mg\sin\theta - \mu mg\cos\theta\,\mathrm{sgn}(v) \\ y &= v + n \end{aligned}}$$

Reminder/
$$\begin{cases} \dot{x} = F(t, x, u) \rightarrow \text{state dynamics} \\ y = H(t, x, u) \rightarrow \text{output.} \end{cases}$$

in this case: $x = v$, $u = f$

Control objective

Design a control algorithm that uses the meas. y to adjust f so that $v(t) \rightarrow v_d(t)$

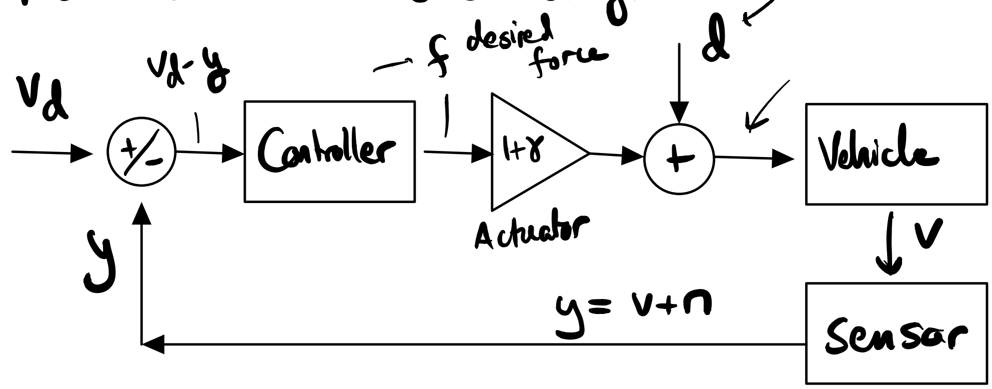
ie. velocity approaches desired velocity

External disturbance

$$d = \underline{w} - \underline{mg\sin\theta} - \underline{\mu mg\cos\theta\,\mathrm{sgn}(v)} - \underline{\beta v|v|}$$

whether or not treat air drag as disturbance or nonlinear piece of dynamics <u>judgement</u> <u>call</u>

more detailed block diagram:



$$m\dot{v} = (1+\gamma)f + d - \beta|v|v$$

$$y = v + n$$

start w/ simple case: $\beta = 0,\ d = 0,\ n = 0,\ \gamma = 0$

m : constant.

$$\begin{cases} m\dot{v} = f \\ y = v \end{cases}$$

want : $v \rightarrow v_d \rightarrow$ not known in advance

how to design controller...

- Simplest controller - Proportional Controller

$$f = -k_p(y - v_d)$$

$$= -k_p(v - v_d), \quad k_p > 0$$

↳ designed "proportional gain"

$$\Rightarrow m\dot{v} = -k_p(v - v_d)$$

let's define: $x \triangleq v - v_d$ → velocity tracking error

$$\Rightarrow \dot{x} = \dot{v} - \dot{v}_d = -\frac{k_p}{m}(\underbrace{v - v_d}_{x}) - \dot{v}_d$$

$$\boxed{\dot{x} = -\frac{k_p}{m}x - \dot{v}_d}$$ → dynamics for tracking error

$$x(0) = v(0) - v_d(0)$$

want x to go to 0

- Suppose v_d = constant $\Rightarrow \dot{v}_d = 0$

$$\Rightarrow \dot{x} = -\frac{k_p}{m}x \qquad x(0) = x_0$$ → initial tracking error

$$\alpha_p \triangleq \frac{k_p}{m} \Rightarrow \dot{x} = -\alpha_p x$$

$$\rightarrow \frac{dx}{dt} = -\alpha_p x,$$

$$\boxed{x(t) = e^{-\alpha_p t} x(0)}$$

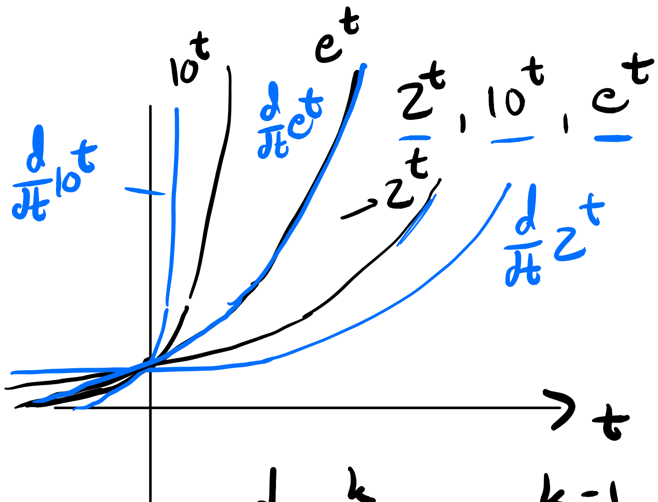
$$\frac{d}{dt}x(t) = \frac{d}{dt}\left(e^{-\alpha_p t} x_0\right) = \frac{d}{dt}\left(e^{-\alpha_p t}\right) x_0 = -\alpha_p \underbrace{e^{-\alpha_p t} x_0}_{x(t)}$$

$$\frac{d}{dt}x(t) = -\alpha_p\, x(t) \Rightarrow \dot{x} = -\alpha_p x$$

Note:

$$\frac{d}{dt}e^t = e^t$$

Definition of e



$$\frac{d}{dt}e^t = e^t$$

$$\frac{d}{dt}t^k = k\,t^{k-1}$$

$$\frac{d}{dt}e^{-\alpha t} = e^{-\alpha t}(-\alpha)$$

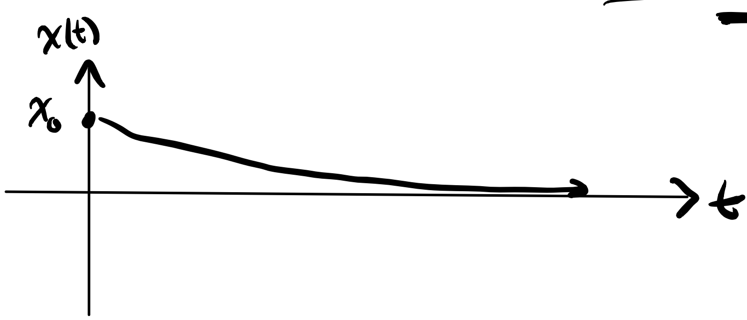
ODE

$$\dot{x} = -\alpha_p x \quad \Rightarrow$$

$$x(0) = x_0$$

Solution

$$x(t) = e^{-\alpha_p t} x_0 \qquad \alpha_p > 0$$



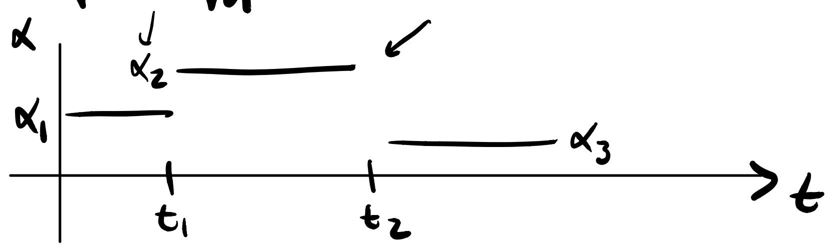
$$\lim_{t\to\infty} x(t) = 0$$

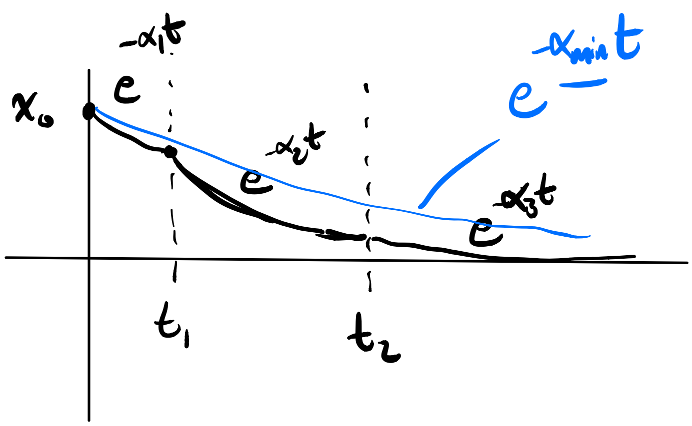
for any x_0

• Suppose m is not constant.

→ $\alpha_p = \frac{k_p}{m}$ → not constant $\alpha_p > 0$

different decay rates





$$\lim_{t\to\infty} x(t) = 0$$

$$\alpha(t) \ge \alpha_{min} > 0$$