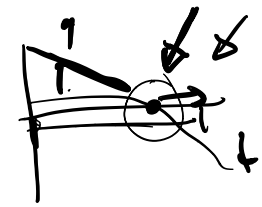
Loop Shaping Process:

- basic controller that
  - stabilizes
  - rejects disturbance (step, ramp)

- Loop shaping.

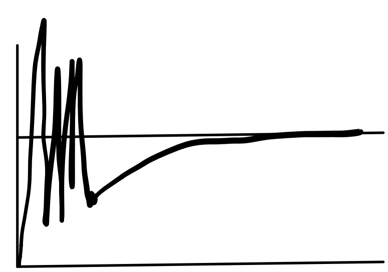
pick order
of denominator

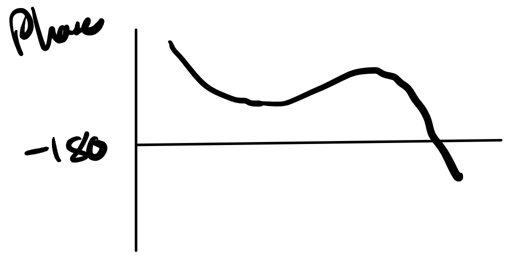


  - increase initial region (PI component
    lag compensator
    increase DC gain)
  - decrease final region (higher order denom. than numerator
    integrator
    lag compensator)
  - improving phase margin (lead compensator)

Note: lower order controllers
$\Rightarrow$ easier to understand.

Note: step response





More on Robustness

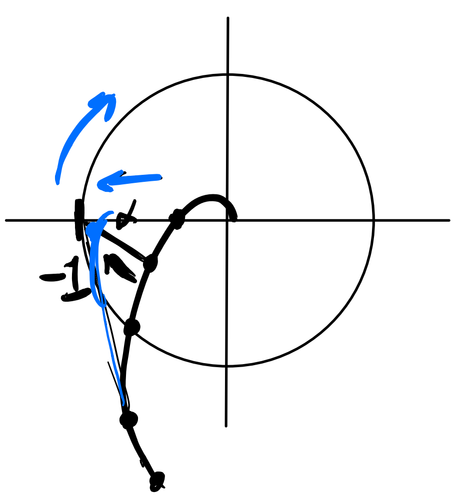
Limits on what you can control

Sensitivity function:

open loop TF : $L(s)$

sensitivity $S(s) = \dfrac{1}{1+L(s)}$

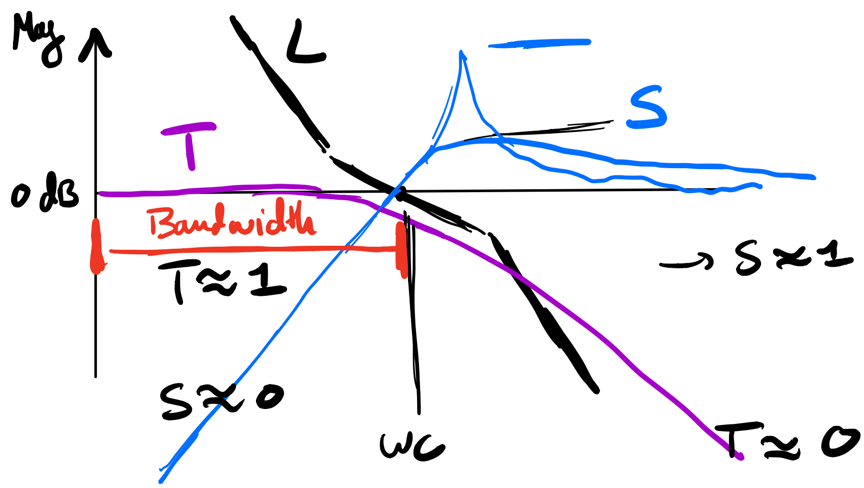
Nyquist Plot:



$$|S(s)| = \frac{1}{|-1 - L(s)|}$$

$$\max_{\omega} |S(i\omega)| = \max_{\omega} \frac{1}{|-1-L(i\omega)|}$$

Bode Plot: $T = \frac{L}{1+L}$ $S + T = 1$



Bandwidth:

set of frequencies w/ good tracking.

depends crossover frequency ω_c

faster system response requires larger bandwidth

How big can we make the bandwidth of the controller w/out screwing up robustness?

Bode Sensitivity Integrals

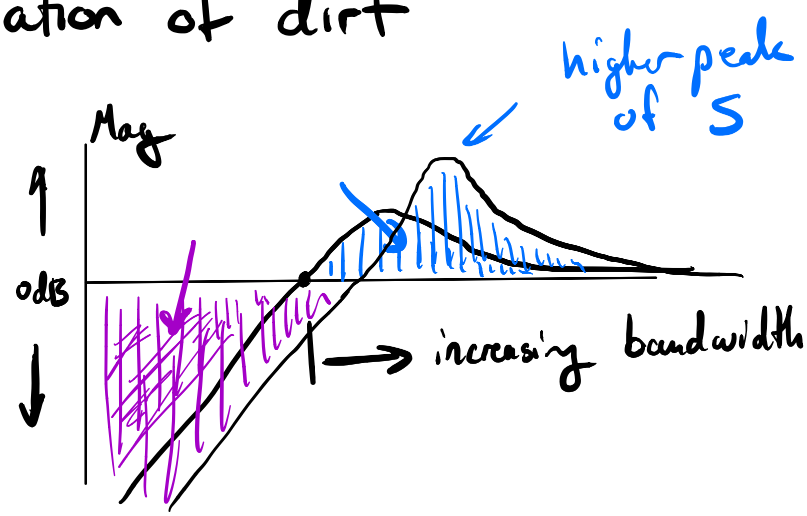
→ Waterbed formulas

→ "conservation of dirt"

Preview:

$$\int_0^\infty \ln|S(i\omega)|\,d\omega$$

bounds on this



$|S(i\omega)| = 1 \Rightarrow \ln|S(i\omega)| = 0$

for stable plant $\int_0^\infty \ln|S(i\omega)|\,d\omega = 0$

for unstable plant (RHP poles) $\int_0^\infty \ln|S(i\omega)|\,d\omega = \text{const} > 0$

for plants w/ RHP zeros

1ST WATERBED FORMULA:

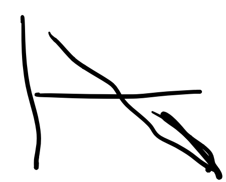
affect of RHP poles...

$$\int_0^\infty \ln|S(i\omega)|\,d\omega = \pi \sum_k Re(p_k) - \underbrace{\frac{\pi}{2}\lim_{s\to\infty} sL(s)}_{0}$$

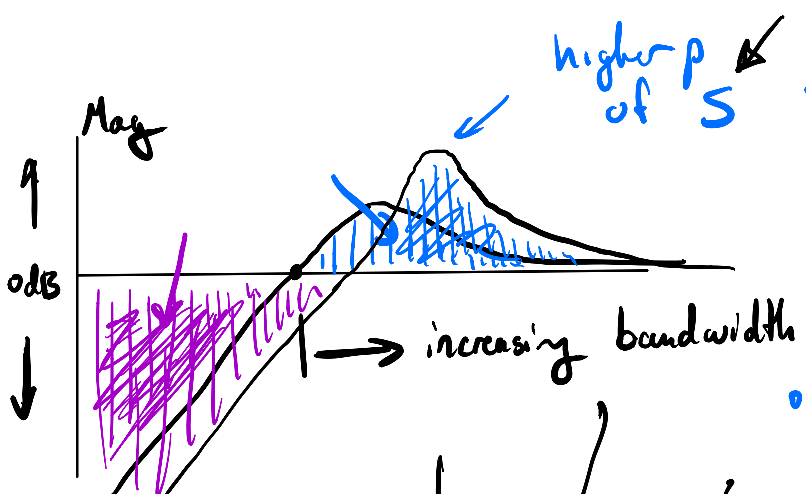
p_k : RHP poles

if stable plant (and controller) $\sum_k Re(P_k) = 0$

if deg den of $L(s)$ 2 or more greater than deg num of $L(s)$ $\lim_{s\to\infty} sL(s) = 0$

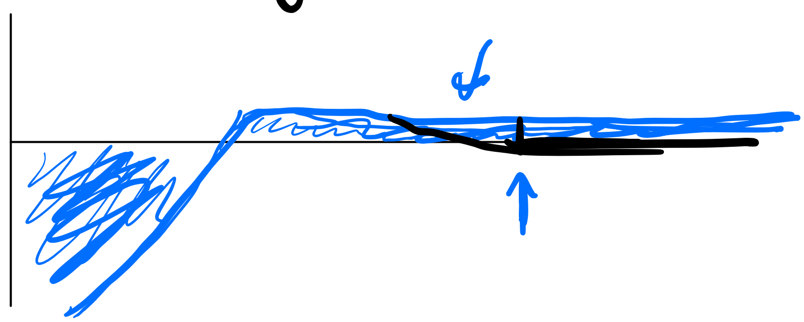


$L(s) = P(s)C(s)$



- unstable poles add mass to the blue region
- more unstable poles $Re(P_k) >> 0$ more mass

Note: possible for this affect to not be very prounounced



RHP zeros can make the dirt pile up closer to the cross over frequency

Second Waterbed Formula:

RHP zeros & poles

Suppose $L(s)$ has a single, real RHP zero or a conjugate pair of RHP zeros

and RHP poles p_k: $k = 1, \dots, N_p$

$$\int_0^\infty \ln|S(i\omega)|\, W(z,\omega)\, d\omega = \pi \cdot \ln \prod_{k=1}^{N_p} \left|\frac{p_k + z}{\bar{p}_k - z}\right|$$

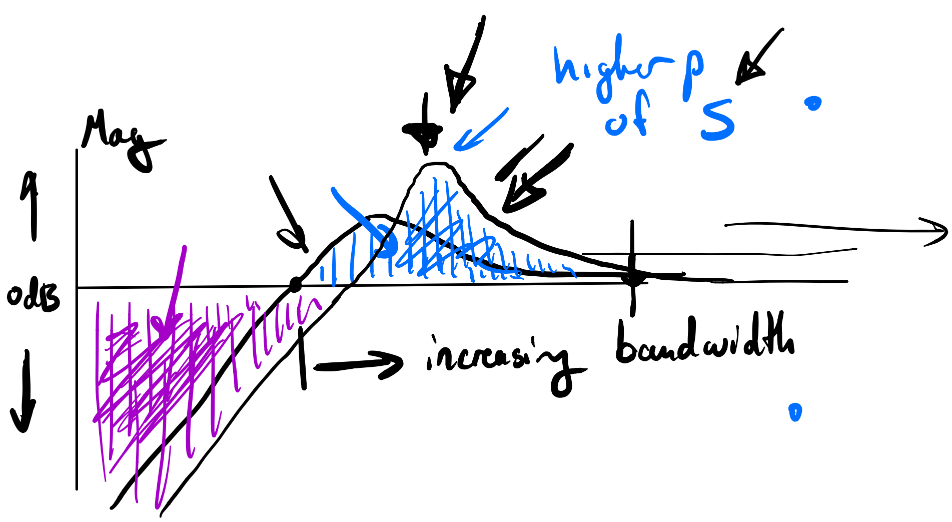
$$\int_0^\infty \ln|S(i\omega)|\, W(z,\omega)\, d\omega = \pi \cdot \sum_{k=1}^{N_p} \ln\left|\frac{p_k + z}{\bar{p}_k - z}\right|$$

if z is real:

$$W(z,\omega) = \frac{2z}{z^2+\omega^2} = \frac{2}{z}\,\frac{1}{1+(\omega/z)^2}$$

if z's is complex: $z = a+bi$ $\quad \bar{z} = a-bi$

$$W(z,\omega) = \frac{a}{a^2+(b-\omega)^2} + \frac{a}{a^2+(b+\omega)^2}$$



$$\int_0^\infty \ln|S(i\omega)|\,W(z,\omega)\,d\omega = \pi \cdot \sum_{k=1}^{N_p} \ln \underbrace{\left| \frac{p_k + z}{\bar{p}_k - z} \right|}$$

if you have a RHP zero close to a RHP pole in your plant $\longrightarrow$ very difficult to control

the affect of these terms will be most pronounced if z is close to p_k

Weighting function:

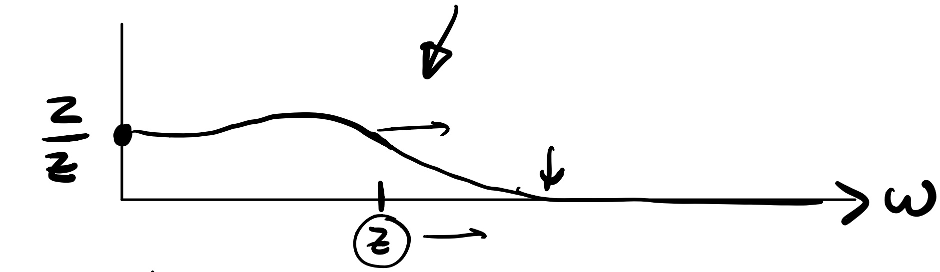
Real z.

$$W(z,\omega) = \frac{2z}{z^2+\omega^2} = \frac{2}{z}\,\frac{1}{1+(\omega/z)^2}$$

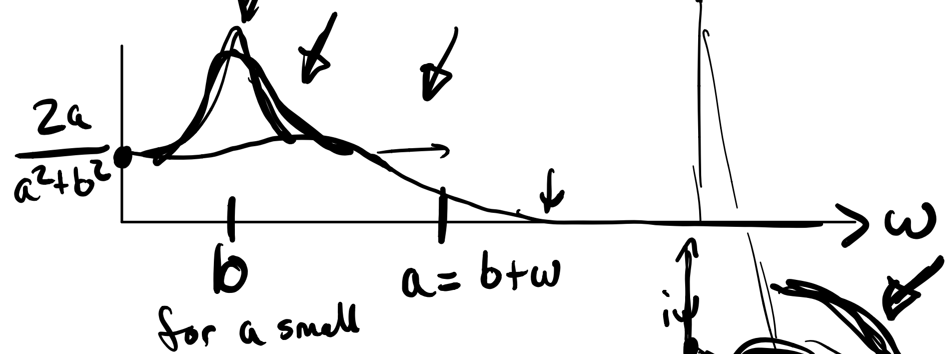
if z's is complex: $z = a + bi \quad \bar{z} = \underline{a - bi}$

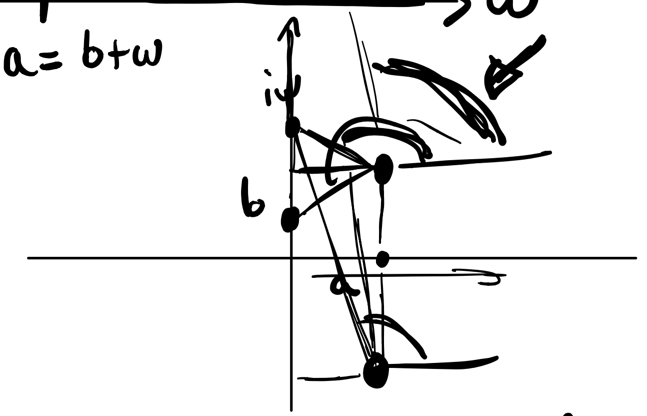
$$W(z,\omega) = \frac{a}{a^2 + (b-\omega)^2} + \frac{a}{a^2+(b+\omega)^2}$$

Real z.



Complex





<u>Intuition:</u> RHP poles are bad because unstable, takes more effort to control

RHP zeros act like time delays in that they add extra phase to the system → bad phase margin.

Diminishing affect of zeros in design:

Ex. state space representation

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

transfer function:

$$G(s) = C(sI - A)^{-1}B$$

matrix

$$= \frac{C\,\mathrm{Adj}(sI-A)\,B}{\underbrace{\det(sI-A)}_{\text{scalar.}}} \quad \begin{matrix} \rightarrow \text{zeros} \\ \rightarrow \text{poles} \end{matrix}$$

poles: only depend on A. ⇐ fund dynamics

zeros: depend on C & B

C → $\boxed{\text{sensors}}$

B → actuators

$$y = [0\ 0 \cdots 1] \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$