AA 549: Estimation and Kalman Filtering
Homework #5
Due: Sunday, May 9, 2021 @ 11:59 pm

For the following problems, feel free to write code that can perform multiple filters. Use some open-loop control $u(t)$ of your choice. Also, experiment with various covariance matrices. Try not to just copy my code. I know everything so I will know. No specific requirements on what plots you produce; the deliverable is to get the code running, but it's worth plotting some things to see the performance.

1. **Linear system:** Run the

   - Discrete time KF: finite & infinite horizon cases
   - Continuous time KF: finite & infinite horizon cases
   - Continuous/discrete KF:

   for the following linear system.

$$\dot{x} = A(t)x + B(t)u + G(t)w, \qquad w \sim \mathcal{N}(0, Q(t))$$
$$y = C(t)x + v, \qquad v \sim \mathcal{N}(0, R(t))$$

   where

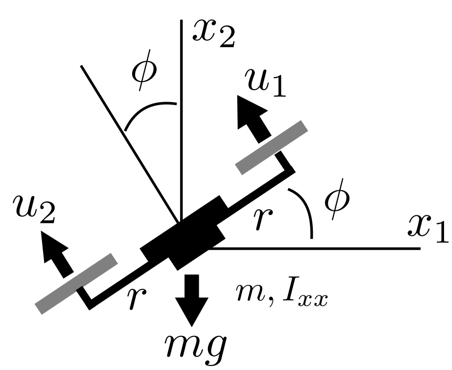
$$A(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -10 & -14 & -11 & -4 \end{bmatrix}, \qquad B(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \qquad G(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \qquad C(t) = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

2. **Non-linear system:**

   Run the

   - Extended Kalman Filter (EKF)

   for the following nonlinear system



$$
\begin{aligned}
\ddot{x}_1 &= -\frac{1}{m}(u_1 + u_2)\sin(\phi) + w_1, \qquad w_1 \sim \mathcal{N}(0, q_1) \\
\ddot{x}_2 &= \frac{1}{m}(u_1 + u_2)\cos(\phi) - mg + w_2, \qquad w_2 \sim \mathcal{N}(0, q_2) \\
\ddot{\phi} &= \frac{r}{I_{xx}}(u_1 - u_2) + w_3, \qquad w_3 \sim \mathcal{N}(0, q_3)
\end{aligned}
$$

Measurement $\qquad y = x + v, \qquad v \sim \mathcal{N}(0, R)$

with state vector $x = [x_1\ \dot{x}_1\ x_2\ \dot{x}_2\ \phi\ \dot{\phi}]^T \in \mathbb{R}^6$. Pick whatever values for m, r, and I_{xx} and some arbitrary control trajectory. Since you're randomly picking an open-loop control trajectory feel free to ignore the gravity term if it's making your quad fall too much.