SLAM:

How do you match observations w/ your state information when we don't know the map of the environment?

Single Object Target Tracking

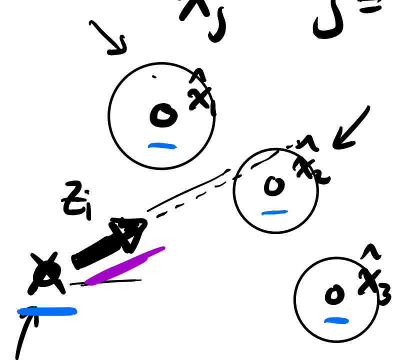
Measurement Model $z_j = h(x_j) + v \qquad v \sim N(0, R)$

a measurement

z_i

set of states

$\hat{x}_j \quad j = 1, \dots, m$



$$z_i = \begin{bmatrix} r_i \\ \theta_i \end{bmatrix}$$

prediction for $\hat{x}_j$

$$\hat{z}_j = h(\hat{x}_j)$$

normalized innovation squared (NIS)

$$M_{ij} = v_{ij}^T \underset{\sim}{S_{ij}}^{-1} v_{ij}$$

Mahalanobis distance

$$v_{ij} = z_i - \hat{z}_j \qquad S_{ij} = \frac{\partial h}{\partial x} P_j \frac{\partial h}{\partial x}^T + R$$

M_{ij} will have a chi-squared distribution.

linearizing h & computing the covariance of V_{ij} could replace w/ Unscented transform

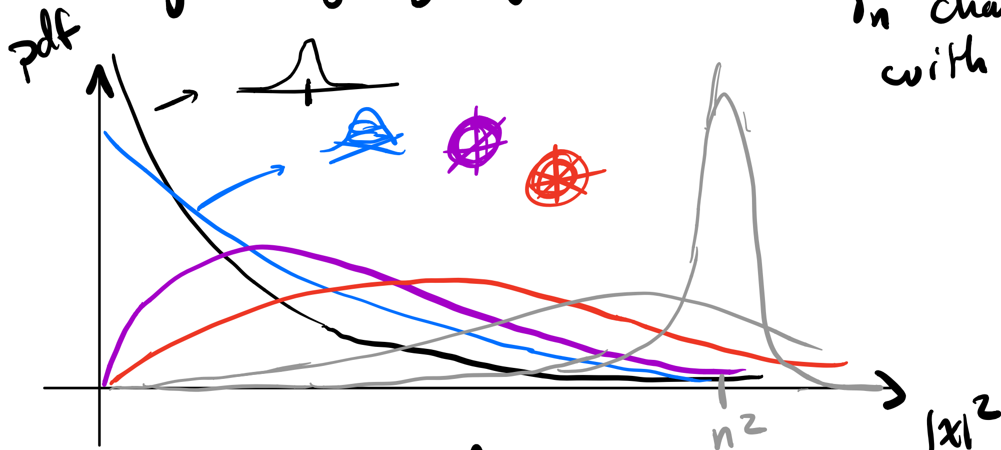
if $x \sim$ Gaussian $\longrightarrow \sum_i x_i^2 \sim$ chi-squared distribution

want

$$M_{ij} = v_{ij}^T S_{ij}^{-1} v_{ij} < \gamma_n$$

if $v_{ij} \in \mathbb{R}^n$

γ_n changes with the dimension



Maximizing Likelihood of z_i

$$\Lambda_{ij} = \frac{1}{(2\pi)^{n/2}\sqrt{|S_{ij}|}} e^{(-\frac{1}{2} v_{ij}^T S_{ij}^{-1} v_{ij})} \leftarrow \text{max}$$

log

$$N_{ij} = v_{ij}^T S_{ij}^{-1} v_{ij} + \ln|S_{ij}| \leftarrow \text{min.}$$

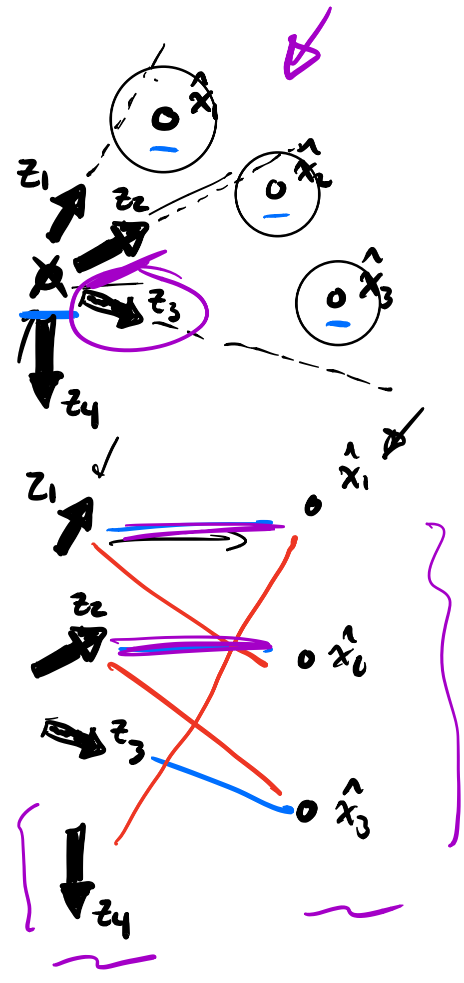
Multiple measurements / states:

Assigned Matching $E_k = \{(z_1, x_1), (z_2, x_2), (z_3, x_3)\}$

$E_k = \{e_1, \dots, e_m\}$

injective matching

ea $\hat{x}_j$ only gets assigned 1 meas z_i



How do we compare matchings?

max
$$\Lambda_{E_k} = \prod_{\{\forall e_m \in E_k\}} \Lambda_{e_m} \quad (*)$$

log

max
$$\sum_{\{\forall e_m \in E_k\}} \ln \Lambda_{e_m}$$

min
$$\sum_{\{\forall e_m \in E_k\}} N_{e_m} = \sum N_{ij}$$

Method: max-weight bipartite graph matching

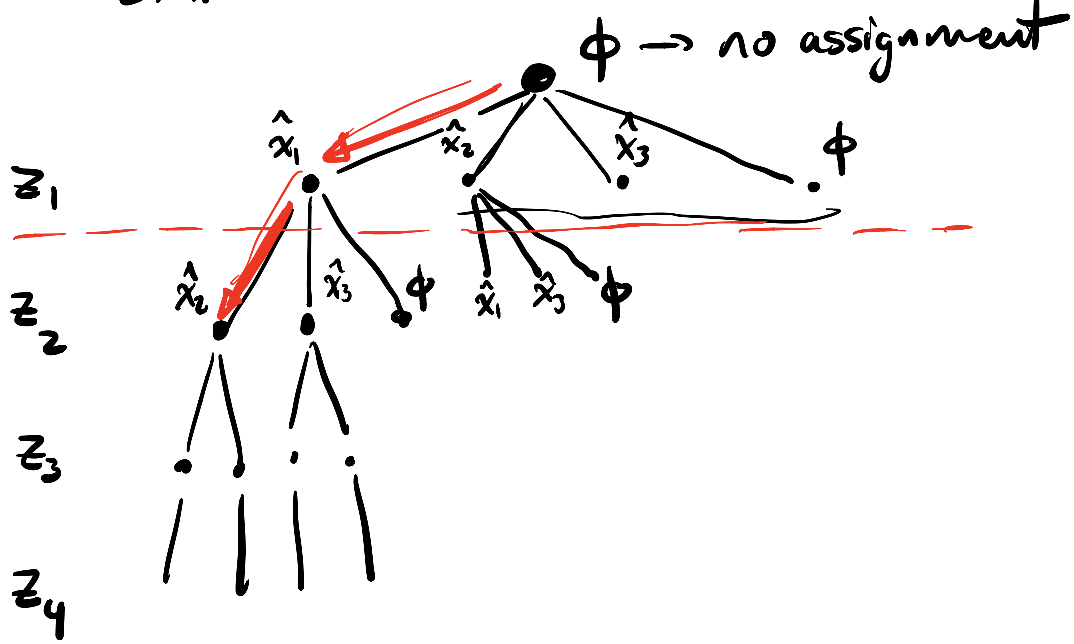
Problem: could have spurious measurements and fake features

<u>Joint</u> <u>Compatibility</u> <u>Branch</u> & <u>Bound</u> (JCBB)

build up matchings one match at a time ...

"interpretation tree"

→ still based on MLE.



Observation space:

$$z = \begin{bmatrix} r \\ \theta \end{bmatrix} \qquad x = \begin{bmatrix} x \\ y \end{bmatrix}$$

meas model $\quad z = h(x) = \begin{bmatrix} \sqrt{x^2 + y^2} \\ \tan^{-1}\left(\frac{y}{x}\right) \end{bmatrix}$

Likelihood of $E_k = \{e_1 \dots, e_m\}$

$$z_{E_k} = [z_{e_1}, \dots, z_{e_m}]$$

noise covariance

$$R_{E_k} = \begin{bmatrix} R_1 & & 0 \\ & \ddots & \\ 0 & & R_m \end{bmatrix}$$

$$\hat{x} = [\hat{x}_1, \dots, \hat{x}_m]$$

state covariance

$$\hat{z}_{E_k} = h_{E_k}(\hat{x}) = \begin{bmatrix} h_{e_1}(\hat{x}) \\ \\ h_{e_m}(\hat{x}) \end{bmatrix}$$

$$P_{E_k} = \begin{bmatrix} P_1 & & 0 \\ & \ddots & \\ 0 & & P_m \end{bmatrix}$$

$$v_{E_k} = z_{E_k} - \hat{z}_{E_k}$$

$$S_{E_k} = \frac{\partial h_{E_k}}{\partial x} P_{E_k} \frac{\partial h_{E_k}}{\partial x}^T + R_{E_k}$$

log likelihood of the full matching is .

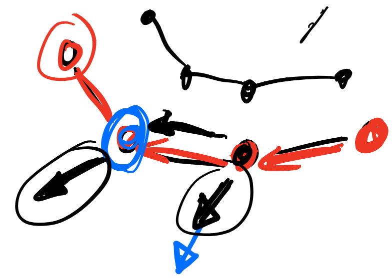
$$M_{E_k} = v_{E_k}^T S_{E_k}^{-1} v_{E_k}$$

Comparing matchings in observation space

$$x_a = \begin{bmatrix} x_v \\ x_M \end{bmatrix} \qquad x_v = \begin{bmatrix} x_v \\ y_v \\ \phi_v \end{bmatrix} \Leftarrow \text{vehicle pose}$$

$$x_M = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \\ \vdots \end{bmatrix} \Leftarrow \text{map features}$$

$$E_k = \{(z_1, x_2), (z_3, x_1)\}$$



$$\hat{z}_{E_k} = h_{E_k}(\hat{x}_a) = \begin{bmatrix} h_2(\hat{x}_a) \\ h_1(\hat{x}_a) \end{bmatrix} = \begin{bmatrix} \sqrt{(\hat{x}_2 - \hat{x}_v)^2 + (\hat{y}_2 - \hat{y}_v)^2} \\ \operatorname{atan}\left(\frac{\hat{y}_2 - \hat{y}_v}{\hat{x}_2 - x_v}\right) - \hat{\phi}_v \\ \sqrt{(\hat{x}_1 - \hat{x}_v)^2 + (\hat{y}_1 - \hat{y}_v)^2} \\ \operatorname{atan}\left(\frac{\hat{y}_1 - \hat{y}_v}{\hat{x}_1 - \hat{x}_v}\right) - \hat{\phi}_v \end{bmatrix}$$