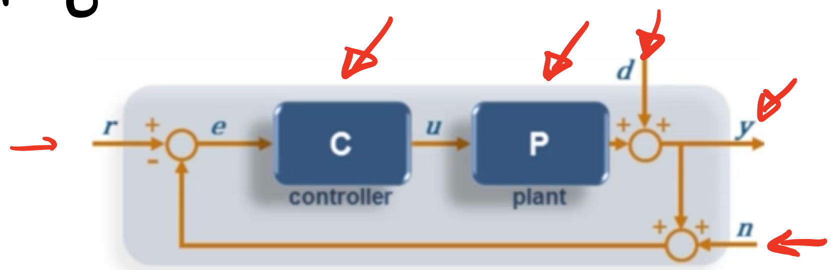
LOOPSHAPING

"shaping the open loop transfer function"



$L = PC$

open loop transfer function

Closed Loop Transfer Functions:

$$y = \boxed{\frac{PC}{(1+PC)}}(r-n) + \boxed{\frac{1}{(1+PC)}}d$$

T complementary sensitivity　　**S** sensitivity

$$e = \boxed{\frac{1}{(1+PC)}}(r-d-n)$$

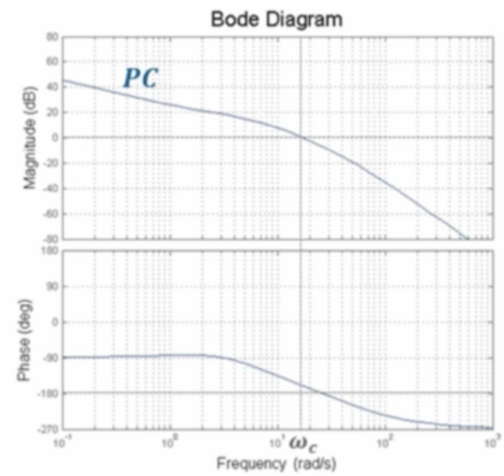
$$T = \frac{PC}{1+PC} \qquad S = \frac{1}{1+PC}$$

Complementary Sensitivity　　Sensitivity

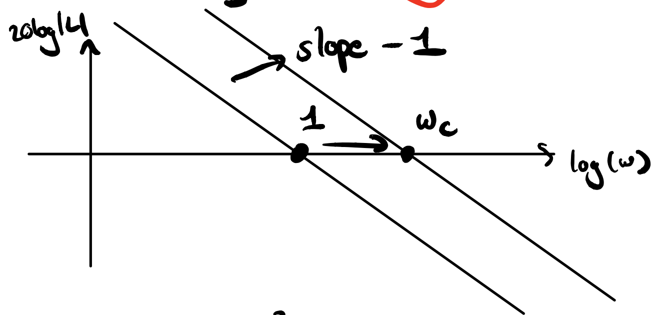
$$S + T = 1$$

$y = PCr$　why not just set $C = P^{-1} \Rightarrow y = r$

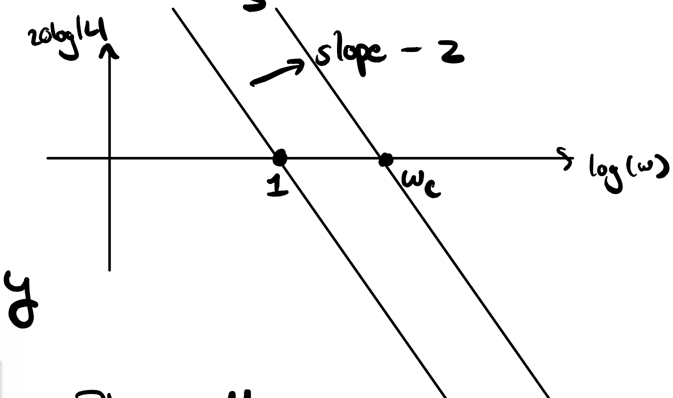
- if denom of P is higher order than the numerator $C = P^{-1}$ is not proper (not causal)
- if P has RHP zeros, $C = P^{-1}$ won't be stable
- P might not be a perfect model



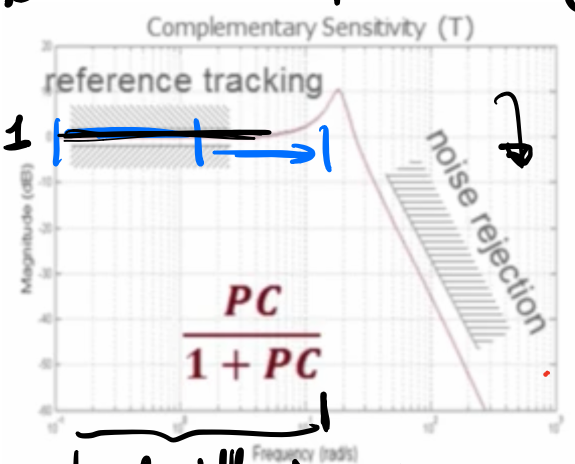
$L(s) = \frac{\omega_c}{s}$



$L(s) = \frac{\omega_c^2}{s^2}$



Reference to output : $r \longrightarrow y$



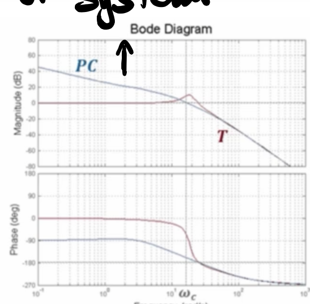
band width of system

Open Loop vs. T

$T = \frac{PC}{(1+PC)}$

for: $PC \gg 1$
$T \approx 1$

for: $PC \ll 1$
$T \approx PC$



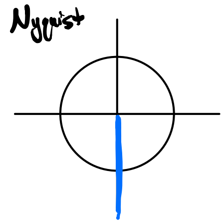
Phase Margin

$L(s) = \frac{1}{s}$

$L(i\omega) = \frac{1}{i\omega} \cdot \frac{i}{i} = -i\omega$

$\measuredangle L(i\omega) = -90^\circ$

phase margin -90°

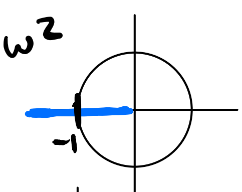


$L(s) = \frac{1}{s^2}$

$L(i\omega) = \frac{1}{i^2\omega^2} = -\omega^2$

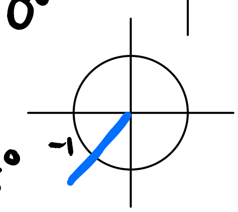
$\measuredangle L(i\omega) = -180^\circ$

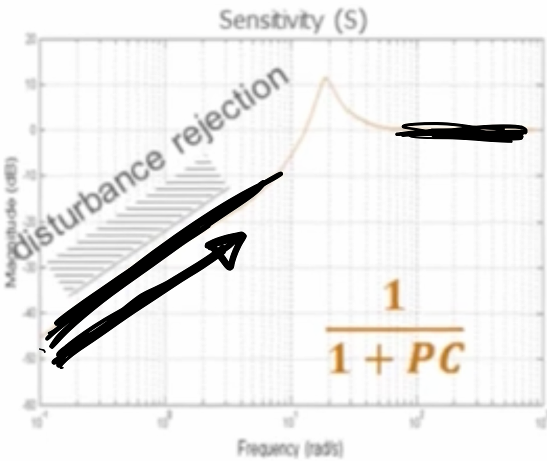
phase margin 0°

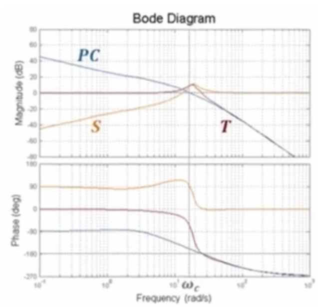


$L(s) = \frac{1}{s^{1.5}}$

phase margin -45°







Open Loop vs. S

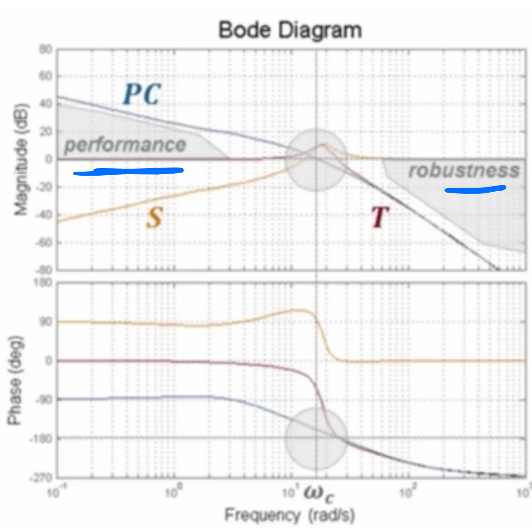
$$S = \frac{1}{(1+PC)}$$

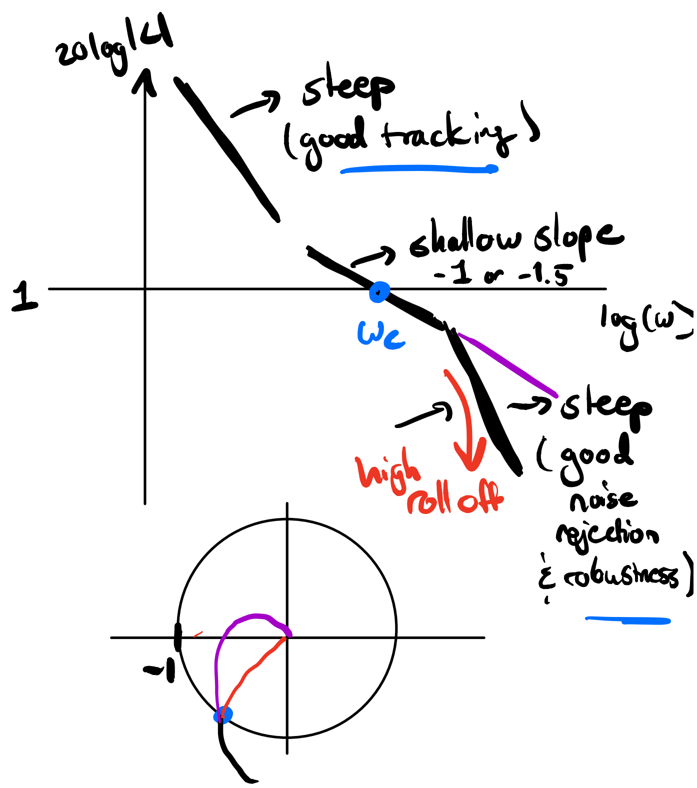
for: $PC \gg 1$

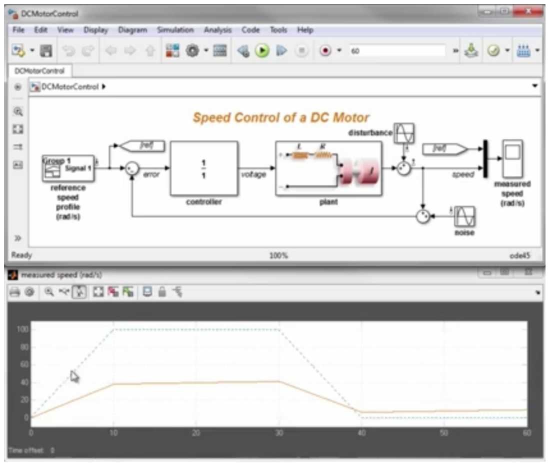
$$S \approx \frac{1}{PC}$$

for: $PC \ll 1$

$$S \approx 1$$





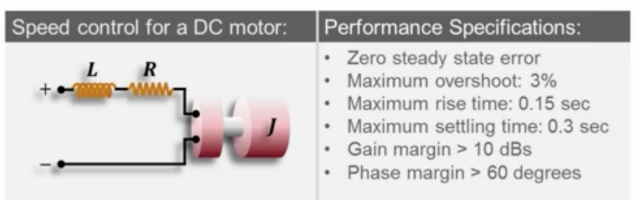


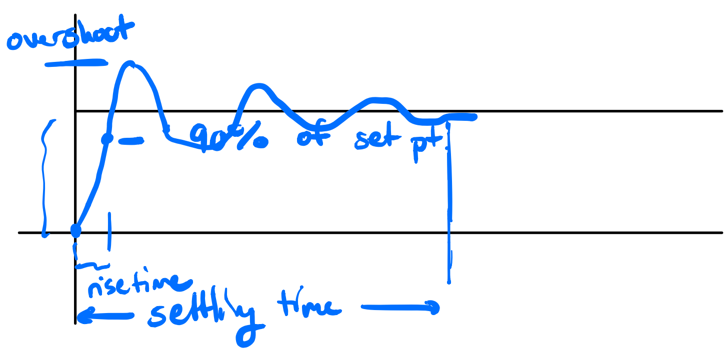
- stability
- zero steady state error — long term
- Max overshoot 3%
- Max rise time 0.15 sec
- Max settling time 0.3 sec

} short term transient

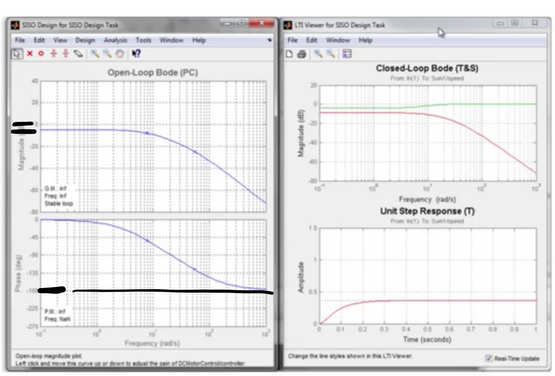
- Gain margin > 10 dBs
- Phase margin > 60 deg

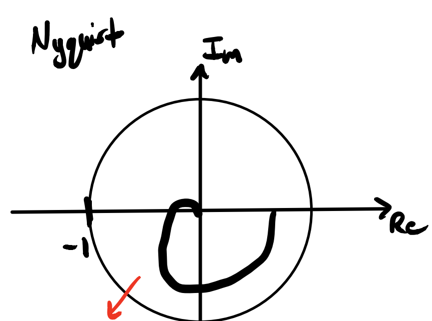
slope of L at cross over of ≈ 1.3



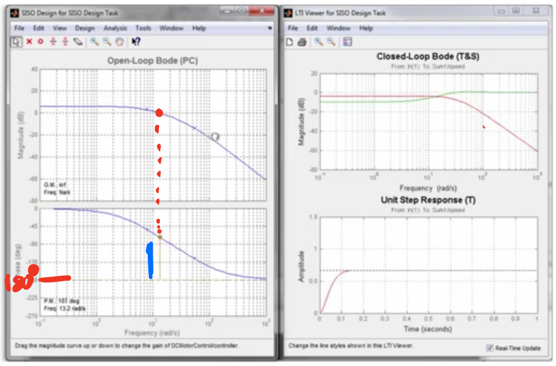


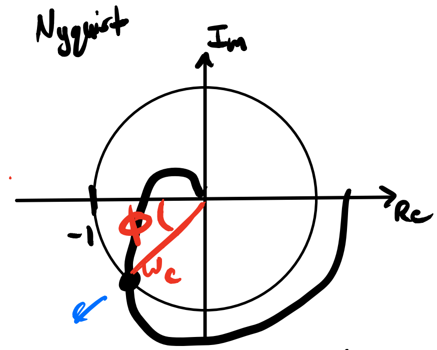
Phase Margin and Transient Specifications





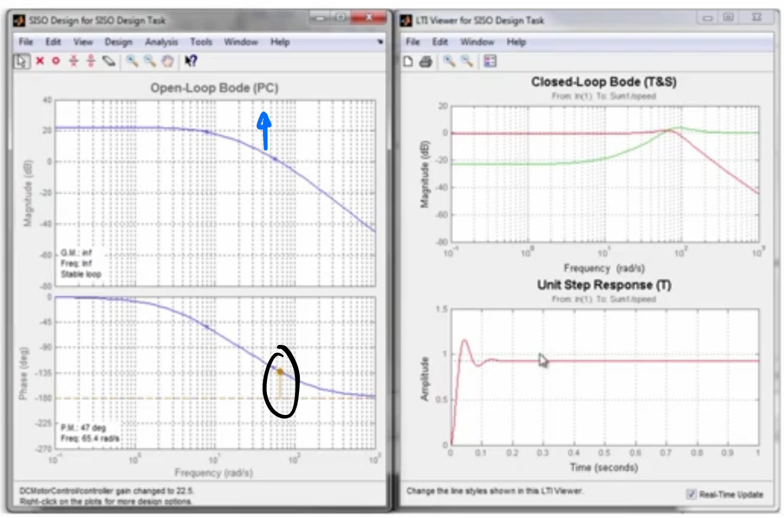
infinite phase margin



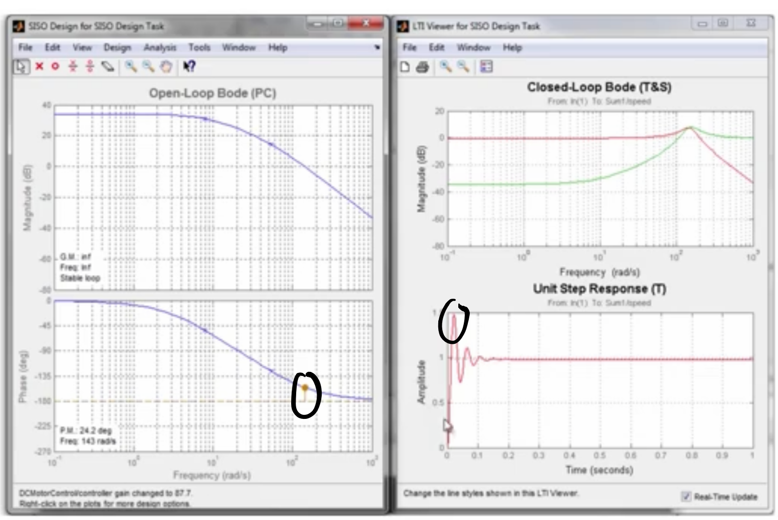


phase margin : ϕ

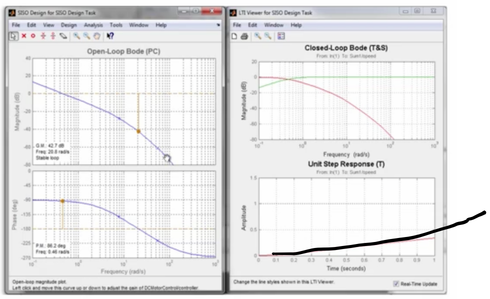
Proportional Controller

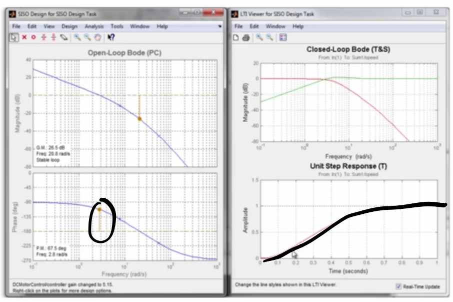


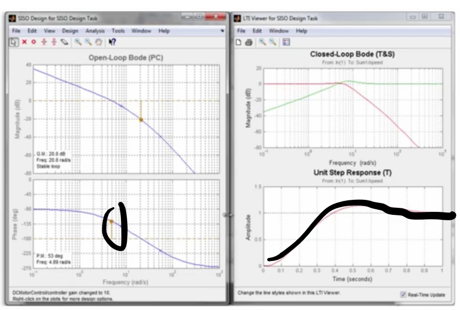
Decreased phase margin results in more oscillations
– more overshoot etc.
and if the phase margin goes to 0 the oscillations go out of control
ie. the system goes unstable

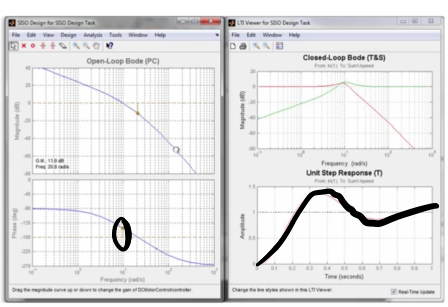


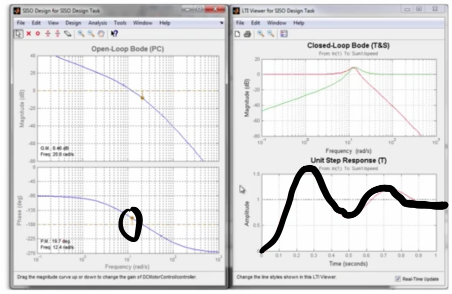
Proportional Integral Controller

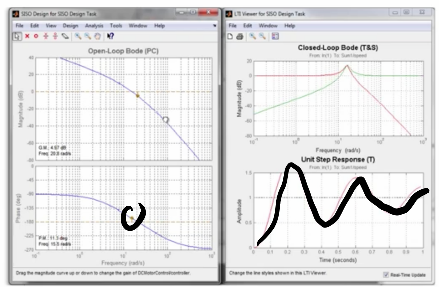




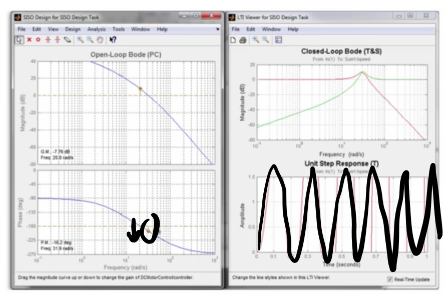








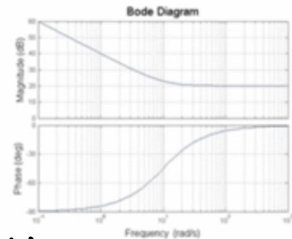
unstable:



Components for Loop Shaping:

$C(s) = C_1(s)C_2(s)\cdots$

PI Compensator:	
drives steady state error to zero, improves disturbance rejection	$C = K_P + \frac{K_I}{s}$



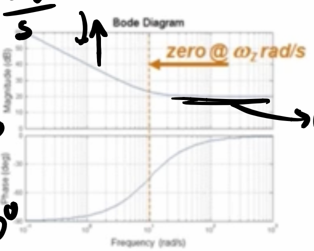
$k_p + \frac{k_I}{s}$

$\frac{k_p s + k_I}{s}$

$\frac{k_p\left(s + \frac{k_I}{k_p}\right)}{s}$

PI Compensator:	
drives steady state error to zero, improves disturbance rejection	$C = G_0\left(\frac{s+\omega_Z}{s}\right)$

$G_0\frac{\omega_z}{s}$



0°

-90°

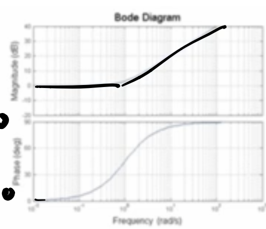
$\to G_0\frac{s}{s}$

steeper slope initially.

PI compensator:
- integrator initial region
- constant final region

90°

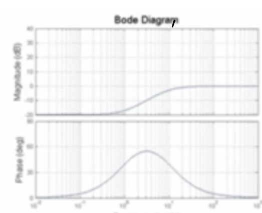
0°



PD Compensator:	
$C = K_P + K_D s$	adds phase lead, improves phase margin, improves damping, speeds up response

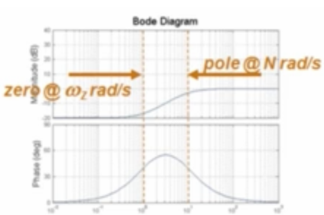
PD compensators: aren't proper
not a realizable controller.

Different version:



PD Compensator:	
$C = K_P + \frac{K_D s}{s+N}$	adds phase lead, improves phase margin, improves damping, speeds up response

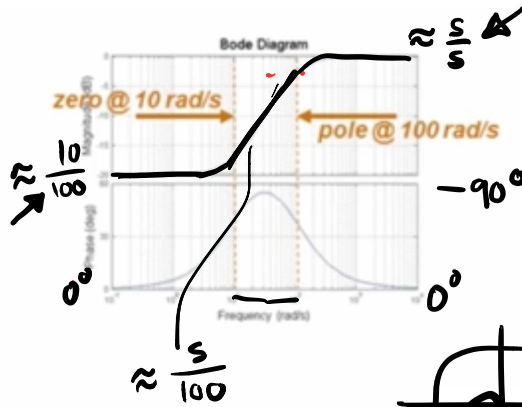
this is proper

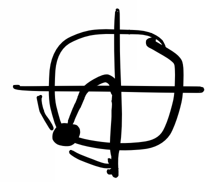


PD Compensator:	
$C = G_0\left(\frac{s+\omega_Z}{s+N}\right)$	adds phase lead, improves phase margin, improves damping, speeds up response

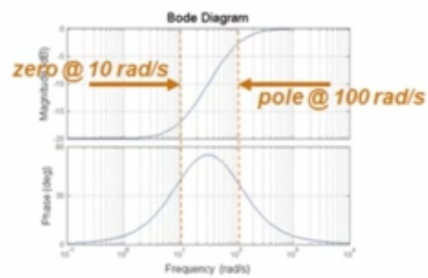
Lead Compensator:	
adds phase lead, improves phase margin, improves damping, speeds up response	$C = \dfrac{s+10}{s+100}$

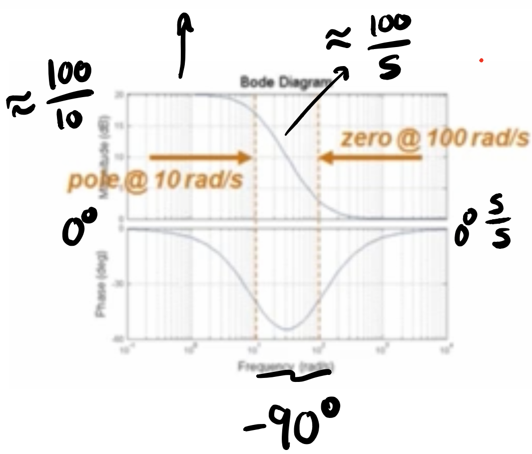
act like PD controllers





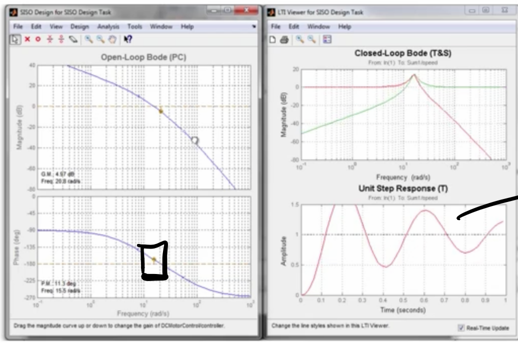
Lead Compensator:	
adds phase lead, improves phase margin, improves damping, speeds up response	$C = \dfrac{10(0.1s+1)}{100(0.01s+1)}$





Lag Compensator:	
$C = \dfrac{s+100}{s+10}$	adds phase lag, improves disturbance rejection, slows down response

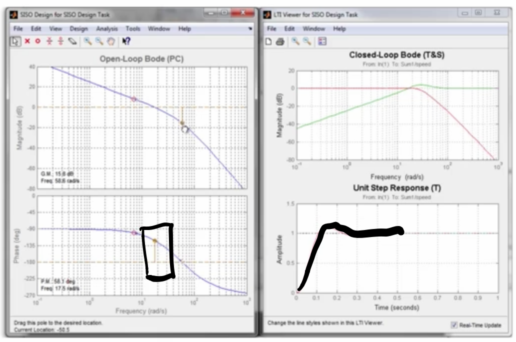
- increase magnitude of L in the tracking region w/out affecting mag in later region (good tracking and disturbance rejection)

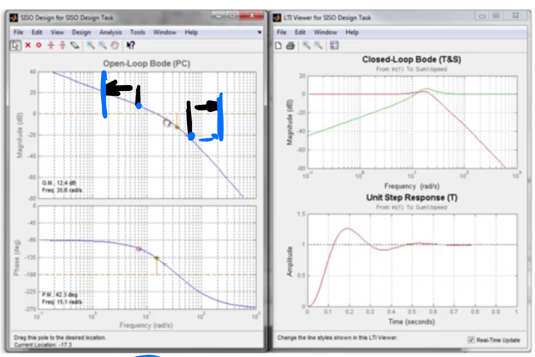


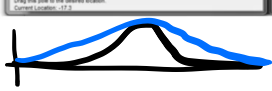
PI controller w/
small phase margin

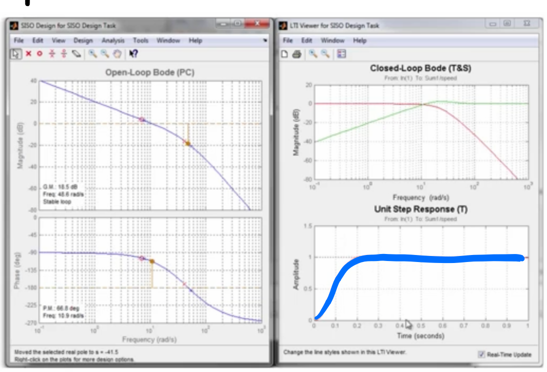
lots of oscillation.

Add a lead compensator









Ex.

$$C(s) = \underbrace{G_0}_{\text{constant gain}} \overbrace{\underbrace{\left(\frac{s+\omega_z}{s}\right)}_{\text{PI compensator}}}^{\frac{\omega_z}{s}} \overbrace{\underbrace{\left(\frac{s+\omega_1}{s+\omega_2}\right)}_{\text{lead compensator}}}^{\frac{\omega_1}{\omega_2}}$$

G_0