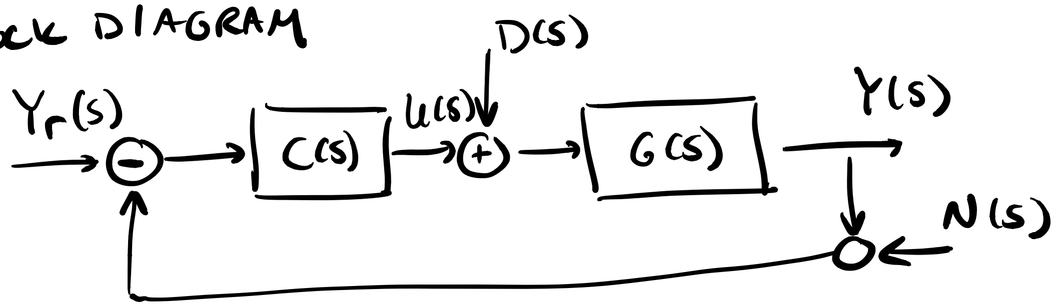
Review:

PLANT & CONTROLLER

BLOCK DIAGRAM



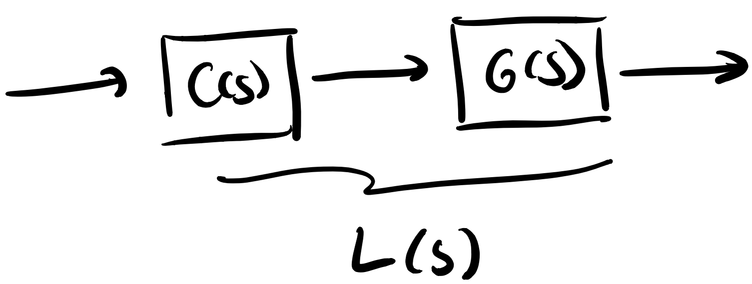
$$Y(s) = G(s)\big(U(s) + D(s)\big)$$

$$U(s) = C(s)\big(Y_r(s) - Y(s) - N(s)\big)$$

$$Y(s) = G(s)C(s)\big(Y_r(s) - Y(s) - N(s)\big) + G(s)D(s)$$

$$\big(1 + G(s)C(s)\big)Y(s) = G(s)C(s)\big[Y_r(s) - N(s)\big] + G(s)D(s)$$

$L(s) = G(s)C(s)$: open loop transfer function



$$Y(s) = \frac{L(s)}{1+L(s)}\left[Y_r(s) - N(s)\right] + \frac{G(s)}{1+L(s)}D(s)$$

$\underbrace{1 + L(s)}_{\text{roots determine the stability...}} = 0$ characteristic eqn.

BIBO stability: roots of $1+L(s)$ in the OLHP.

Track a reference signal $Y_r(s)$...

Error: $E(s) = Y(s) - Y_r(s)$

$$E(s) = \frac{L(s)}{1+L(s)}\left[Y_r(s) - N(s)\right] + \frac{G(s)}{1+L(s)}D(s) - Y_r(s)$$

$$= \left(\frac{L(s)}{1+L(s)} - 1\right)Y_r(s) - \frac{L(s)}{1+L(s)}N(s) + \frac{G(s)}{1+L(s)}D(s)$$

$$= \underbrace{\frac{-1}{1+L(s)}Y_r(s)}_{\substack{\text{reference}\\\text{signal}\\\text{(tracking)}}} + \underbrace{\frac{G(s)}{1+L(s)}D(s)}_{\substack{\text{disturbance}\\\text{(disturbance}\\\text{rejection)}}} - \frac{L(s)}{1+L(s)}N(s)$$

want no steady state error from

FINAL VALUE THM:

$f(t)$ $\bar{w}$ Laplace Transform $F(s)$

$F(s)$ has poles in OLHP $\bar{w}$ at most 1 pole at the origin.

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

$$= \lim_{s \to 0} \frac{\beta_k s^k + \cdots + \beta_1 s + \boxed{\beta_0}}{\alpha_n s^n + \cdots + \alpha_1 s + \boxed{\alpha_0}}$$

$$E(s) = \frac{-1}{1+L(s)} Y_r(s) + \frac{G(s)}{1+L(s)} D(s) - \frac{L(s)}{1+L(s)} N(s)$$

$$L(s) = G(s)\,C(s) = \frac{num_G}{den_G} \frac{num_C}{den_C}$$

Tracking

$$\frac{-1}{1+L(s)} Y_r(s) = \frac{-den_G den_C}{den_G den_C + num_G num_C} Y_r(s)$$

Disturbance Rejection

$$\frac{G(s)}{1+L(s)} D(s) = \frac{num_G den_C}{den_G den_C + num_G num_C} D(s)$$

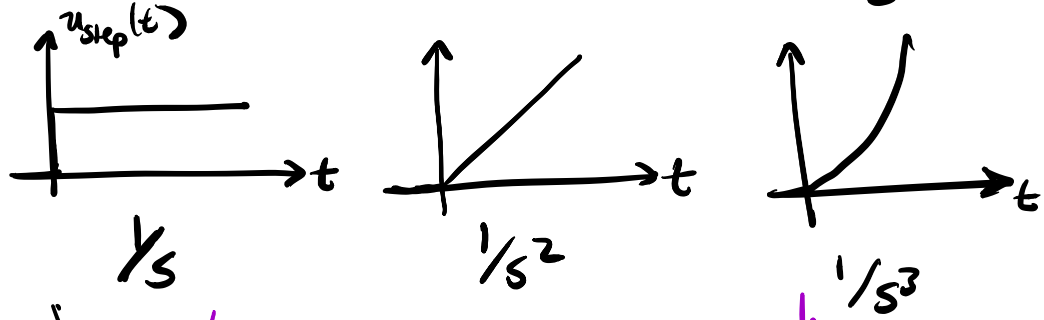
WANT

$$\lim_{s \to 0} s \frac{-1}{1+L(s)} Y_r(s) = 0$$

$$\lim_{s \to 0} s \frac{G(s)}{1+L(s)} D(s) = 0$$

Lets assume $Y_r(s) = \frac{1}{s^k}$ or $D(s) = \frac{1}{s^k}$

$\frac{1}{s^k}$:



want

$$\lim_{s\to 0} \frac{\boxed{den_G}\,\boxed{den_C}}{\left(\boxed{den_G}\,den_C + \boxed{num_G}\,num_C\right.} \frac{s}{\boxed{s^k}} = 0 \qquad \lim_{s\to 0} \frac{\boxed{num_G}\,den_C}{\boxed{den_G}\,den_C + \boxed{num_G}\,\boxed{num_C}} \frac{s}{\boxed{s^k}} = 0$$

trying to get a constant term in the denominator of these expressions & no constant term in the top...

set $den_C(s) = \boxed{s^k}$

what if $den_C(s) = s^{k-1}$

works for tracking but not for disturbance rejection

1

get

$$\lim_{s\to 0} \frac{\cancel{den_G}\ \ ms \text{ or } ms^2}{den_G den_C + num_G num_C} \qquad \lim_{s\to 0} \frac{\cancel{num_G}\,\cancel{den_C}\,s}{den_G den_C + num_G num_C} \frac{\cancel{s}}{\cancel{s^k}}$$

1

single integrator or double integrator

$$\frac{1}{ms}, \quad \frac{1}{ms^2} \Big\}\ \text{Newton's 2nd law.}$$

velocity control $\qquad$ position control

once we've picked the den_c
(for disturbance rejection / tracking)
pick the $num_c(s)$ for stability:

$$den_G(s)\underbrace{\boxed{den_c(s)}}_{\text{picked}} + num_G(s)\underbrace{num_c(s)}_{\text{DOF for stability.}} = 0$$

want roots in OLHP

Ex. $G(s) = \frac{1}{ms^2}$

$$ms^2(s^k) + 1(num_c(s)) = 0$$

$$ms^{k+2} + num_c(s) = 0.$$

- all terms of $num_c(s)$ need positive coeffs.

$$num_c(s) = \beta_{k+1}s^{k+1} + \cdots + \beta_1 s + \beta_0$$

$\beta_{k+1}, \ldots, \beta_1, \beta_0 > 0$ necessary for stability.

determine conditions on β's using the Routh-Hurwitz test.

RH Test:

$$s^5 + bs^4 + cs^3 + ds^2 + es + f = 0$$

$$
\begin{array}{lllll}
s^5 & 1 & c & e & 0 \\
s^4 & \underline{b} & d & f & 0 \\
s^3 & \boxed{c-\frac{d}{b}} \; \frac{bc-1d}{b} & \boxed{e-\frac{f}{b}} \; \frac{be-f}{b} & 0 & \\
s^2 & \boxed{d-\frac{b^2e-fb}{bc-d}} & f & 0 & \\
s^1 & \boxed{\frac{eb-f}{b}-\frac{(bcf-df)(bc-d)}{bd(bc-d)-(b^2e-fb)b}} & 0 & & \\
s^0 & f & & &
\end{array}
$$

$$d-\frac{be-f}{c-d/b}$$

$$e-\frac{f}{b}-\frac{cf-\frac{df}{b}}{d-\frac{b^2e-fb}{bc-d}}$$

$$\underline{bcf-df}$$

$$\frac{eb-f}{b}-\frac{(bcf-df)(bc-d)}{bd(bc-d)-(b^2e-fb)b}$$

Conditions

$1 > 0$

$b > 0$

$c > \frac{d}{b}$

$d - \frac{b^2e - fb}{bc - d} > 0$

$\frac{eb-f}{b} - \frac{(bcf - df)(bc-d)}{bd(bc-d) - (b^2e - fb)b} > 0$

$f > 0$

could show that these conditions imply that $b, c, d, e, f > 0$ $\Leftarrow$

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stability for	Nec & suff
• $s^2 + bs + c$	$b, c > 0$
• $s^3 + as^2 + bs + c$	$a, b, c > 0$ $ab > c$

Disturbance Rejection for Sinusoidal Input and Tracking.

$$D(s) = \frac{\omega}{s^2 + \omega^2} = \frac{\omega}{(s + i\omega)(s - i\omega)}$$

$$d(t) = \sin(\omega t)$$

Tracking

$$\lim_{s\to 0} \frac{den_G\, den_C}{(den_G\, den_C + num_G\, num_C)} \frac{s\,\omega}{s^2+\omega^2}$$

Disturbance Rejection

$$\lim_{s\to 0} \frac{num_G\, den_C}{den_G\, den_C + num_G\, num_C} \frac{s\,\omega}{s^2+\omega^2}$$

these are only marginally stable

want to set $den_C = s^2+\omega^2$

what if now there is a step input?

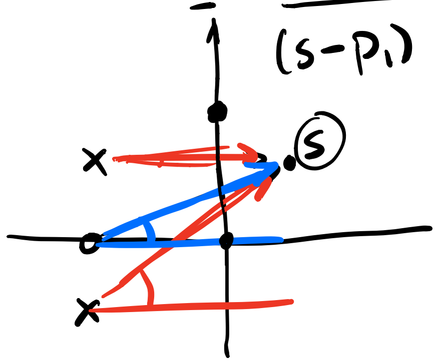
$$\lim_{s\to 0} \frac{\cancelto{1}{num_G}\,(s^2+\omega^2)}{den_G\, den_C + num_G\, num_C} \frac{\cancel{s}\,\omega}{s^{k-1}} \rightarrow$$ won't have disturbance rejection for step inputs

need $den_C(s) = s^k(s^2+\omega^2)$

Transfer Functions

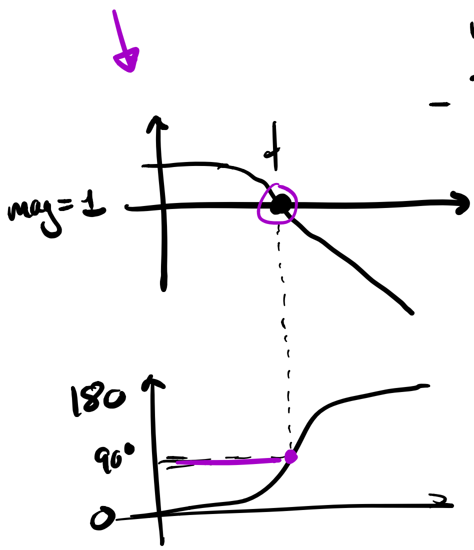
$$G(s) = \frac{\beta_k s^k + \cdots + \beta_1 s + \beta_0}{\alpha_n s^n + \cdots + \alpha_1 s + \alpha_0}$$

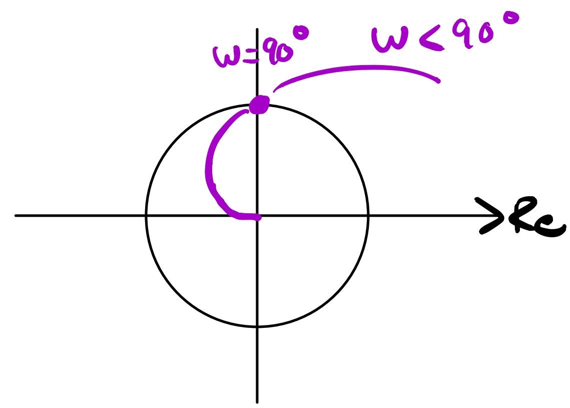
$$= \frac{(s-z_1)\cdots(s-z_k)}{(s-p_1)\cdots(s-p_n)}$$



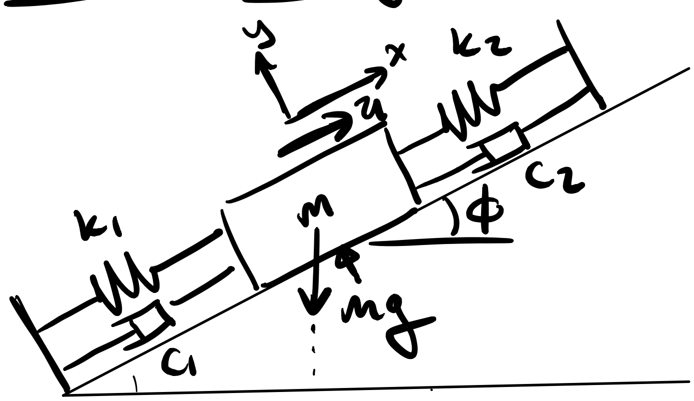
$G(i\omega)$

$$G(s) = \underbrace{\frac{\boxed{\prod_j |s-z_j|}}{\boxed{\prod_{j'} |s-p_{j'}|}}}_{\text{magnitude - Bode plot}} \; e^{\,i\left(\underbrace{\boxed{\sum_j \angle(s-z_j)} - \boxed{\sum_{j'} (s-p_{j'})}}_{\text{phase Bode plot.}}\right)}$$





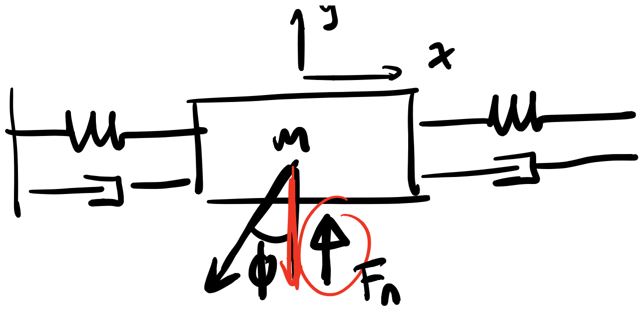
Dynamics Modeling:



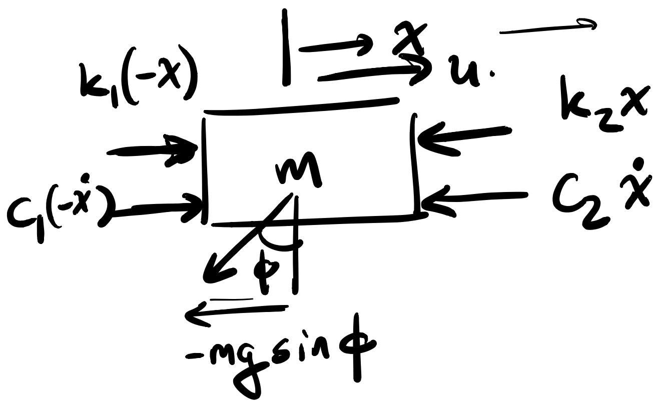
Deriving Eqns of Motion:

FBD: $\sum F_x = m\ddot{x}$

$\sum F_y = m\ddot{y}$



$$\Sigma F_y = F_n - mg\cos\phi = 0 \quad \rightarrow$$
$$F_n = mg\cos\phi$$



$$\Sigma F_x = k_1(-x) - k_2 x$$
$$c_1(-\dot{x}) - c_2\dot{x} - mg\sin\phi + u = m\ddot{x}$$

$$\ddot{x} = \frac{1}{m}\left(-(k_1+k_2)x - (c_1+c_2)\dot{x} - mg\sin\phi + u\right)$$

$$z = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{(k_1+k_2)}{m} & -\frac{(c_1+c_2)}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u + \begin{bmatrix} 0 \\ g\sin\phi \end{bmatrix}$$

$$s^2 X(s) - \dot{x}(0) - s\,x(0) = \frac{-(k_1+k_2)}{m} X(s) - \frac{c_1+c_2}{m}\left(sX(s) - x(0)\right) - \frac{1}{s} g \sin\phi + \frac{1}{m} U(s)$$

$$\left(s^2 + \frac{c_1+c_2}{m}s + \frac{k_1+k_2}{m}\right) X(s) = \dot{x}(0) + s\,x(0) + \frac{c_1+c_2}{m}x(0) - g\sin\phi\,\frac{1}{s} + \frac{1}{m}U(s)$$

$$X(s) = \frac{1}{\left(s^2 + \frac{c_1+c_2}{m}s + \frac{k_1+k_2}{m}\right)}\left(\cancel{\dot{x}(0)} + s\,\cancel{x(0)} + \frac{c_1+c_2}{m}\cancel{x(0)} - \frac{g\sin\phi}{s}\right) + \frac{1}{\underbrace{m\left(s^2 + \frac{c_1+c_2}{m}s + \frac{k_1+k_2}{m}\right)}} U(s)$$