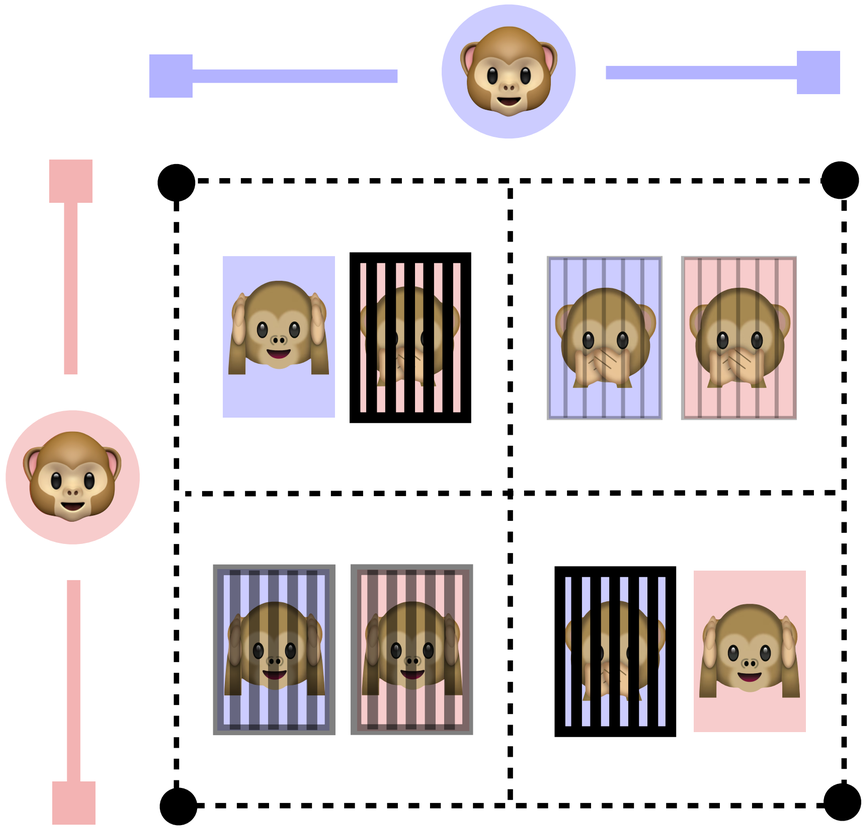
Prisoner’s Dilemma

Matrix Game: Prisoner's Dilemma

$+1$	0
-1	-3

-3	0
-1	$+1$



- Two prisoners have to decide to confess or not.

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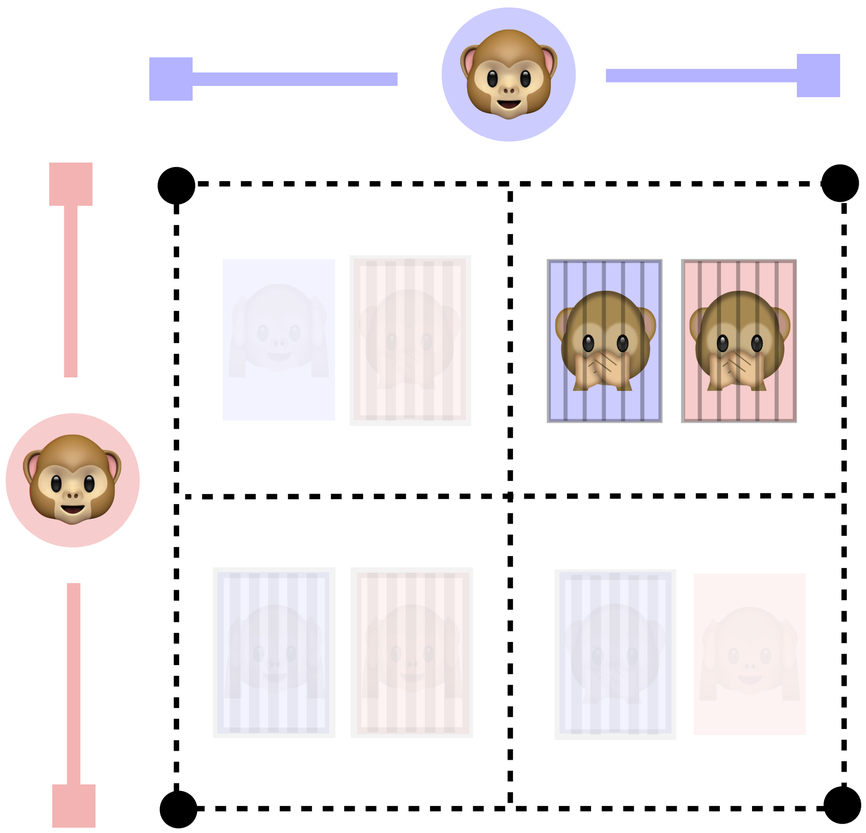

- **Two prisoners have to decide to confess or not.**
- If they both stay silent, they go to prison for a year. reward = (0,0)

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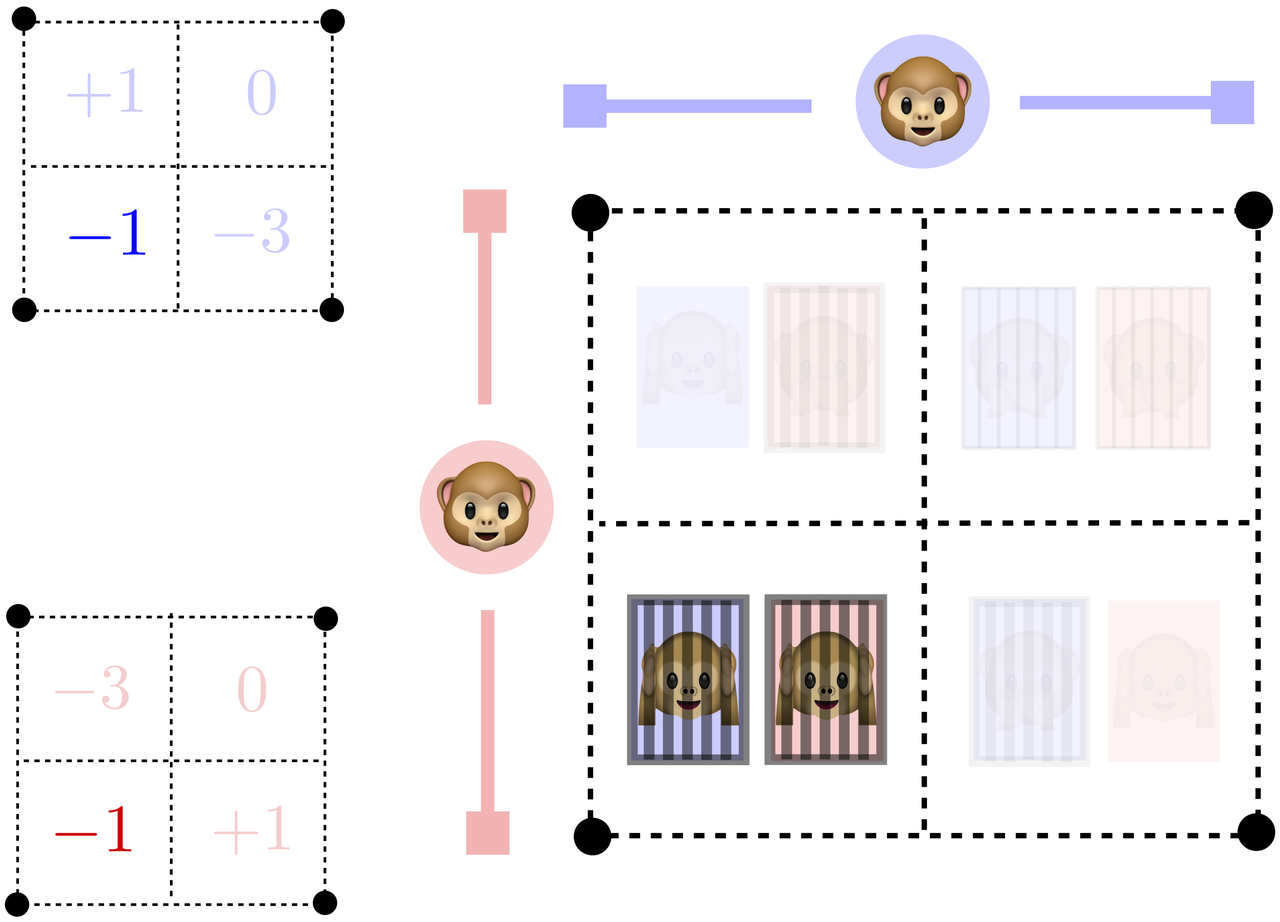

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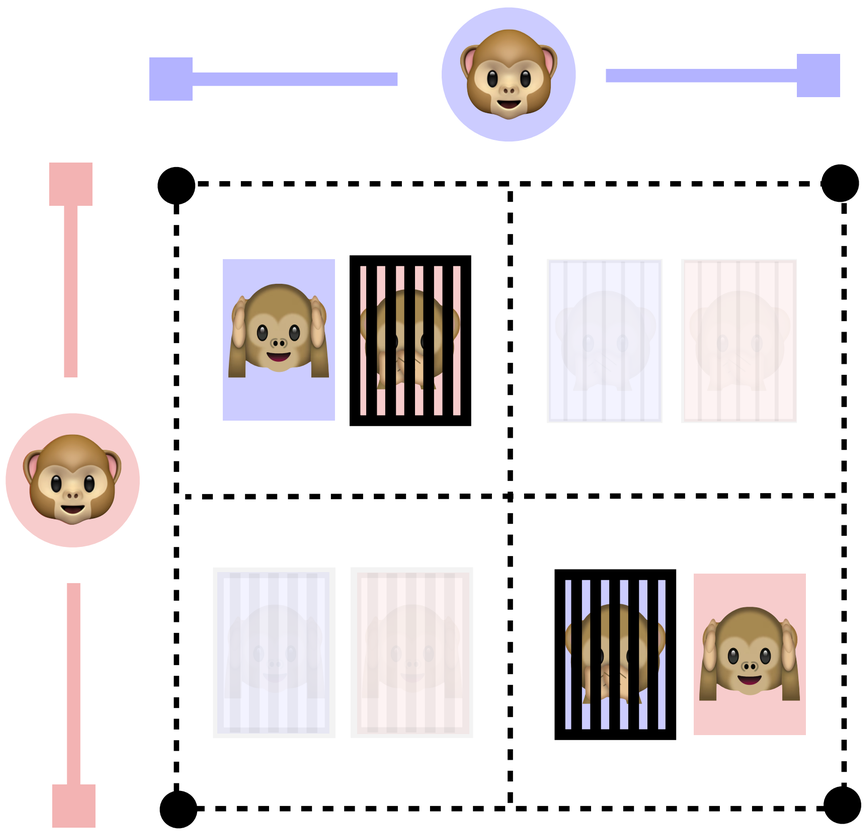

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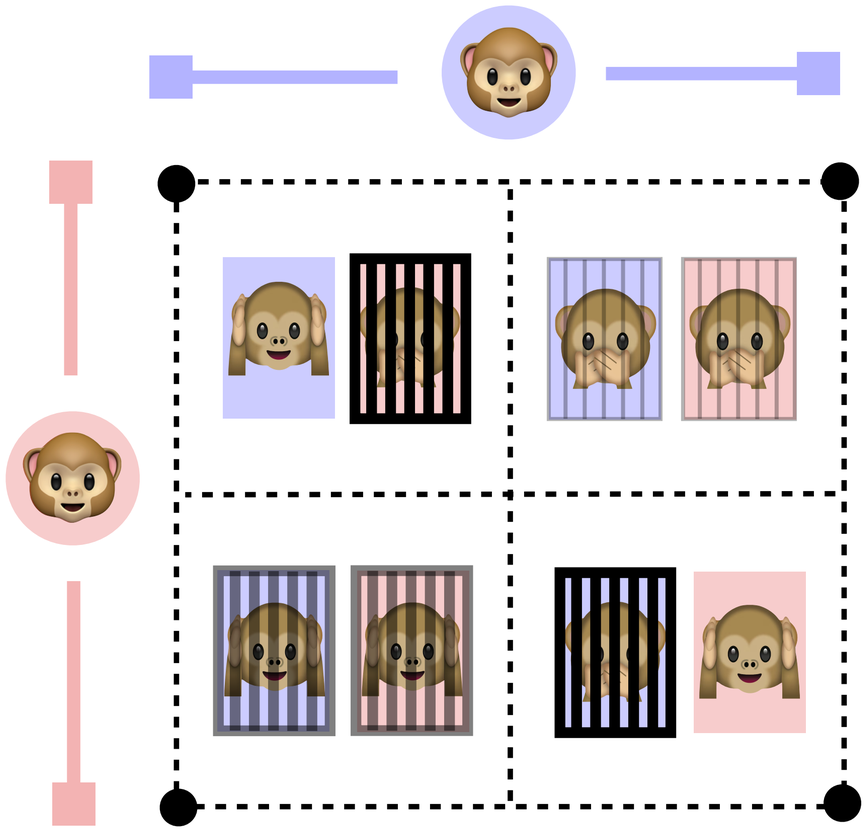

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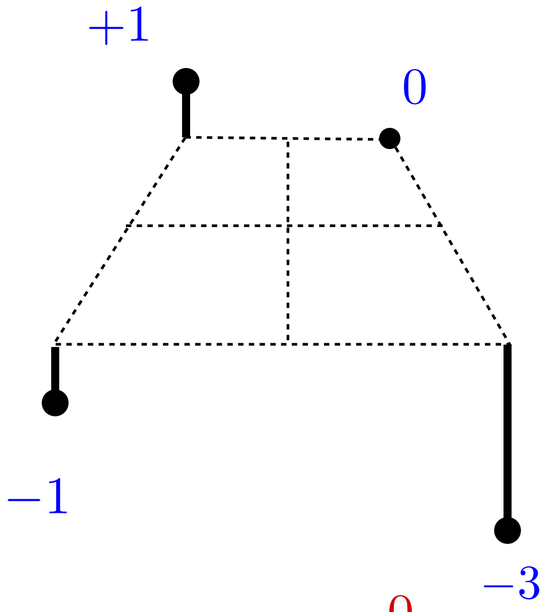
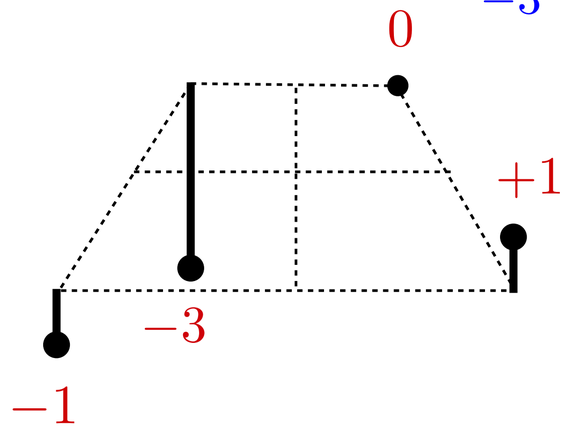
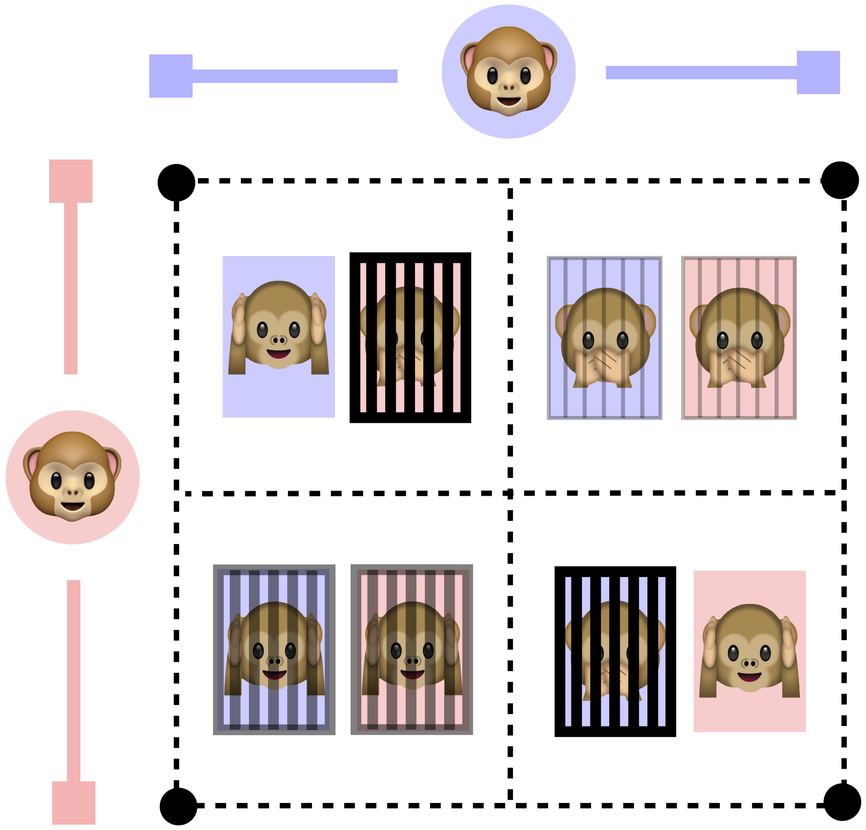




+1	0
−1	−3

−3	0
−1	+1



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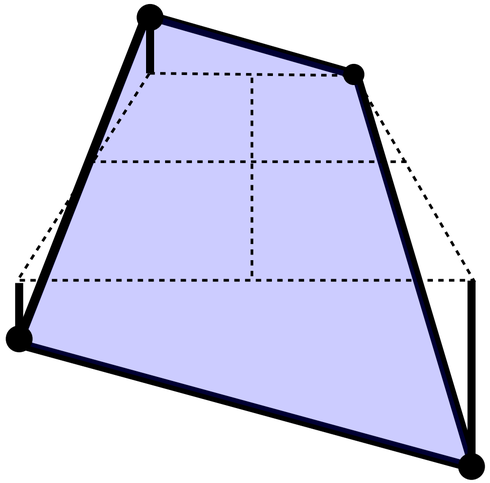
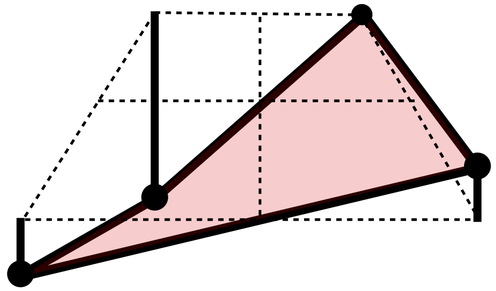
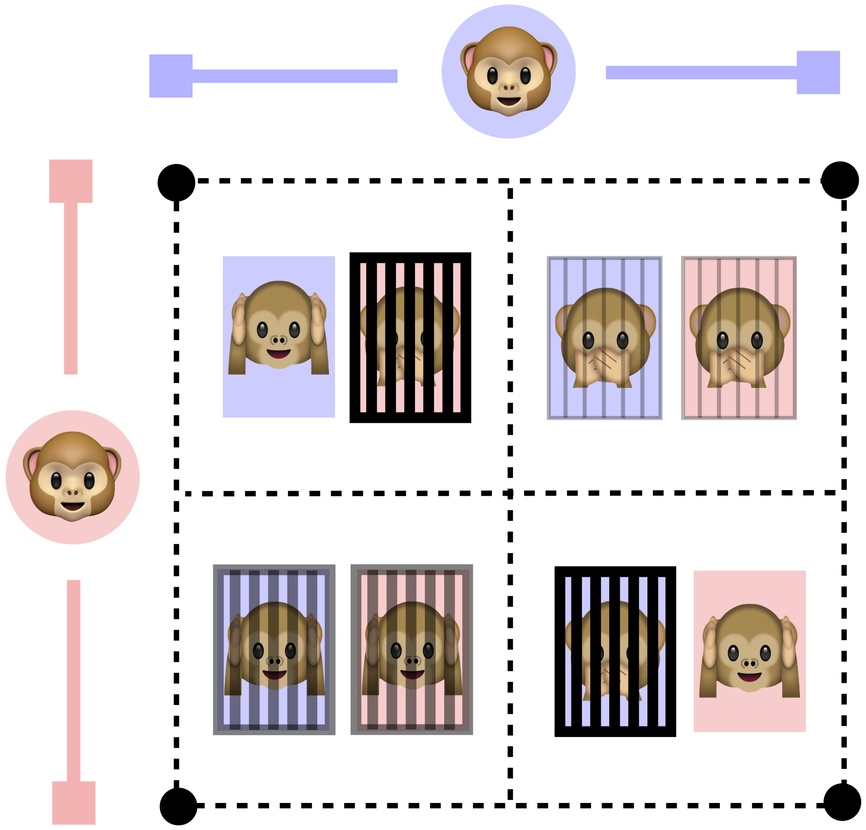


$+1$	0
-1	-3



-3	0
-1	$+1$

Possible Payoffs
from Mixed Strategies



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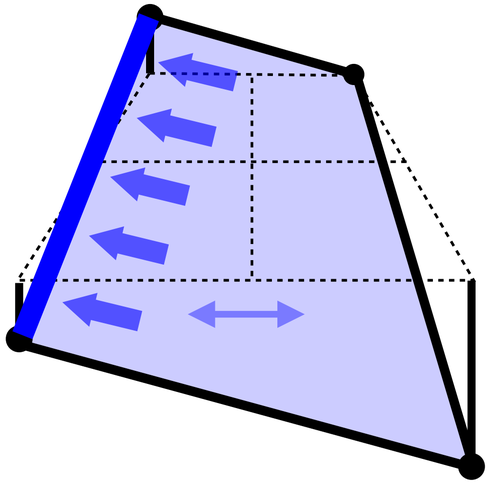
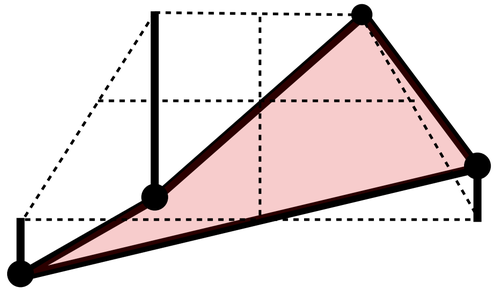
Matrix Game: Prisoner's Dilemma - Best Responses

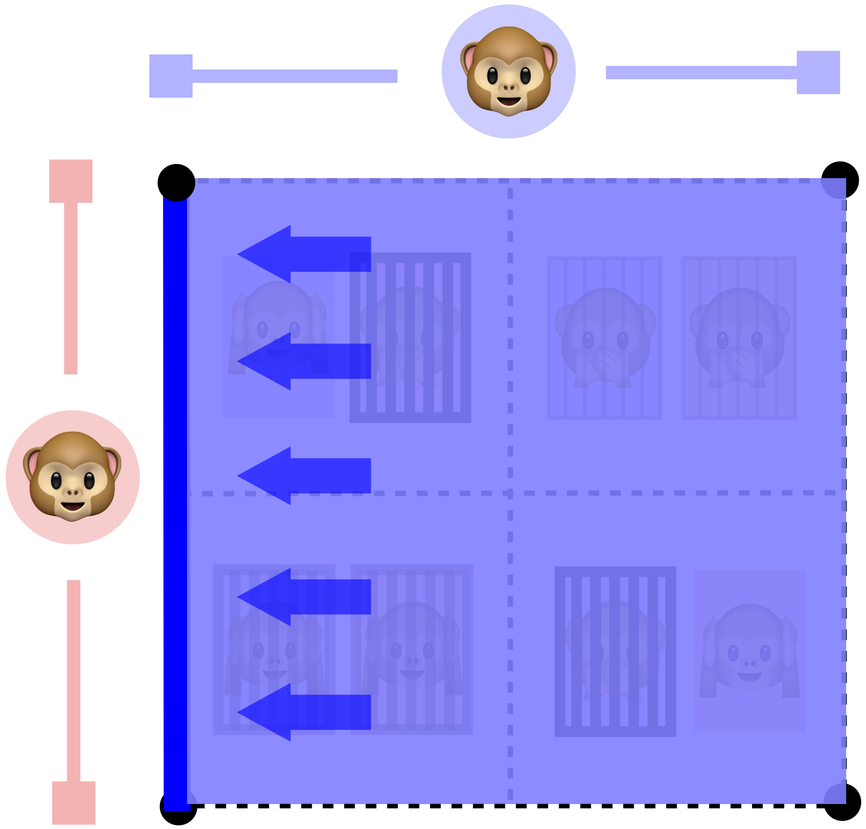


$+1$	0
-1	-3



-3	0
-1	$+1$

Best Responses
for Blue Player…



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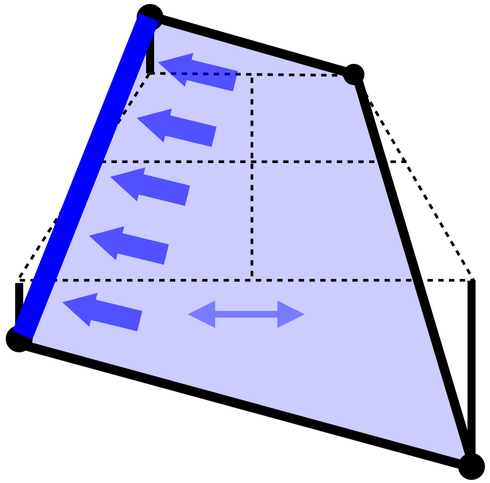
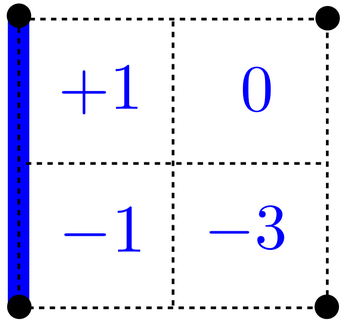
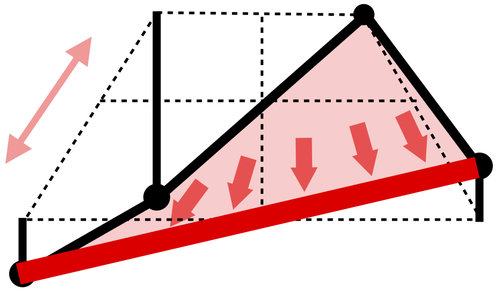
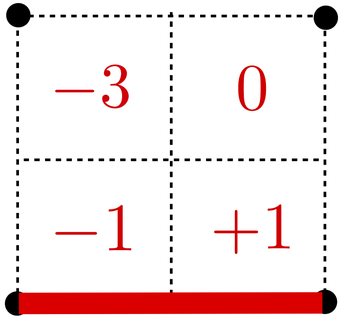
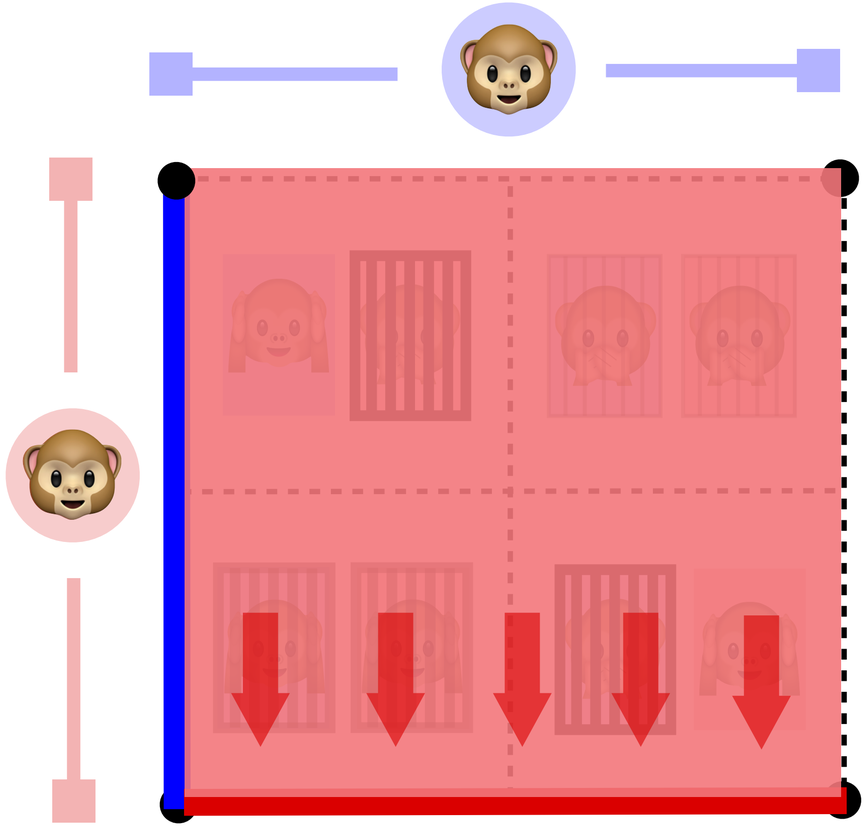








Best Responses
for Red Player…



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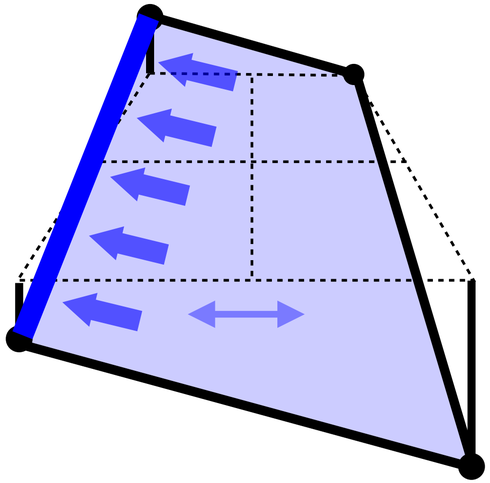
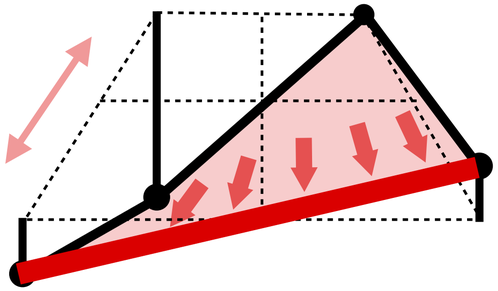
- If only one of them, confesses, that one goes free and the other goes to jail for 3 years.
  ex. reward = (+1, -3)
- **What will they do?**

Matrix Game: Prisoner's Dilemma - Nash Equilibrium

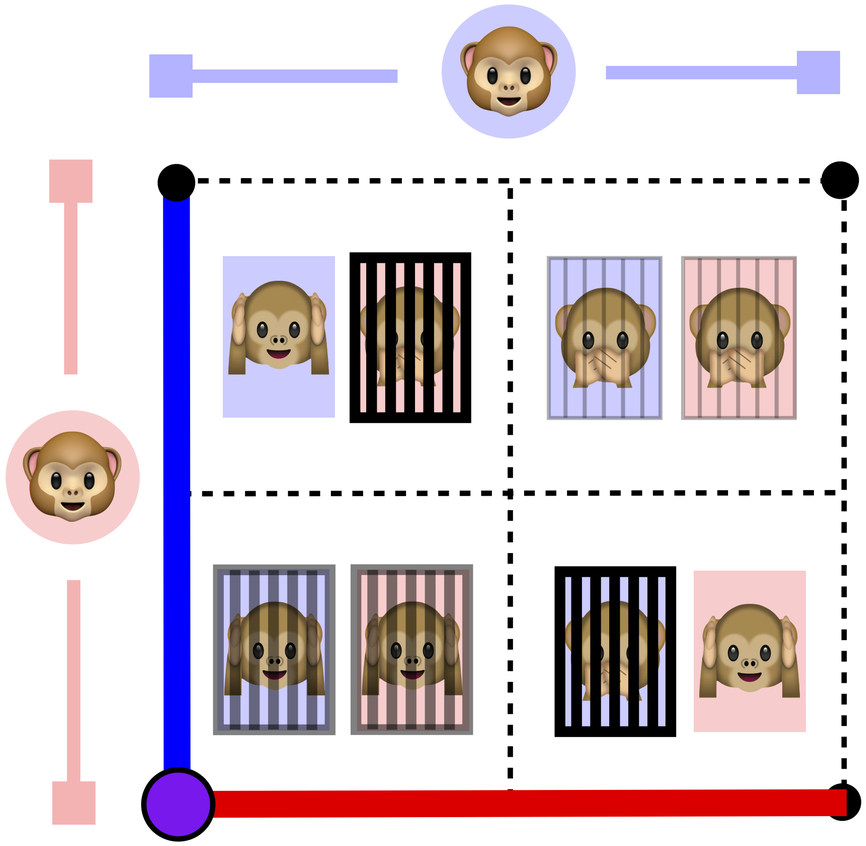


$+1$	0
-1	-3



-3	0
-1	$+1$

Nash must be best response for both



Nash = they both confess!

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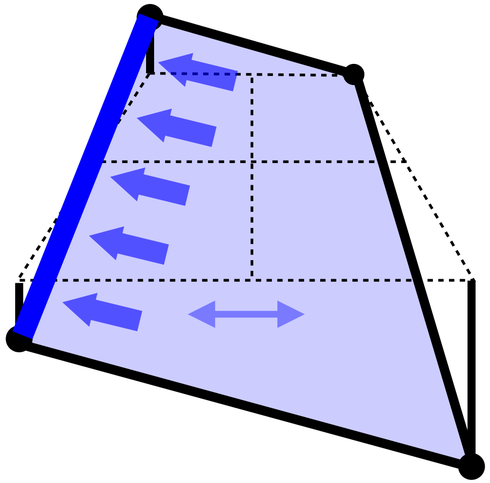
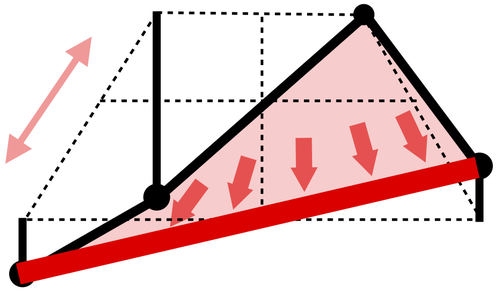
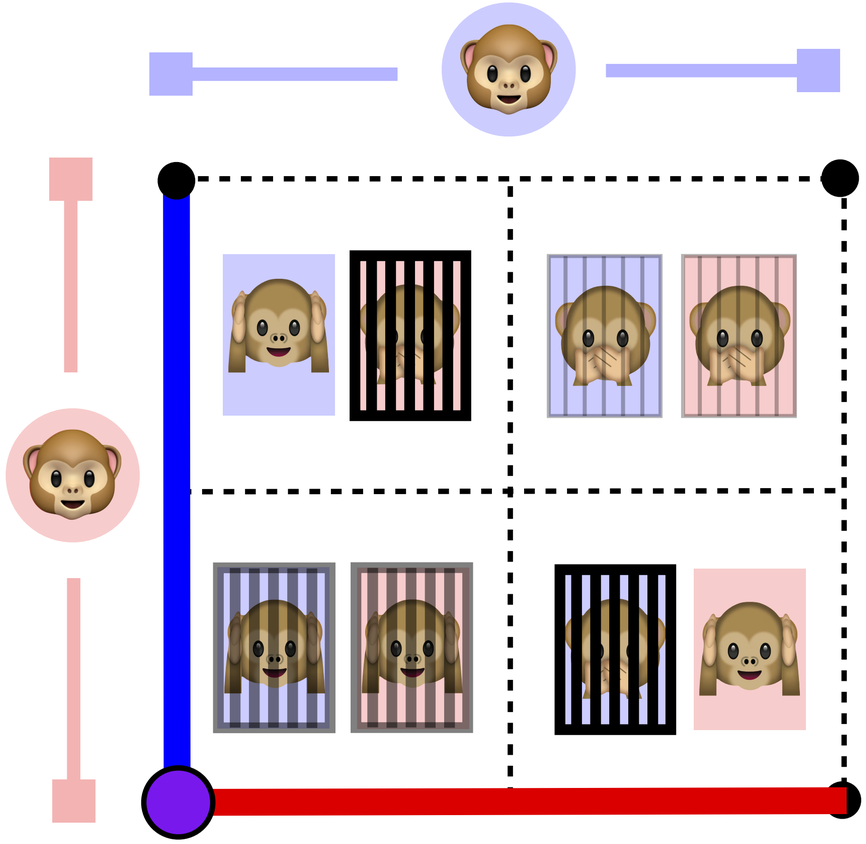
Matrix Game: Prisoner's Dilemma - SVO Nash

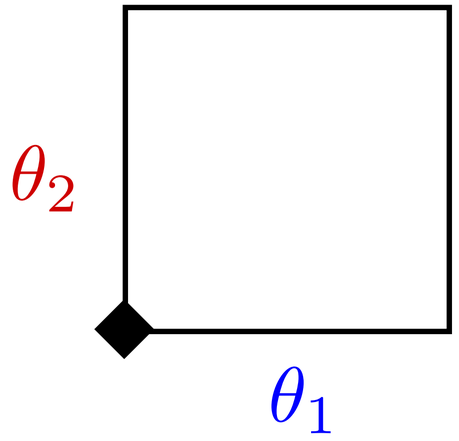


$+1$	0
-1	-3



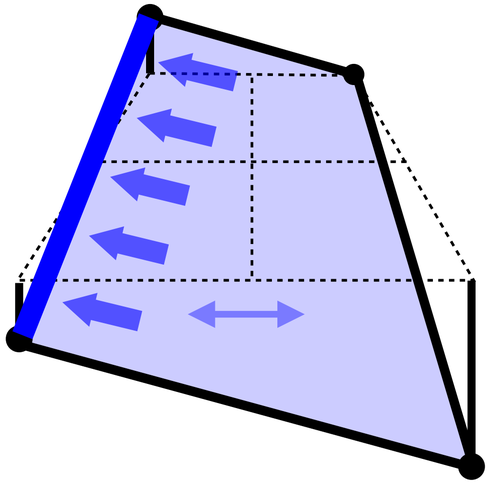
-3	0
-1	$+1$





- What happens if agents consider some portion of their opponent's outcome?

Matrix Game: Prisoner's Dilemma - SVO Nash



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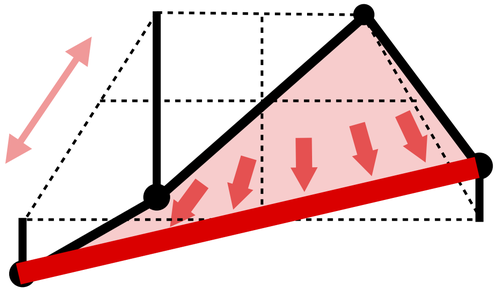

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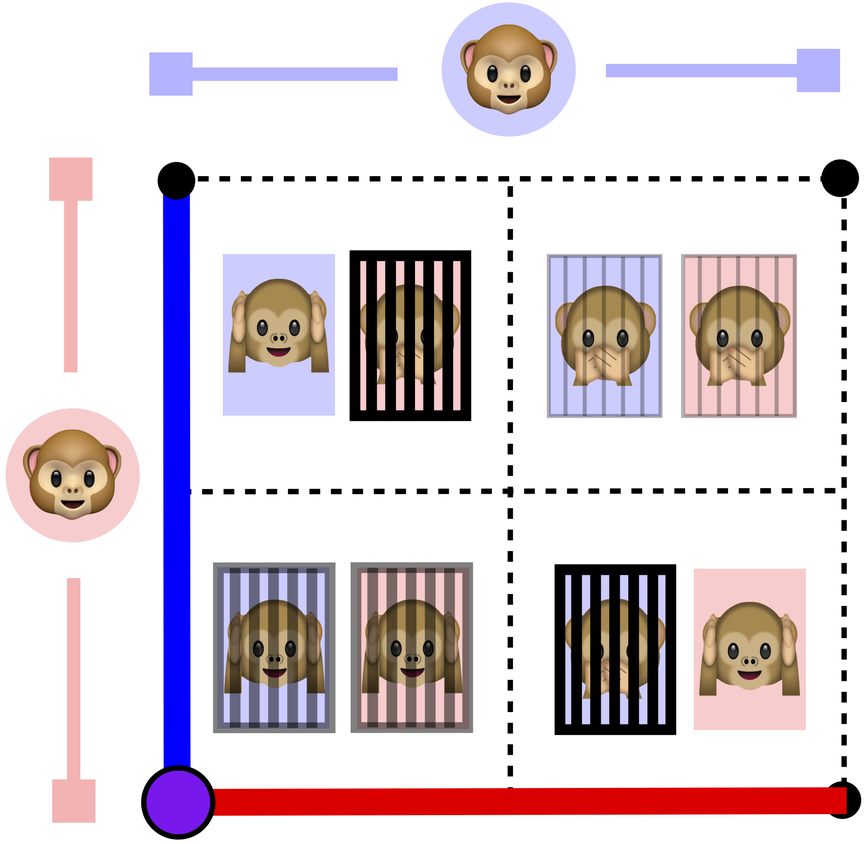

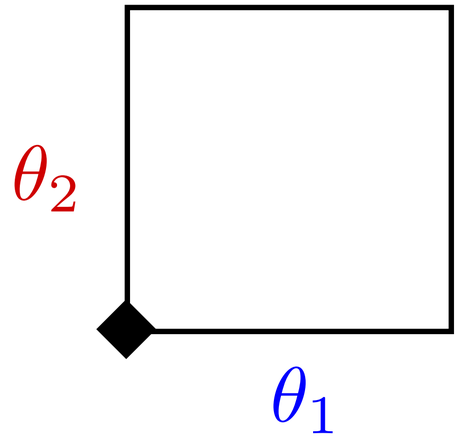

- **What happens if agents consider some portion of their opponent's outcome?**
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$$\mathbf{J}_i = (1 - \theta_i)J_i + \theta_i J_{-i}$$

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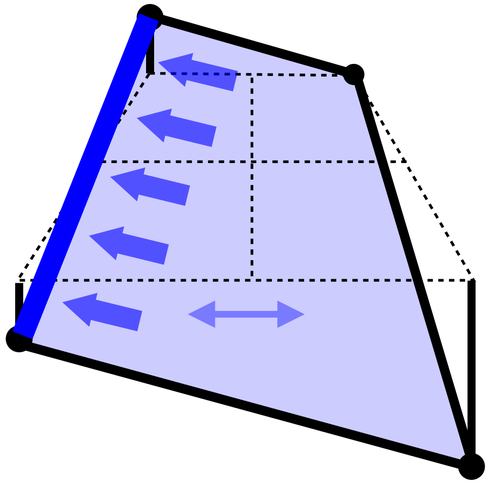
Matrix Game: Prisoner's Dilemma - SVO Nash



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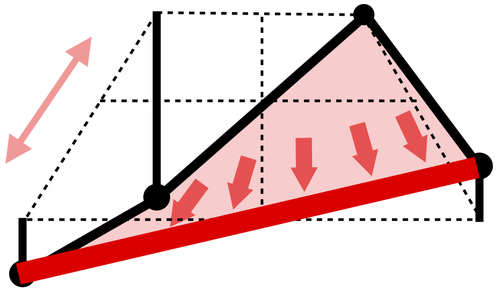

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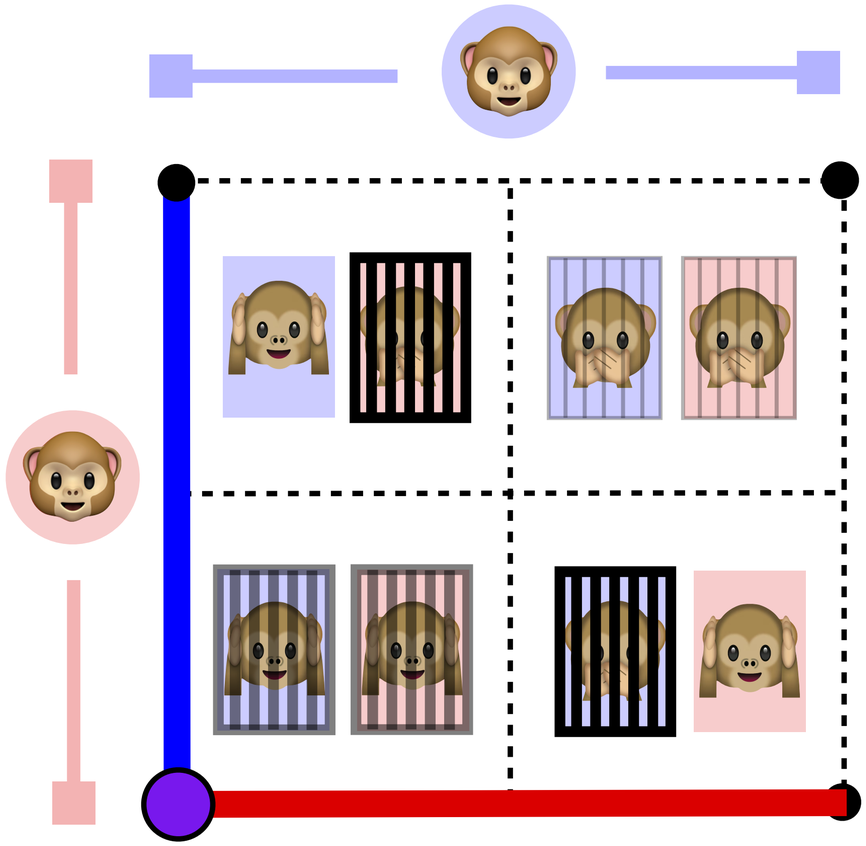

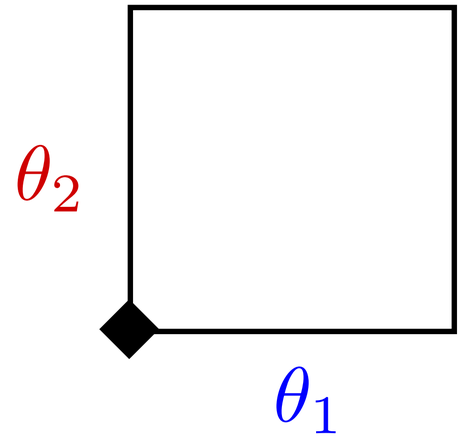

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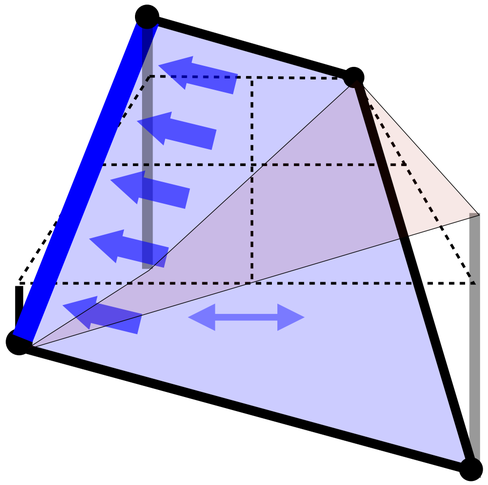
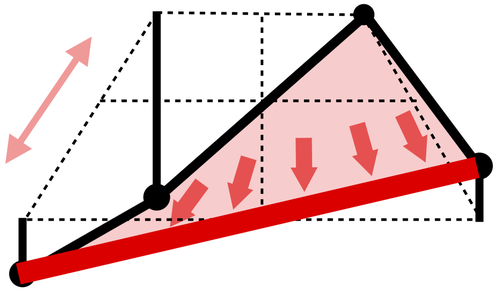




as Blue player becomes more altruistic…

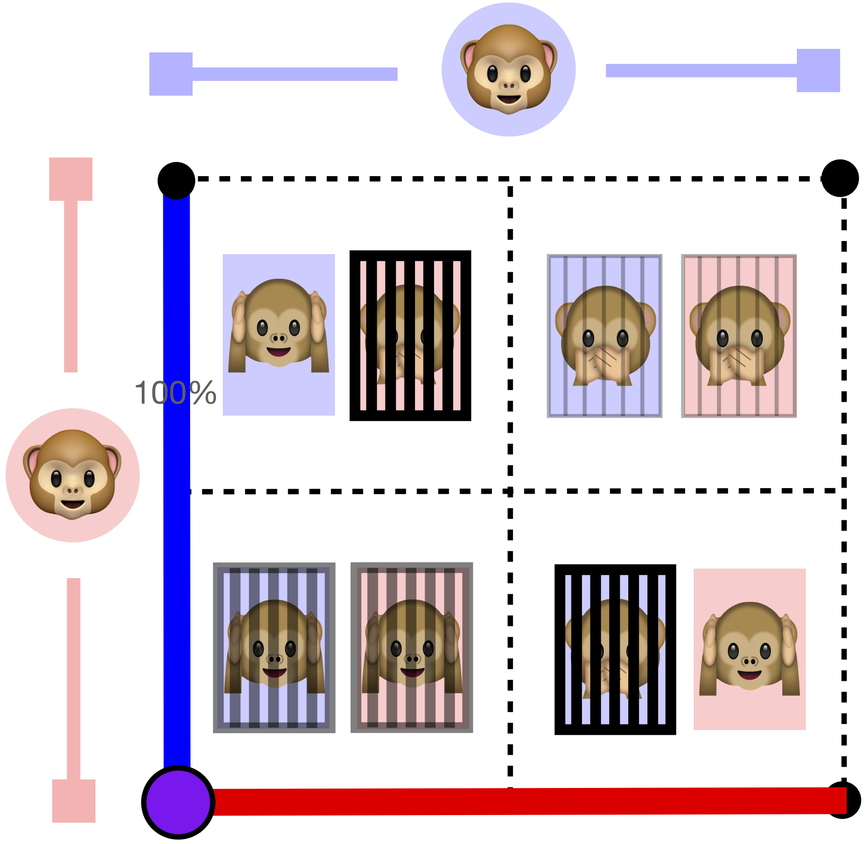
$(1-\theta_1)\times$

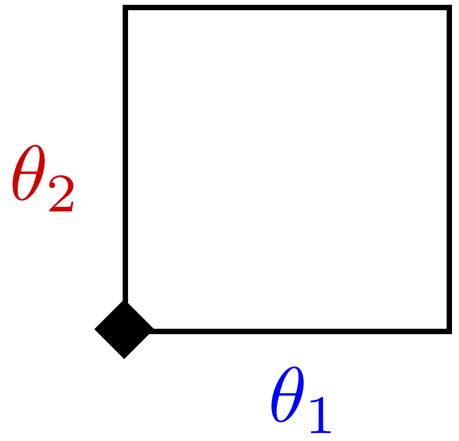
$+1$	0
-1	-3

$+\theta_1\times$

-3	0
-1	$+1$

-3	0
-1	$+1$





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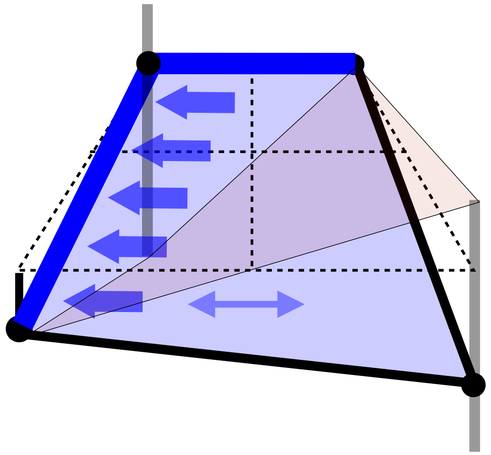

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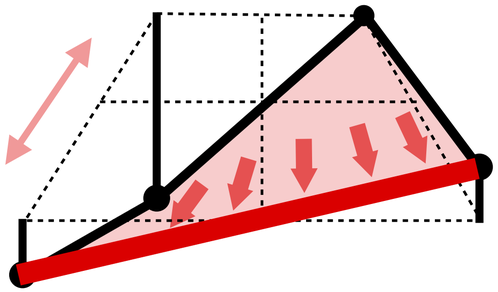

-3	0
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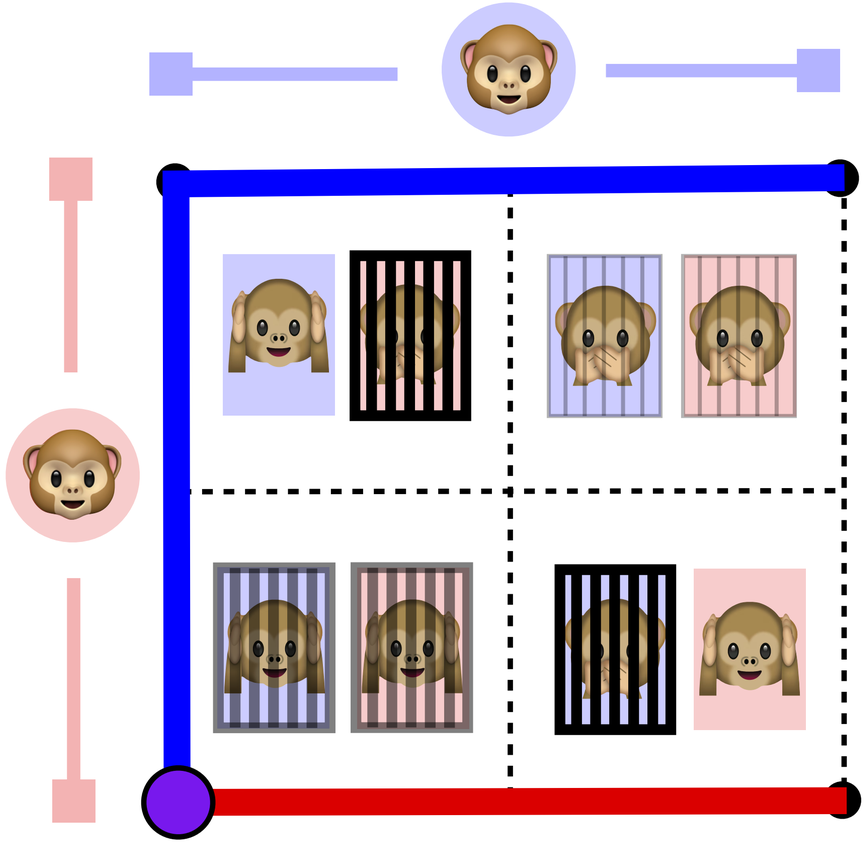
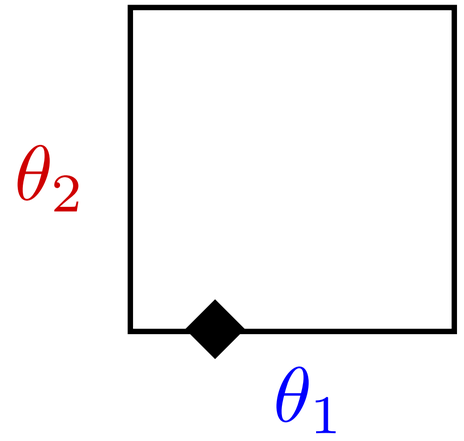
as Blue player becomes more altruistic…





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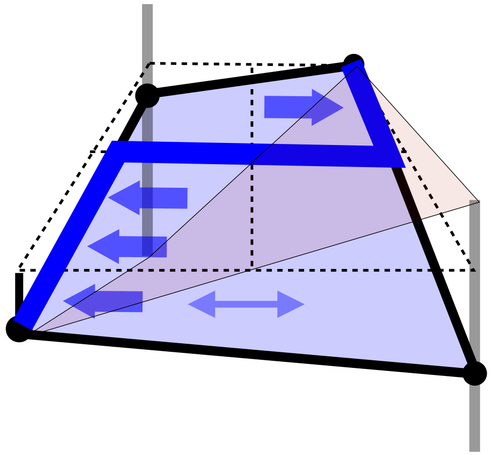

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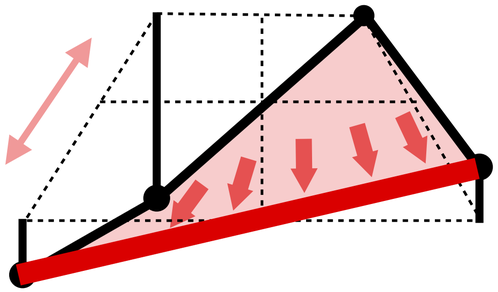
-1	-3

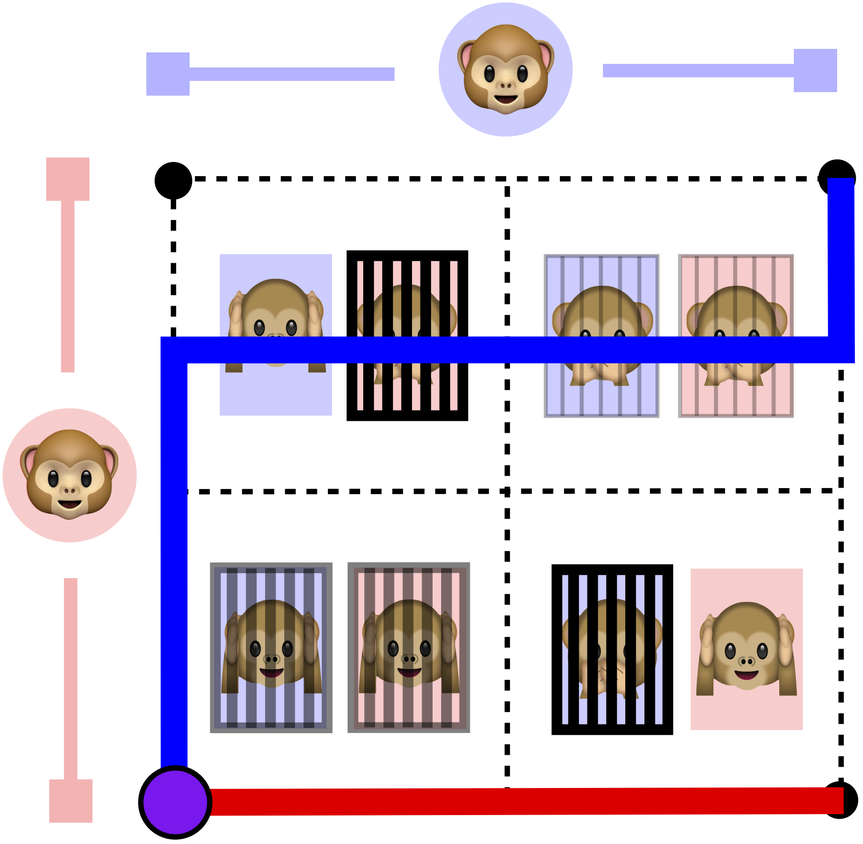
$+\theta_1\times$

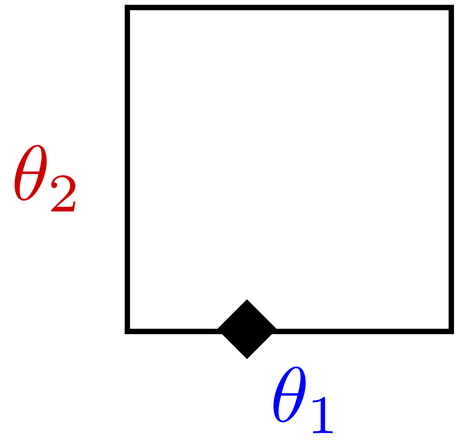
-3	0
-1	$+1$



-3	0
-1	$+1$

as Blue player becomes more altruistic…





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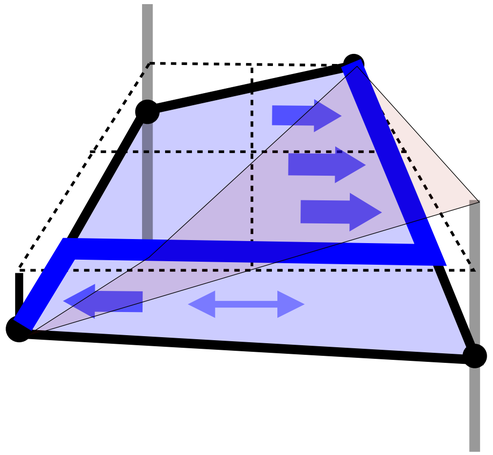
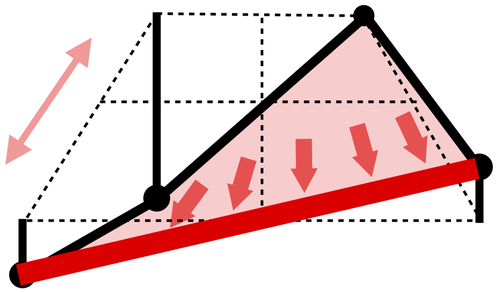
Matrix Game: Prisoner's Dilemma - SVO Nash





as Blue player becomes more altruistic…

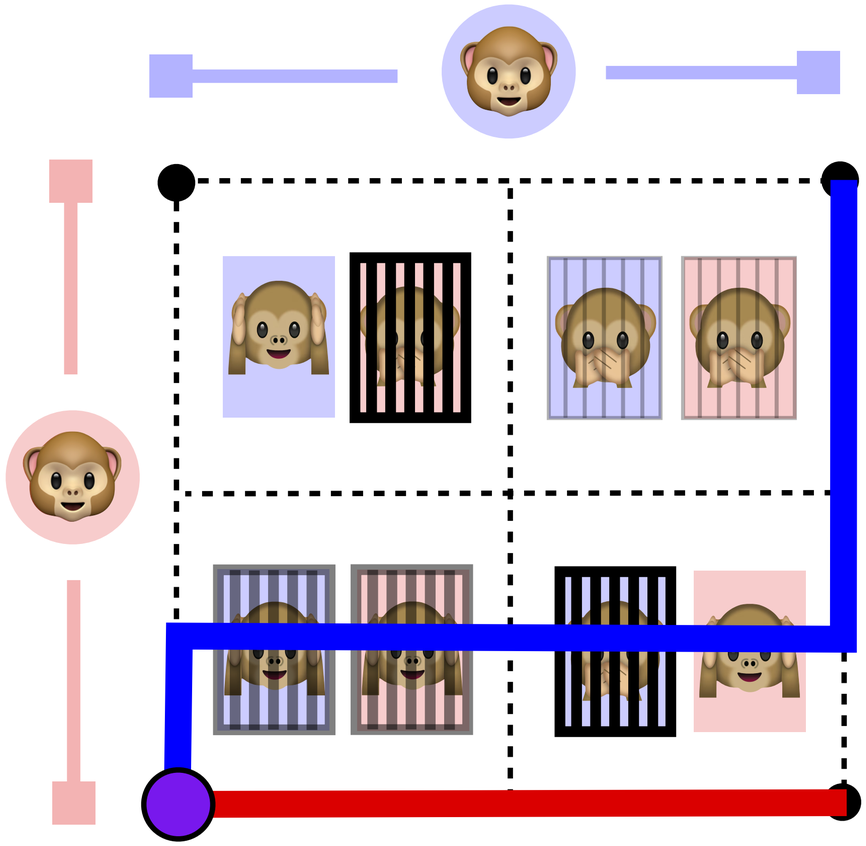
$(1-\theta_1)\times$

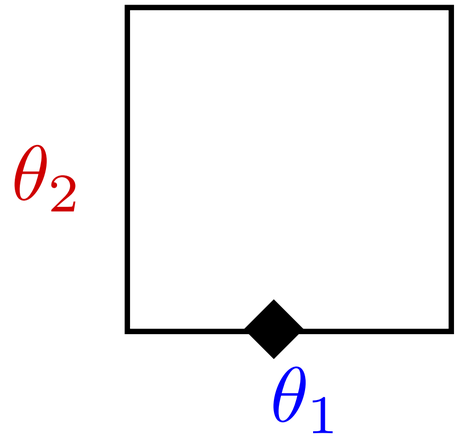
$+1$	0
-1	-3

$+\theta_1\times$

-3	0
-1	$+1$

-3	0
-1	$+1$





- **What happens if agents consider some portion of their opponent's outcome?**
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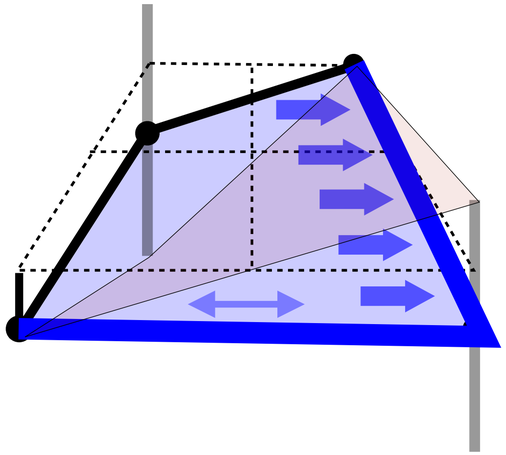
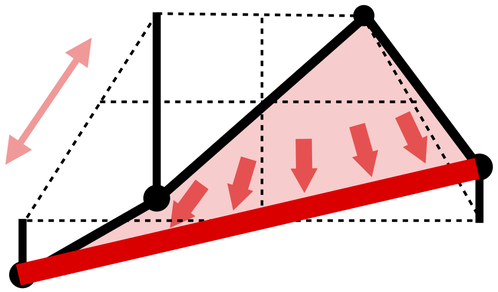


$(1-\theta_1)\times$

$+1$	0
-1	-3

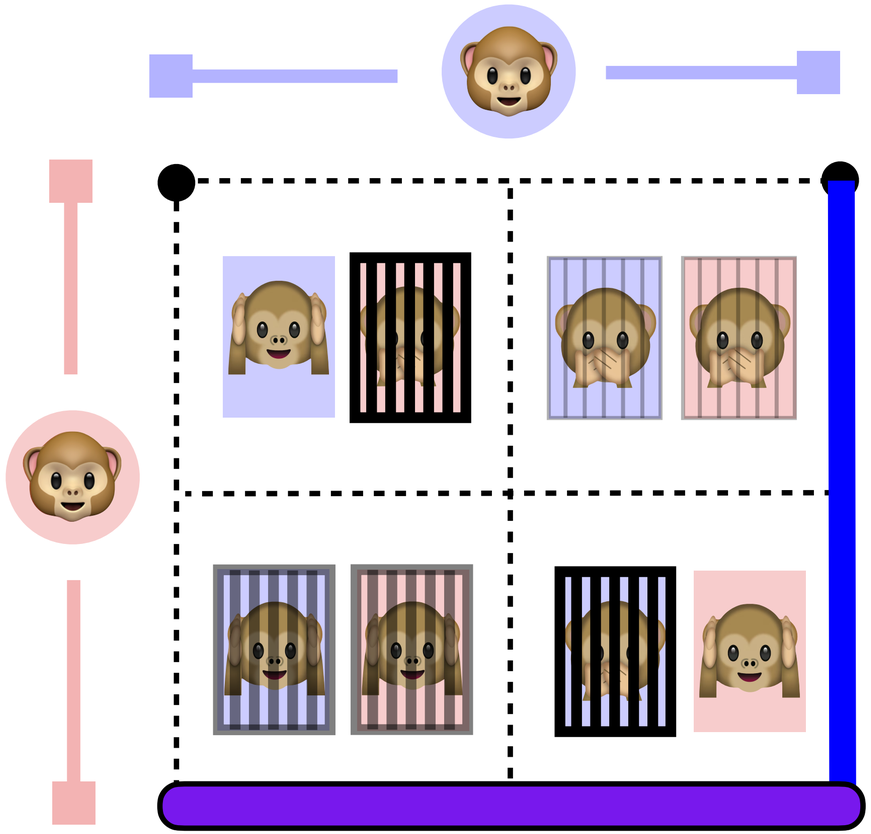
$+\theta_1\times$

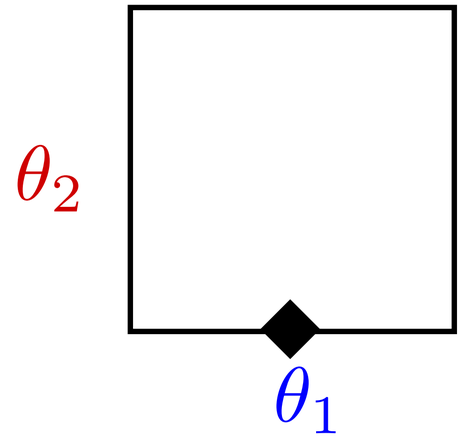
-3	0
-1	$+1$



-3	0
-1	$+1$

as Blue player becomes more altruistic…





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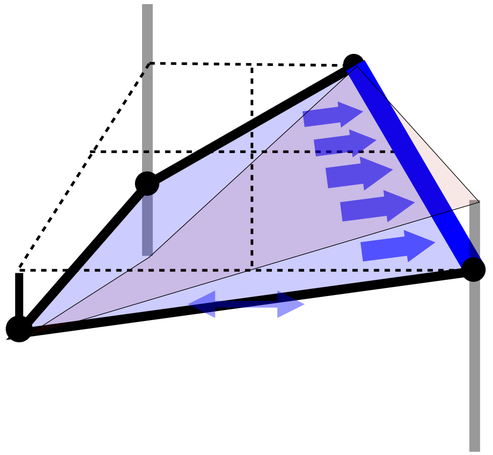
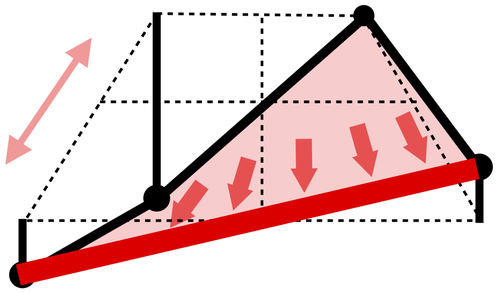


$(1-\theta_1)\times$

$+1$	0
-1	-3

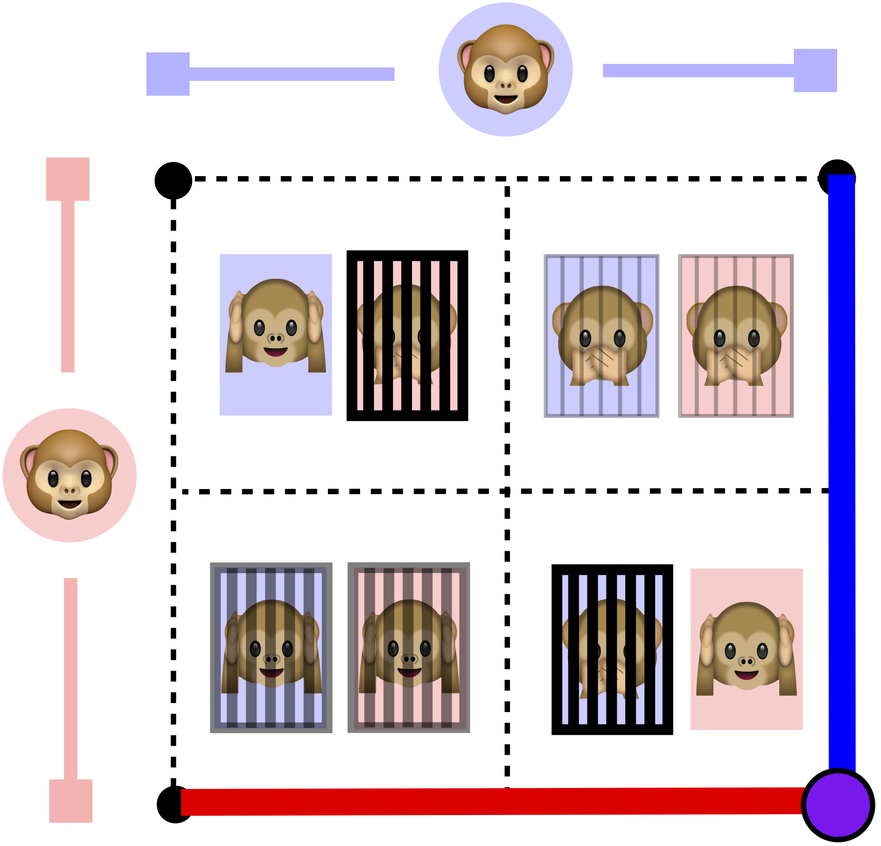
$+\theta_1\times$

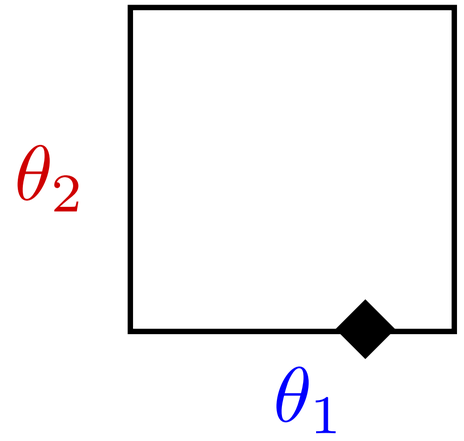
-3	0
-1	$+1$



-3	0
-1	$+1$

as Blue player becomes more altruistic…





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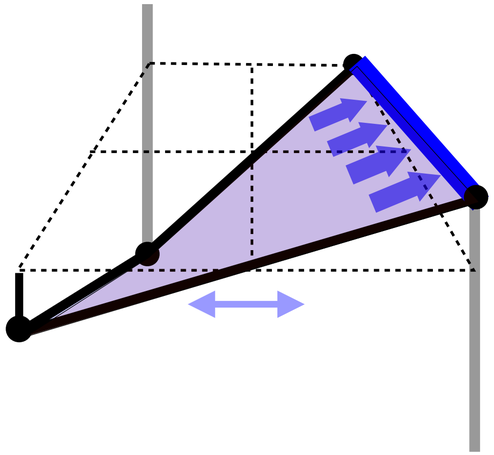

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-1	-3

$+\theta_1\times$

-3	0
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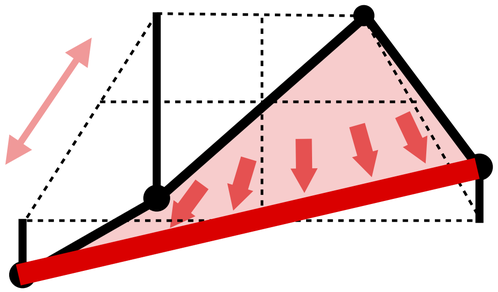

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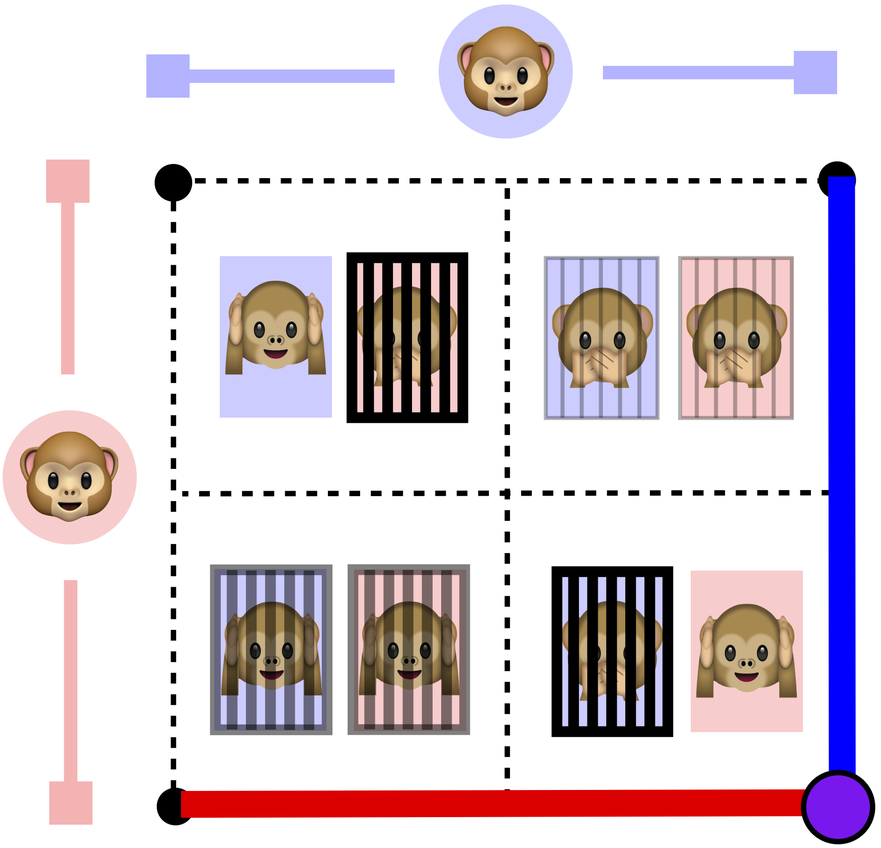

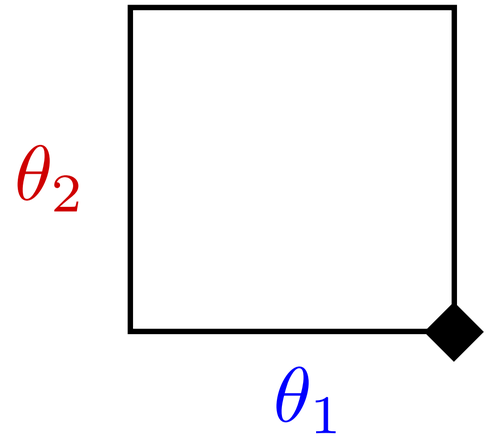

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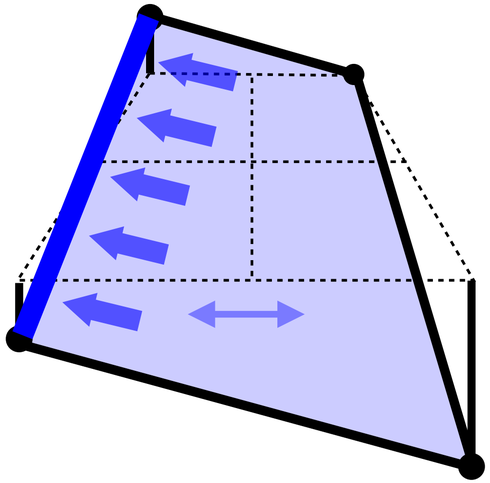
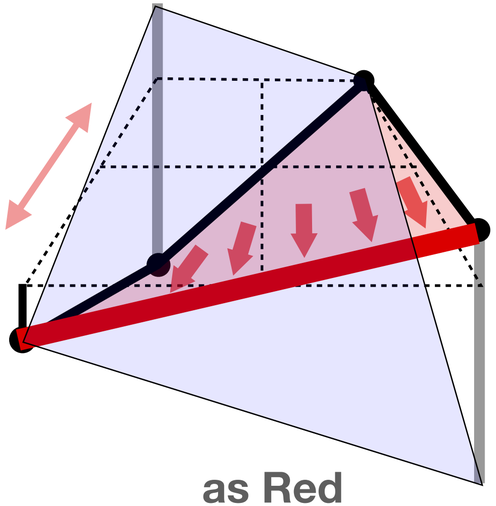


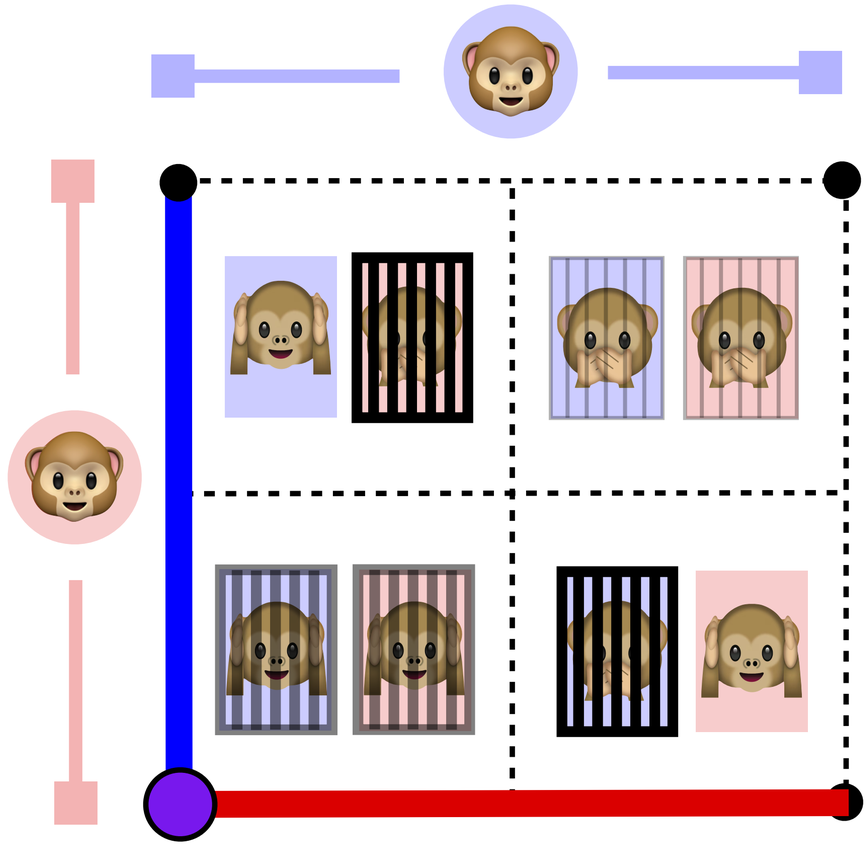


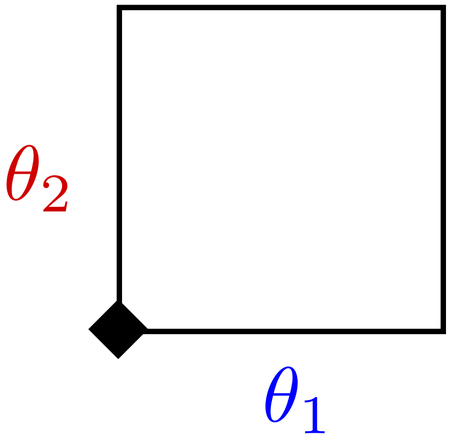
as Red player becomes more altruistic...

$$\begin{array}{|c|c|}\hline +1 & 0 \\ \hline -1 & -3 \\ \hline \end{array}$$

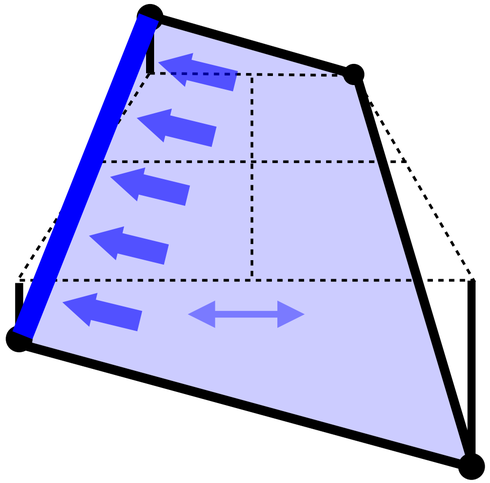
$$(1-\theta_2)\times \begin{array}{|c|c|}\hline -3 & 0 \\ \hline -1 & +1 \\ \hline \end{array}$$

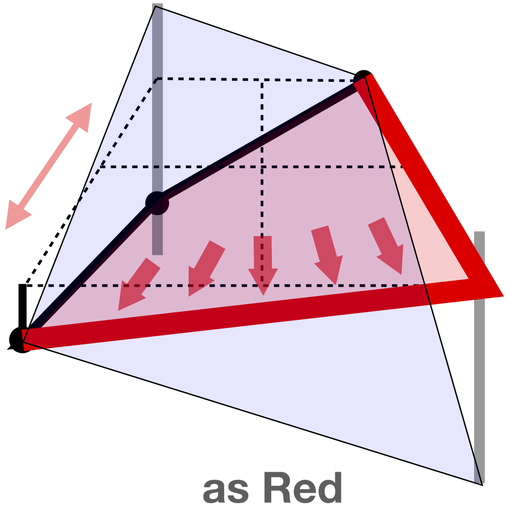
$$+\theta_2\times \begin{array}{|c|c|}\hline +1 & 0 \\ \hline -1 & -3 \\ \hline \end{array}$$





Matrix Game: Prisoner's Dilemma - SVO Nash





as Red player becomes more altruistic…

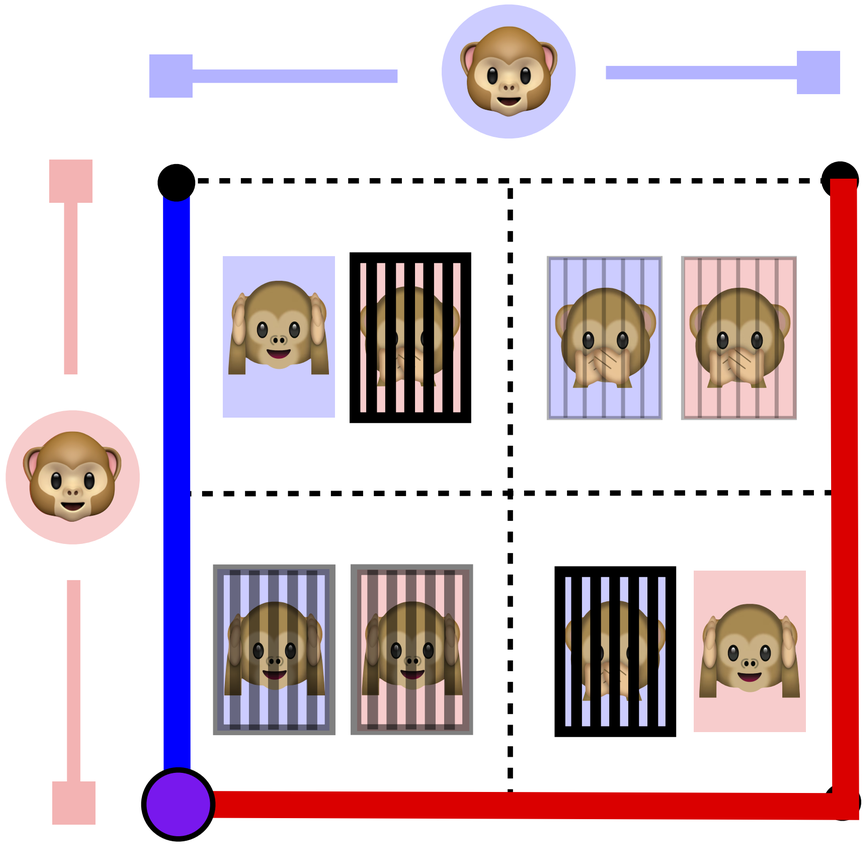
$+1$	0
-1	-3

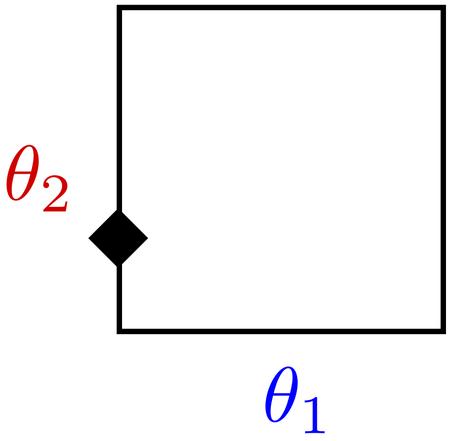
$(1-\theta_2)\times$

-3	0
-1	$+1$

$+\theta_2\times$

$+1$	0
-1	-3





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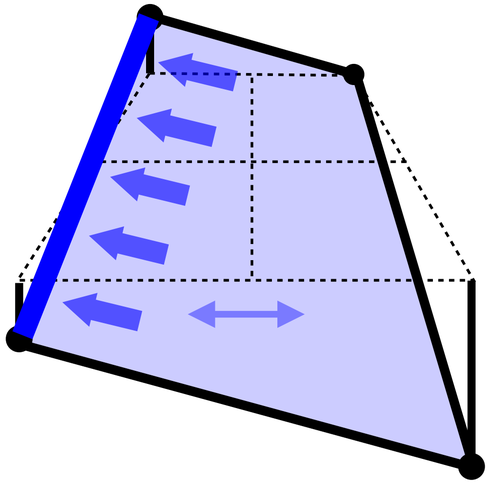

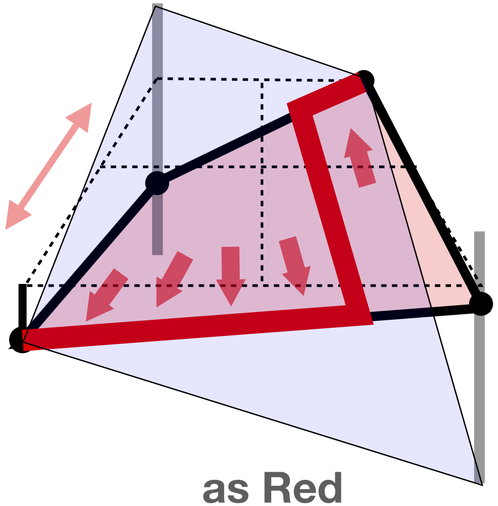

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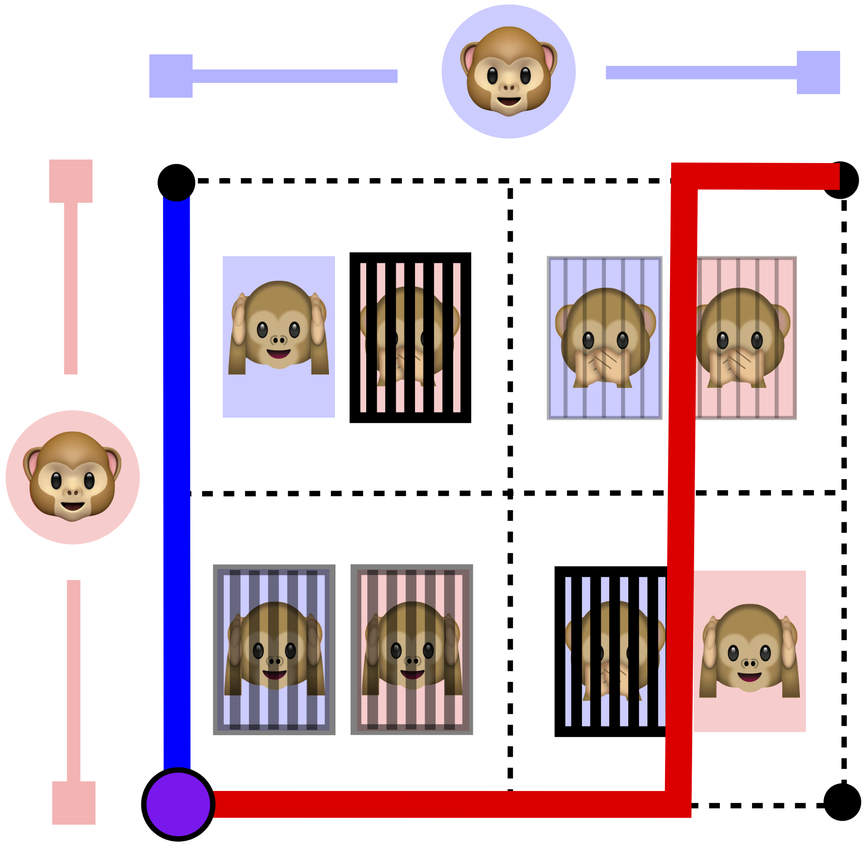

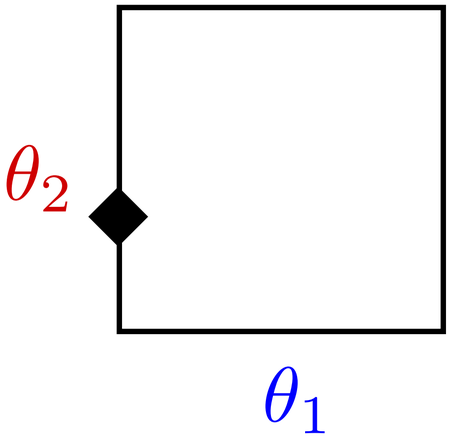

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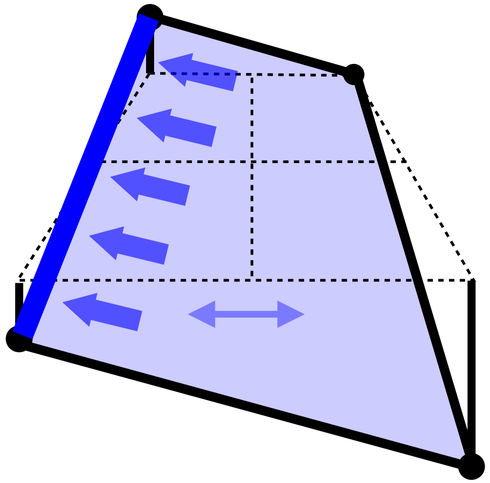

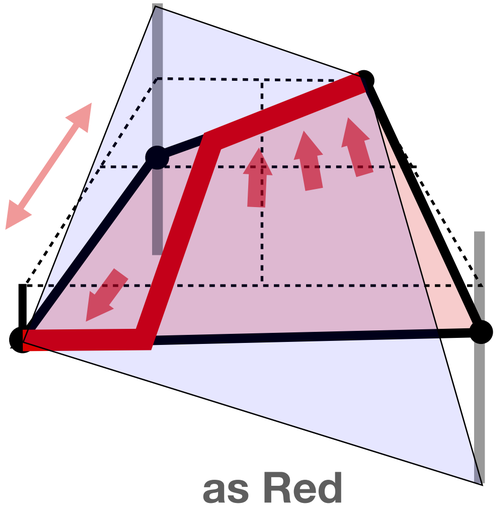

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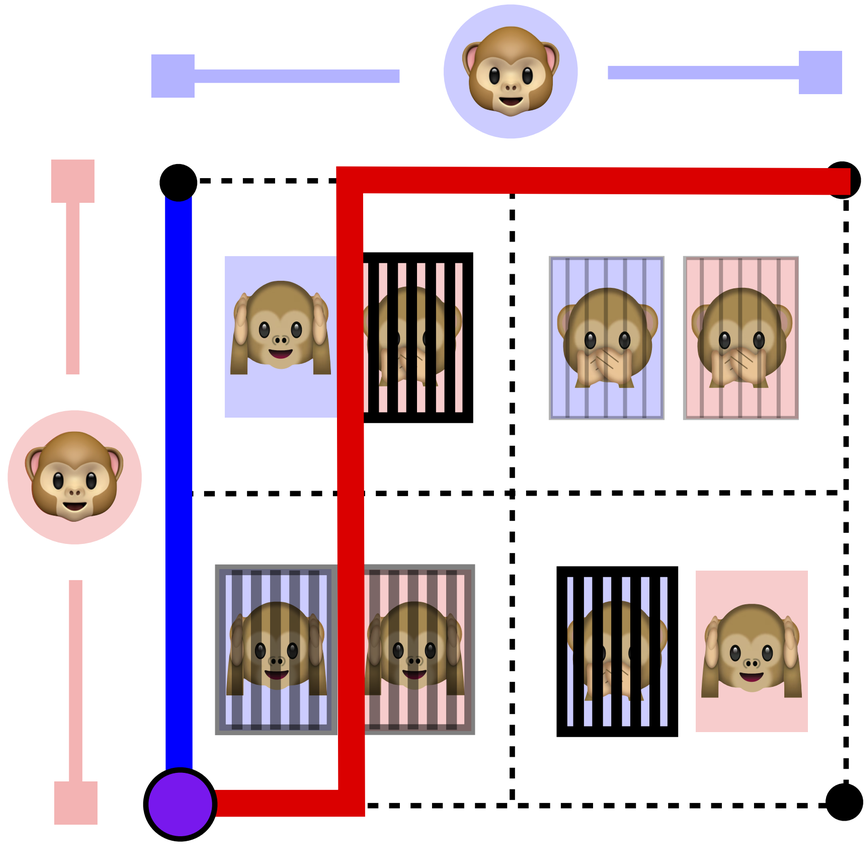

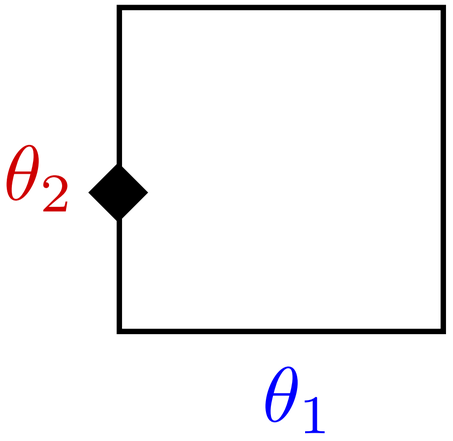

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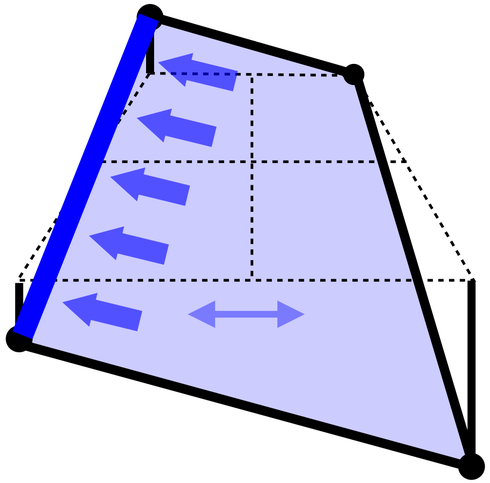
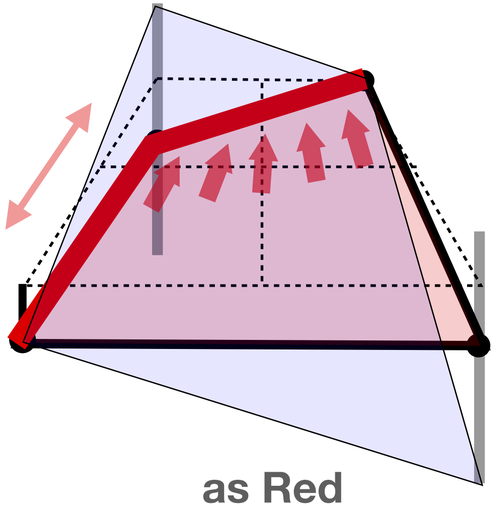


$+1$	0
-1	-3



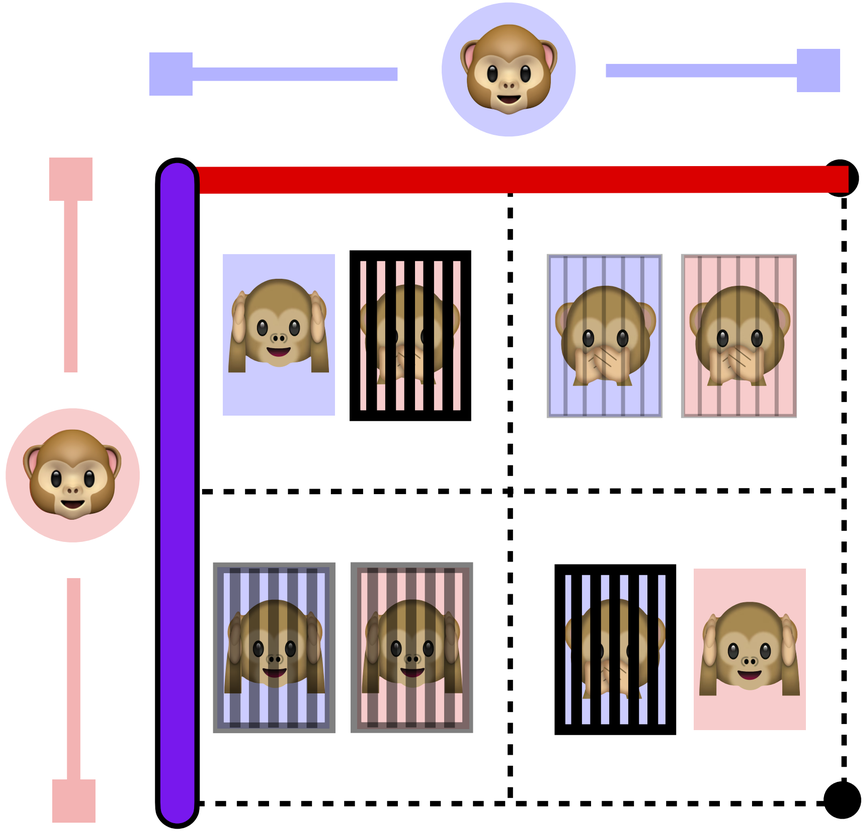
$(1-\theta_2)\times$

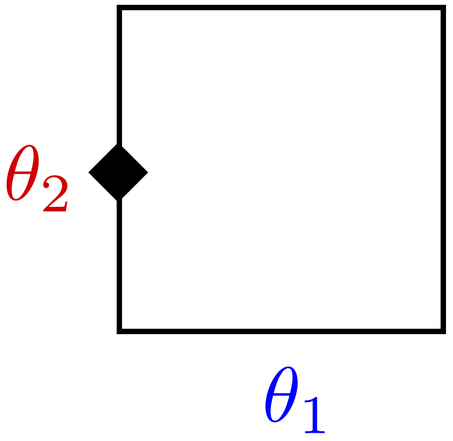
-3	0
-1	$+1$

$+\theta_2\times$

$+1$	0
-1	-3

as Red player becomes more altruistic…





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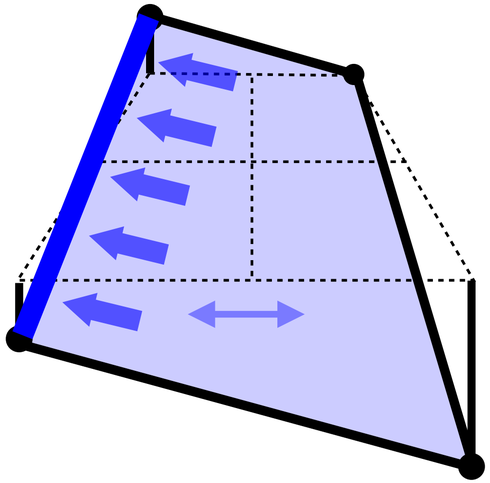

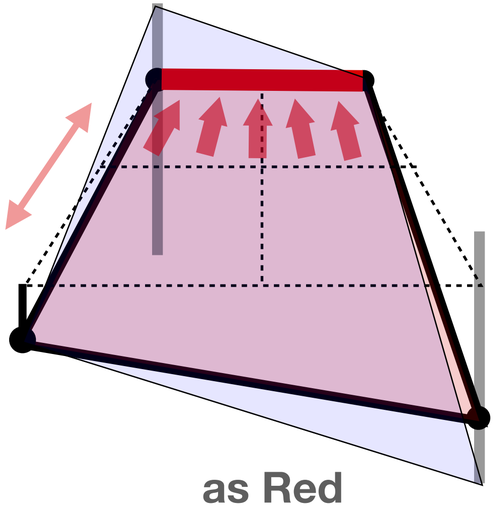

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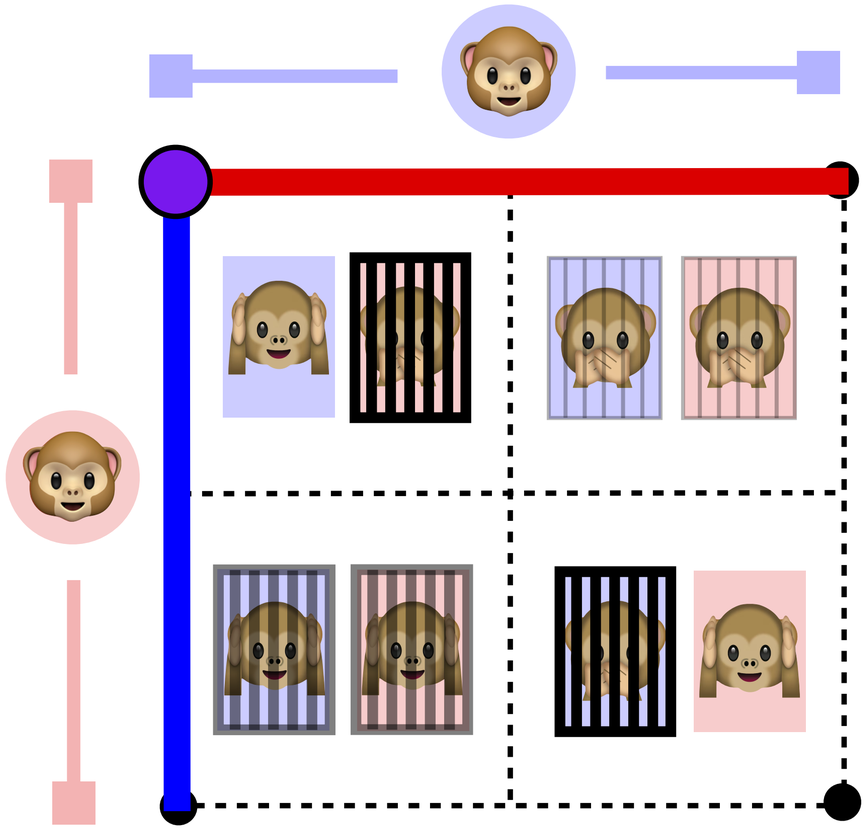

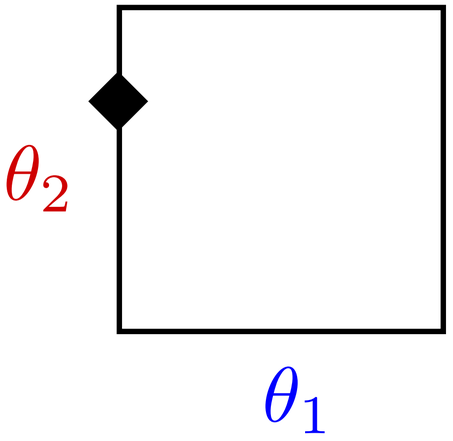

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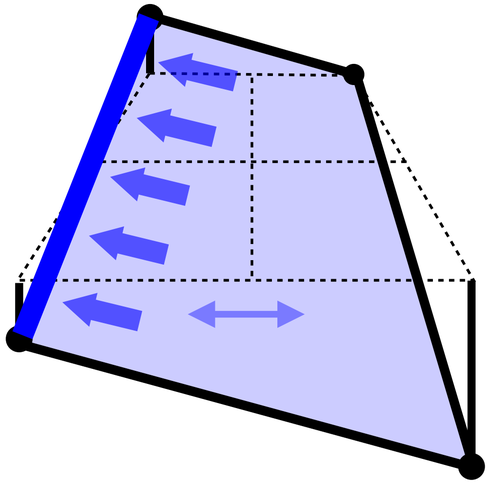

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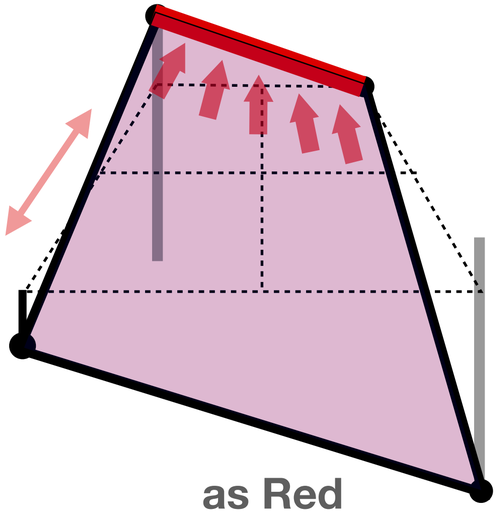
-1	-3



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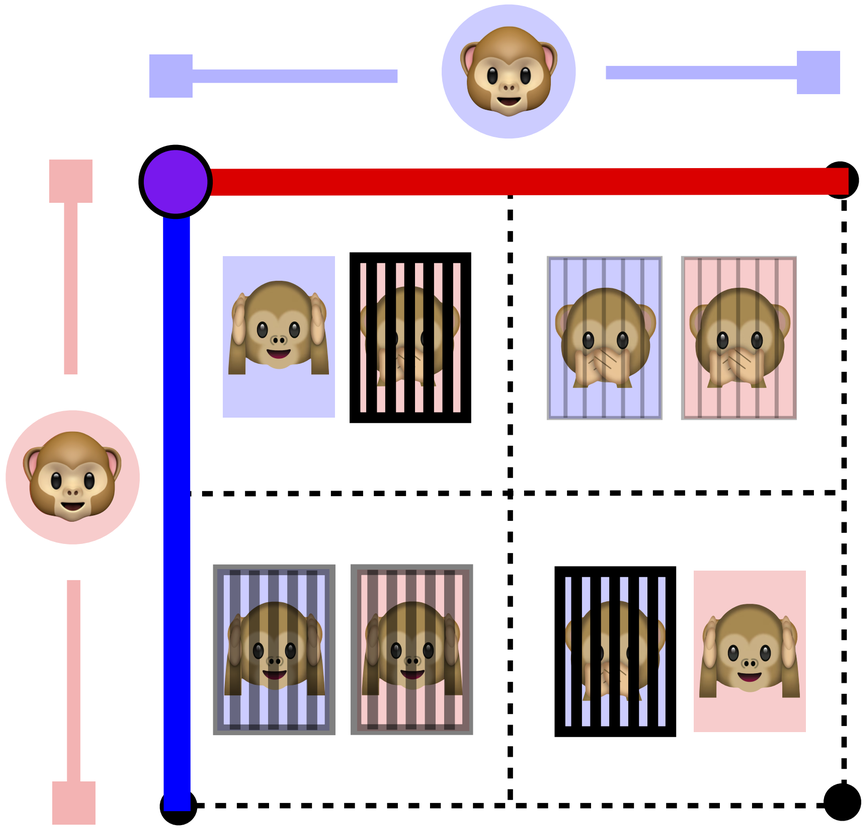

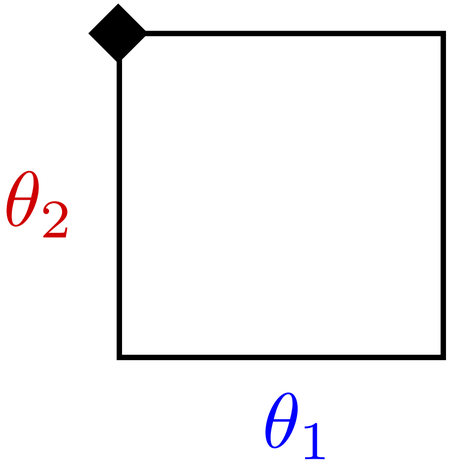

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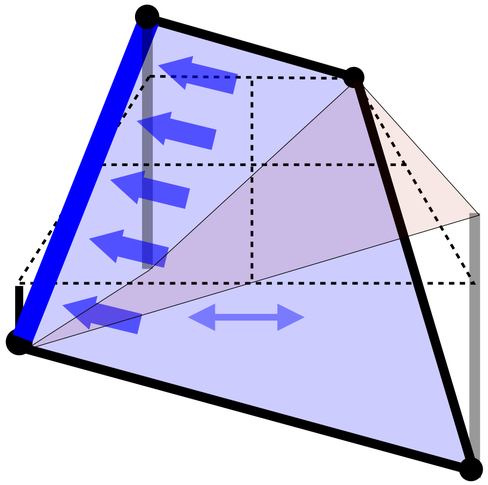
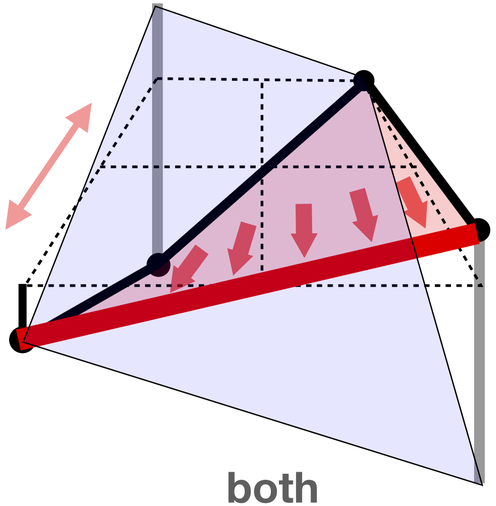




both
become more
altruistic (equally)...

$(1-\theta_1)\times$

$+1$	0
-1	-3

$+\theta_1\times$

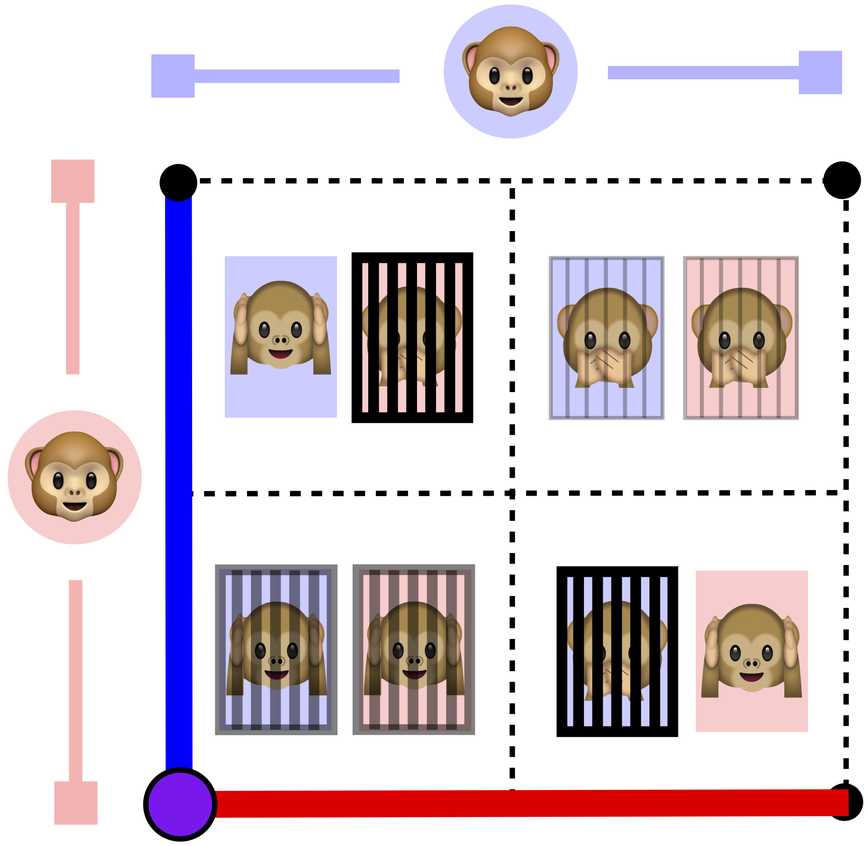
-3	0
-1	$+1$

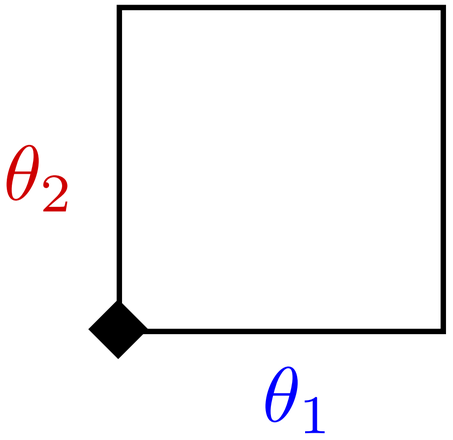
$(1-\theta_2)\times$

-3	0
-1	$+1$

$+\theta_2\times$

$+1$	0
-1	-3





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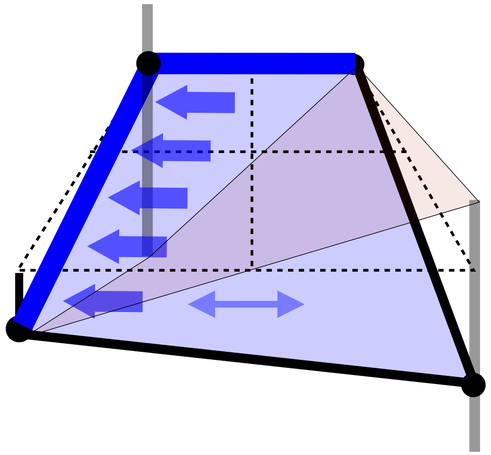

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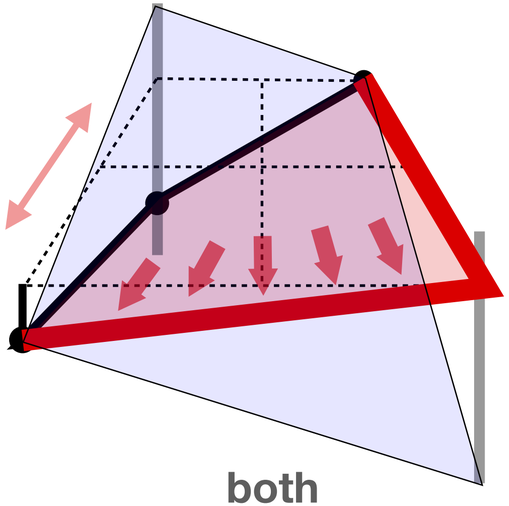
-1	-3

$+\theta_1\times$

-3	0
-1	$+1$



$(1-\theta_2)\times$

-3	0
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$+\theta_2\times$

$+1$	0
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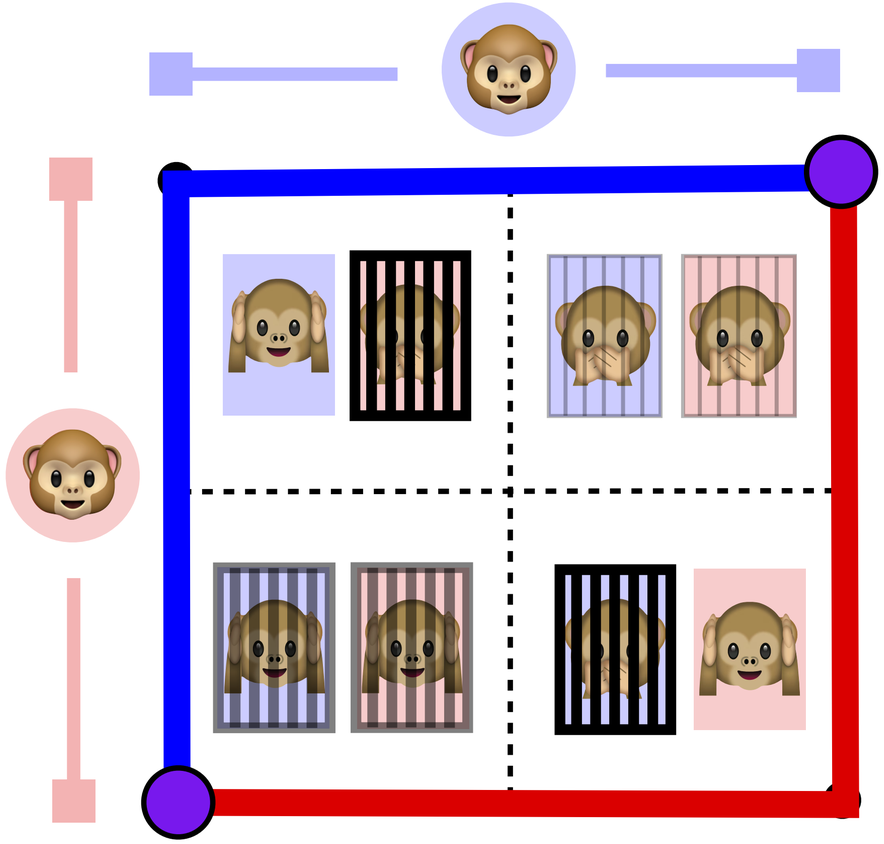
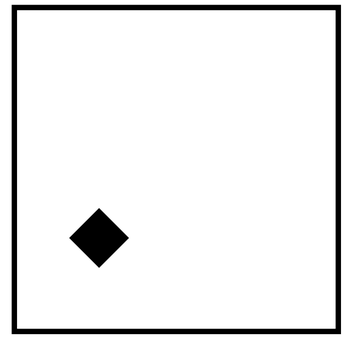
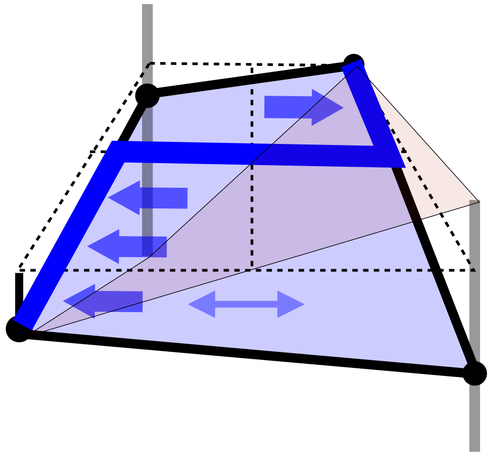
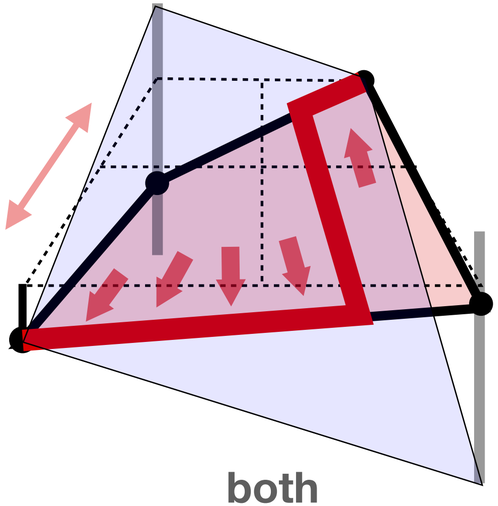
altruistic (equally)...





Matrix Game: Prisoner's Dilemma - SVO Nash





both
become more
altruistic (equally)...

$(1-\theta_1)\times$

$+1$	0
-1	-3

$+\theta_1\times$

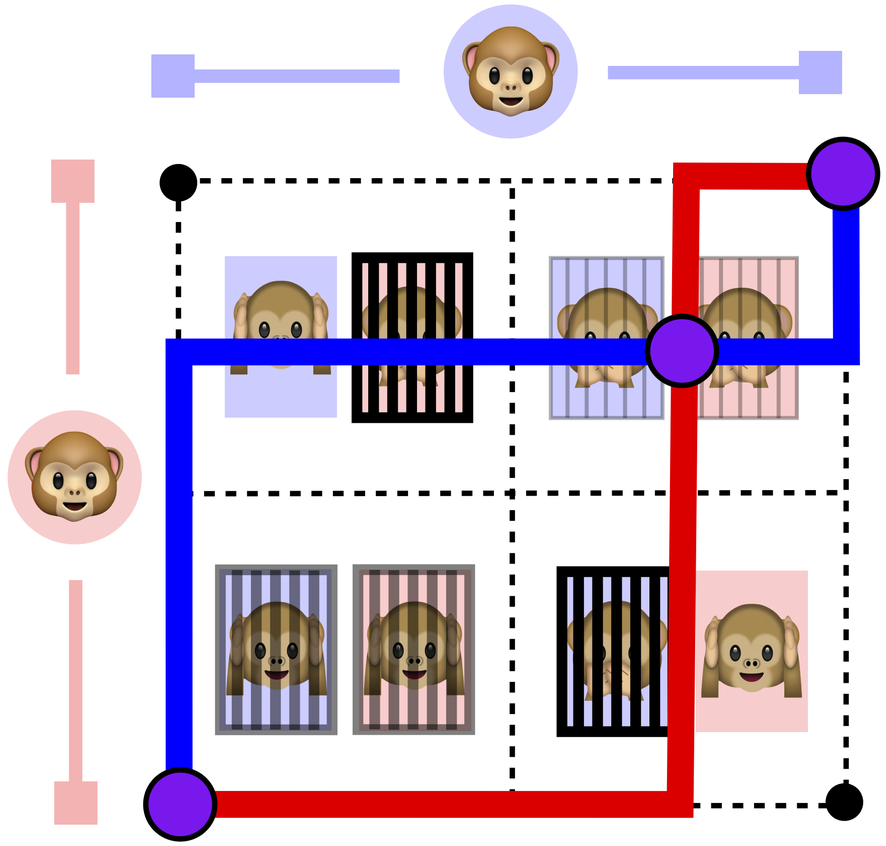
-3	0
-1	$+1$

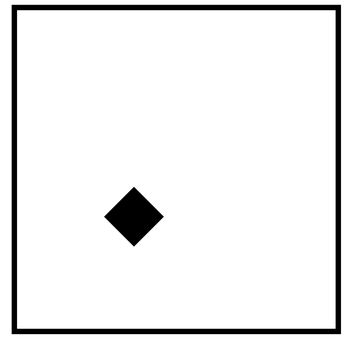
$(1-\theta_2)\times$

-3	0
-1	$+1$

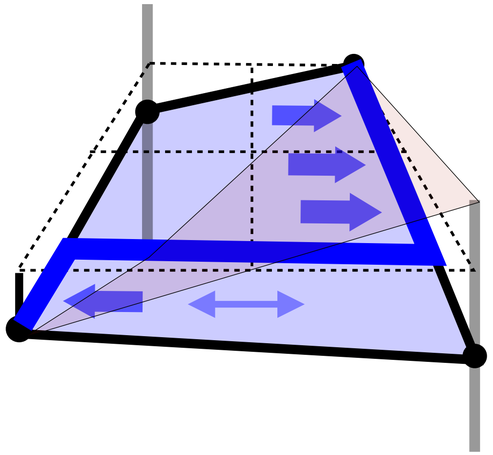
$+\theta_2\times$

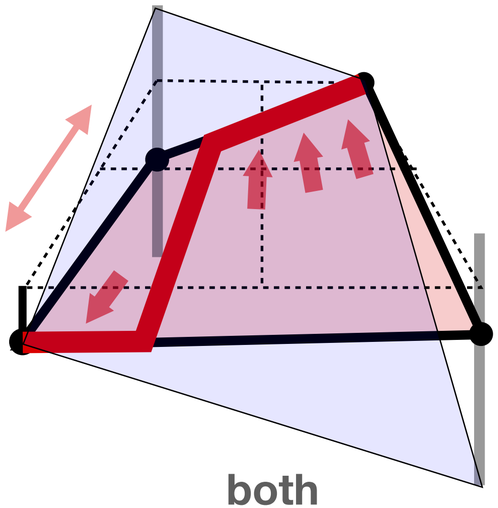
$+1$	0
-1	-3





Matrix Game: Prisoner's Dilemma - SVO Nash





both
become more
altruistic (equally)...

$(1-\theta_1)\times$

$+1$	0
-1	-3

$+\theta_1\times$

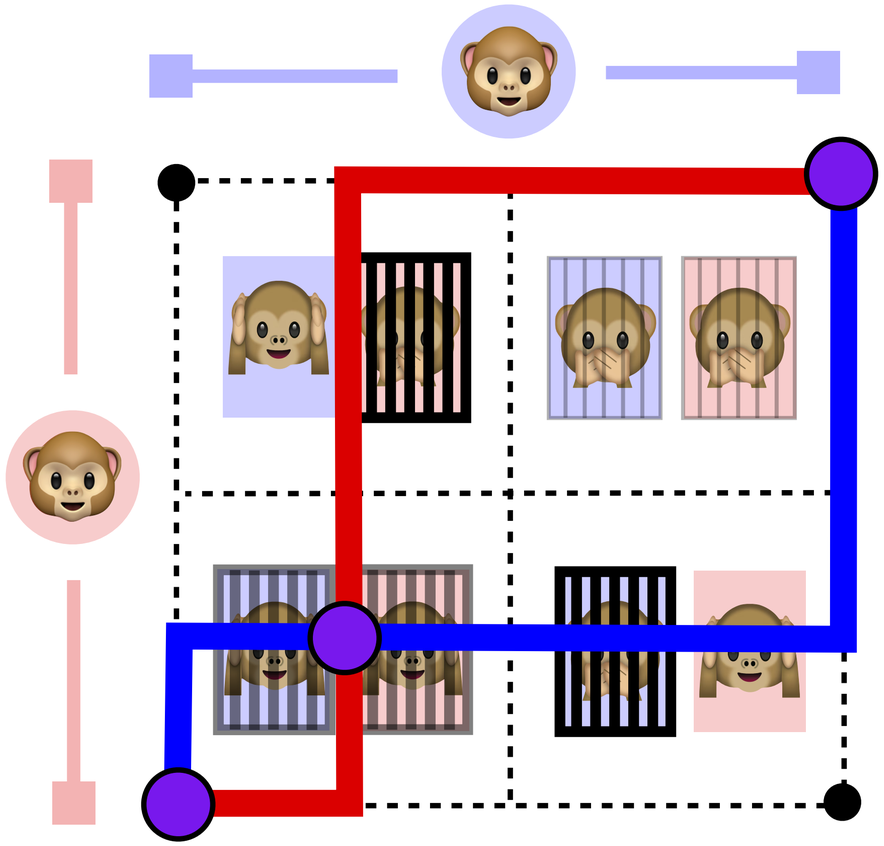
-3	0
-1	$+1$

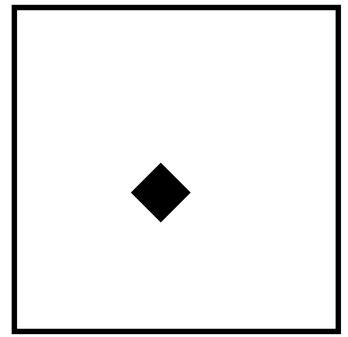
$(1-\theta_2)\times$

-3	0
-1	$+1$

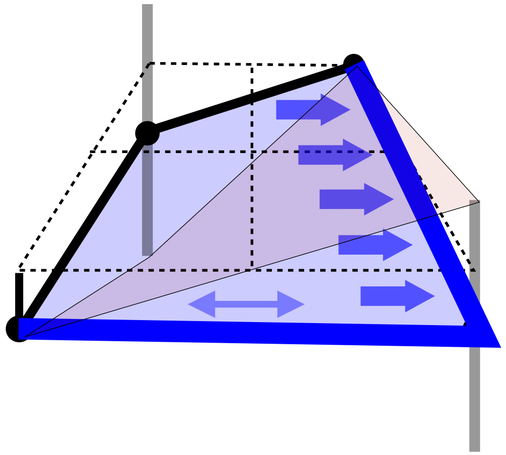
$+\theta_2\times$

$+1$	0
-1	-3





Matrix Game: Prisoner's Dilemma - SVO Nash

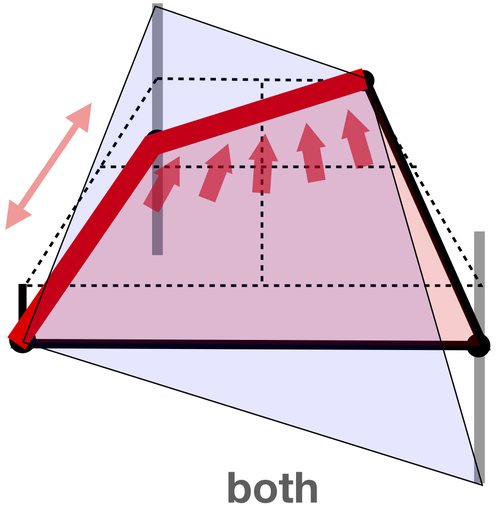


$(1-\theta_1)\times$

$+1$	0
-1	-3

$+\theta_1\times$

-3	0
-1	$+1$



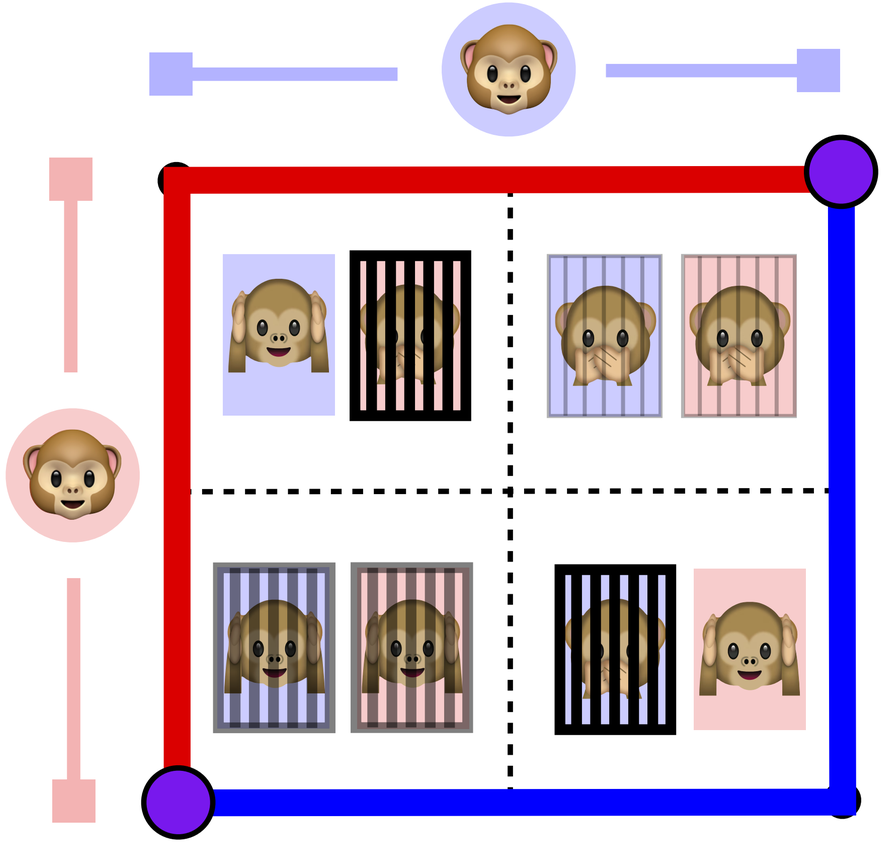
$(1-\theta_2)\times$

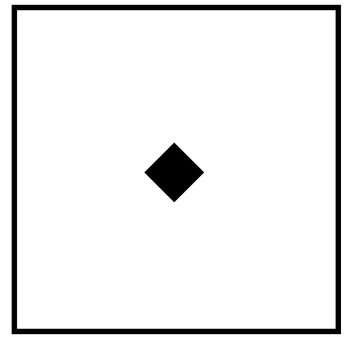
-3	0
-1	$+1$

$+\theta_2\times$

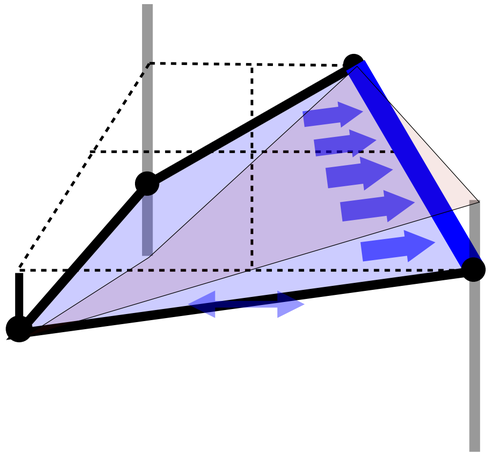
$+1$	0
-1	-3

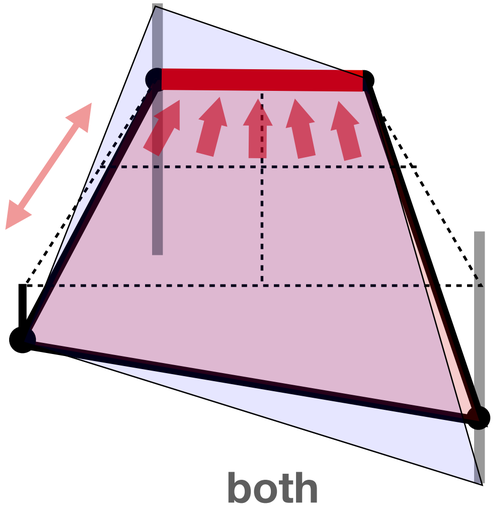
both
become more
altruistic (equally)...





Matrix Game: Prisoner's Dilemma - SVO Nash





both
become more
altruistic (equally)...

$(1-\theta_1)\times$

$+1$	0
-1	-3

$+\theta_1\times$

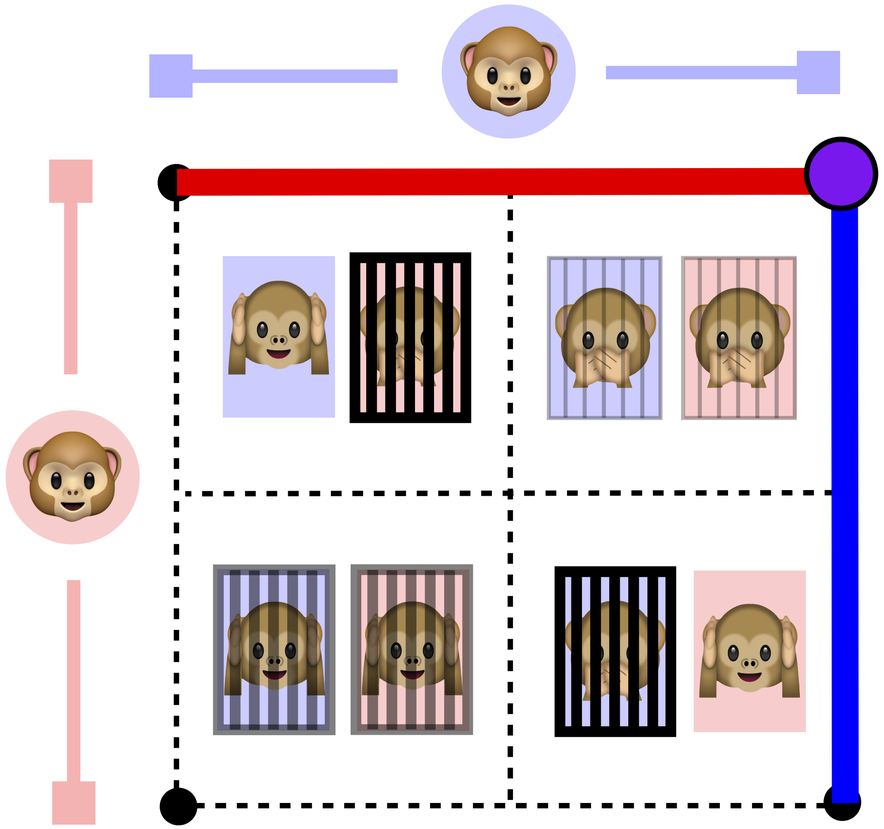
-3	0
-1	$+1$

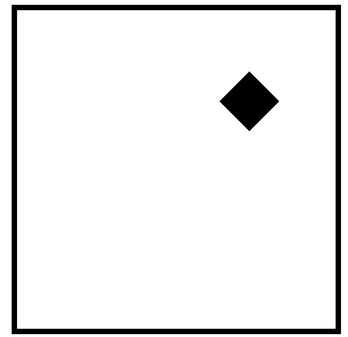
$(1-\theta_2)\times$

-3	0
-1	$+1$

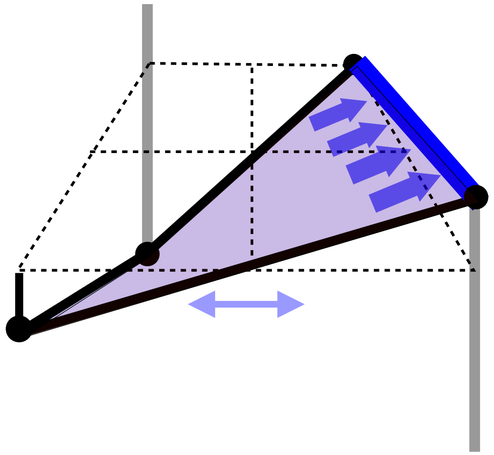
$+\theta_2\times$

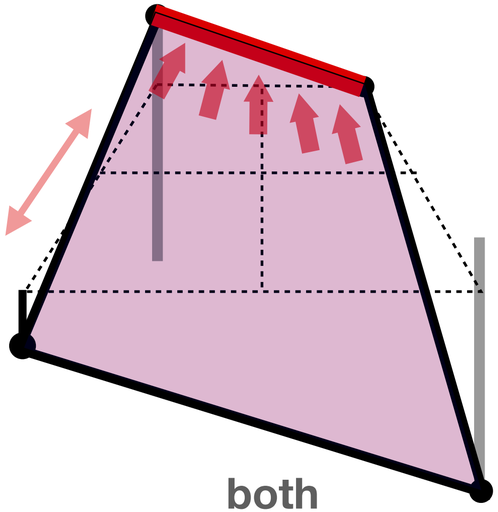
$+1$	0
-1	-3





Matrix Game: Prisoner's Dilemma - SVO Nash





both
become more
altruistic (equally)...

$(1-\theta_1)\times$

$+1$	0
-1	-3

$+\theta_1\times$

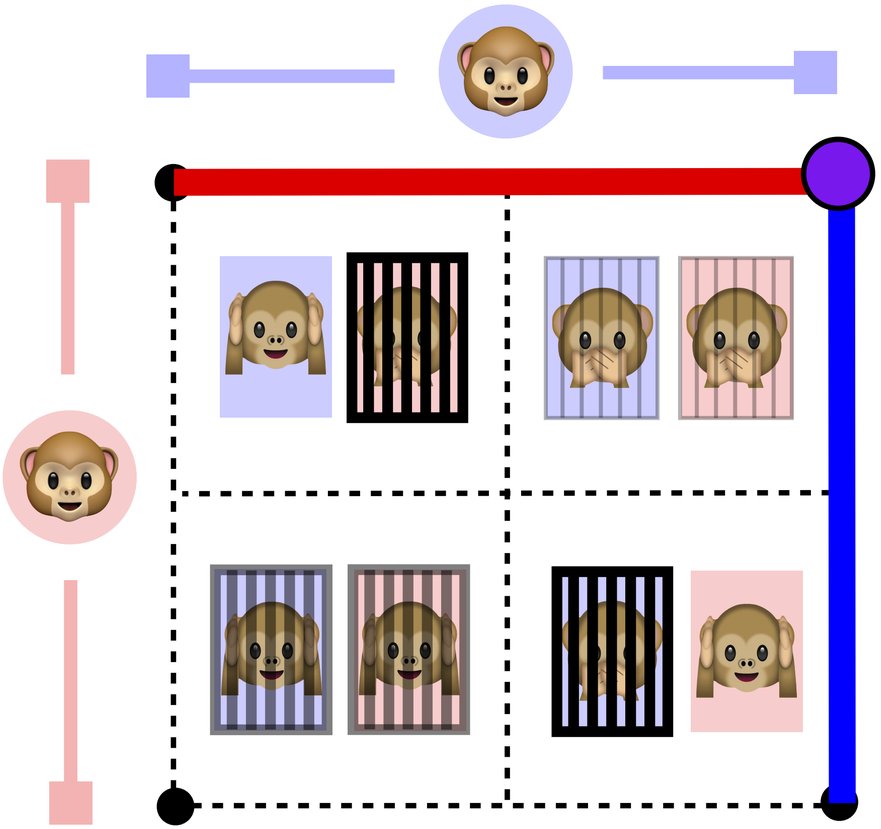
-3	0
-1	$+1$

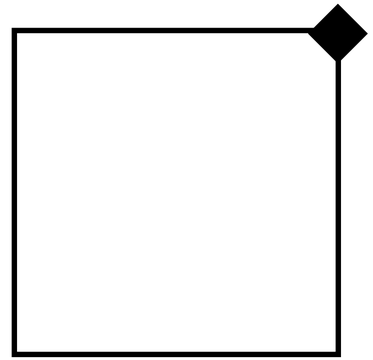
$(1-\theta_2)\times$

-3	0
-1	$+1$

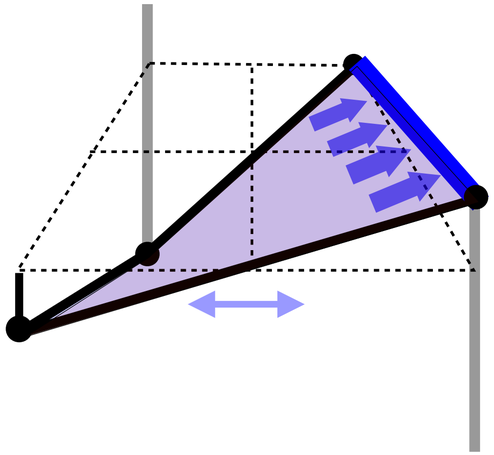
$+\theta_2\times$

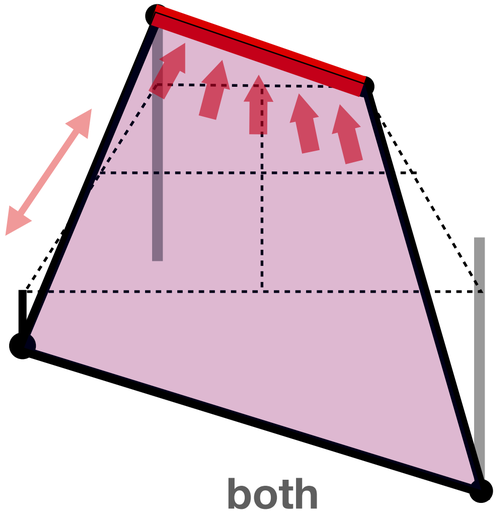
$+1$	0
-1	-3





Matrix Game: Prisoner's Dilemma - SVO Nash





both
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$(1-\theta_1)\times$

$+1$	0
-1	-3

$+\theta_1\times$

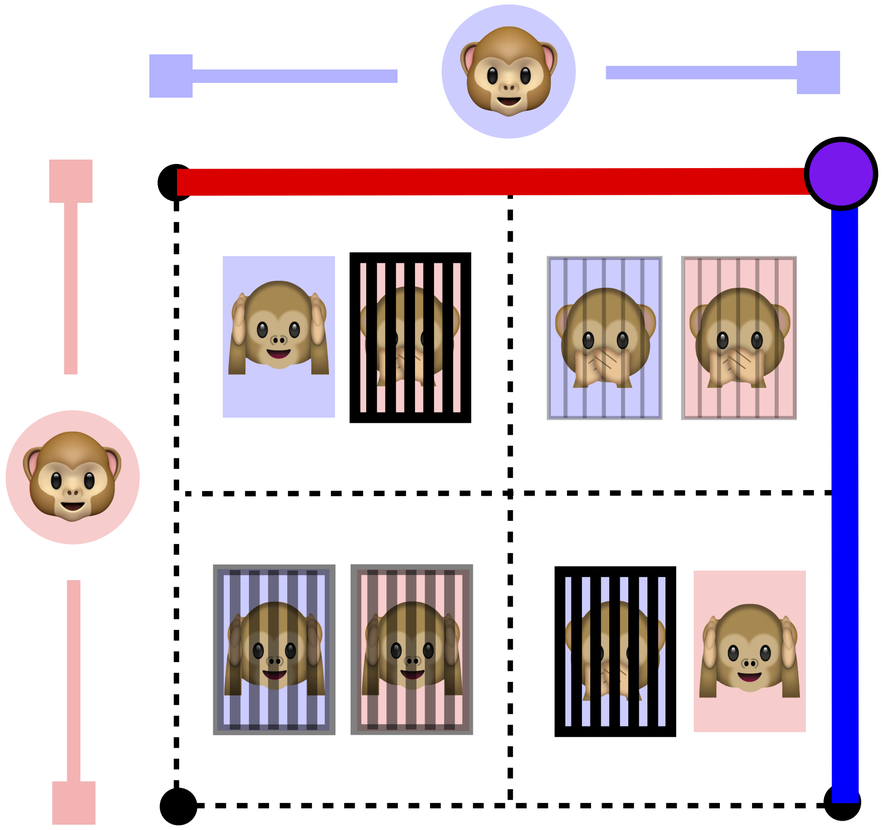
-3	0
-1	$+1$

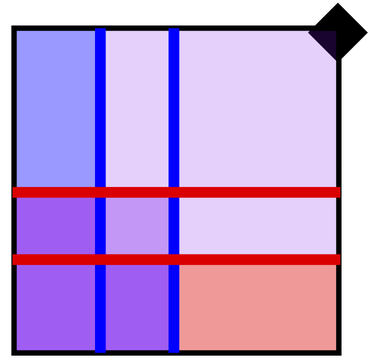
$(1-\theta_2)\times$

-3	0
-1	$+1$

$+\theta_2\times$

$+1$	0
-1	-3

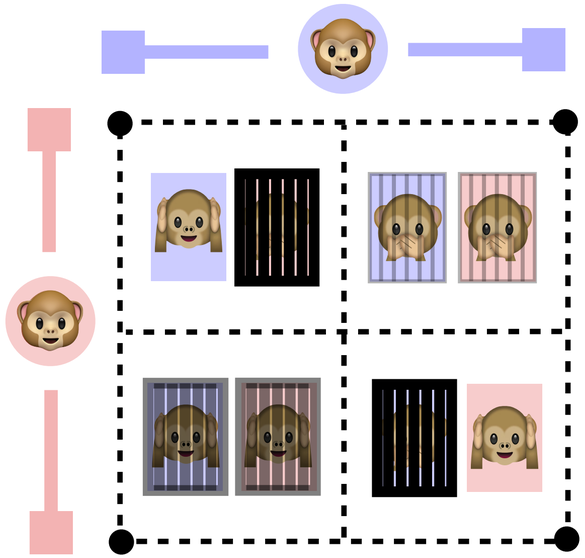


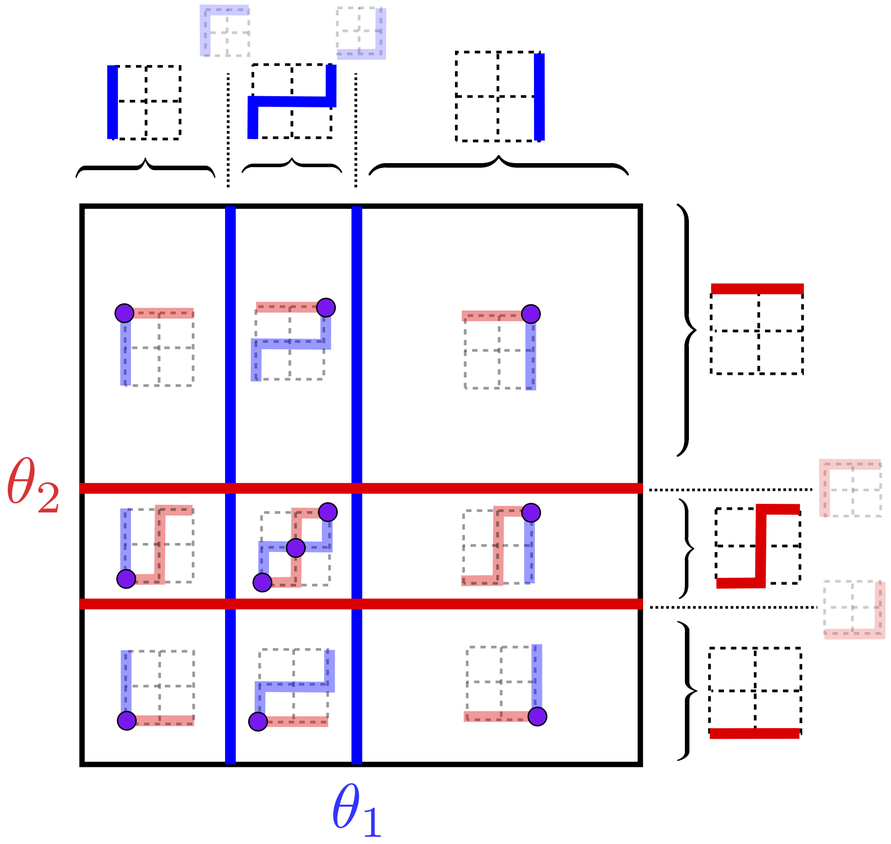


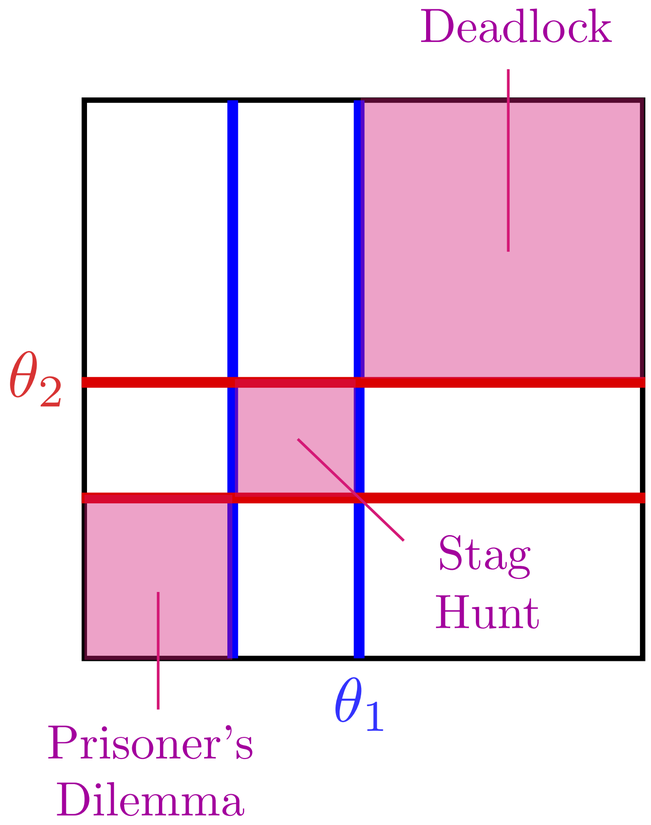
Prisoner's Dilemma
Stag Hunt
Deadlock
Player 1 Optimal
Player 2 Optimal

Matrix Game: Prisoner's Dilemma - SVO Nash Structure

"Prisoner's dilemma morphs into other well-known game types as the players' SVO change."

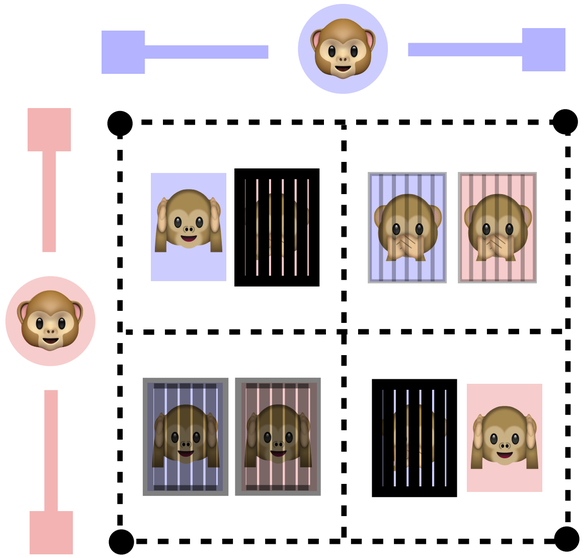


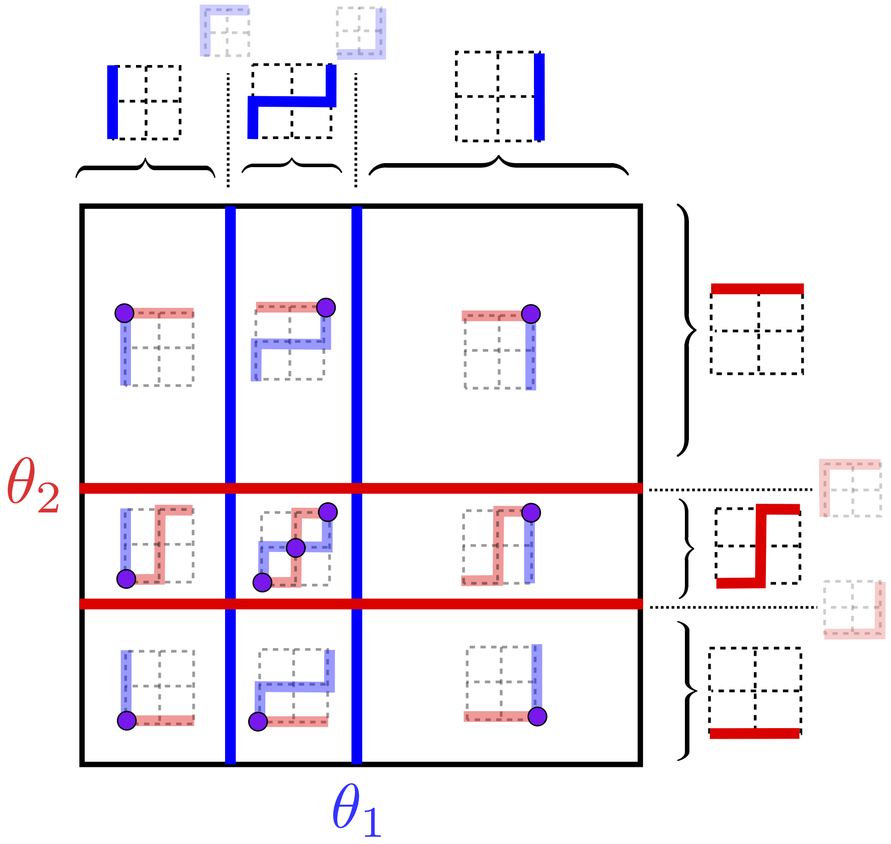




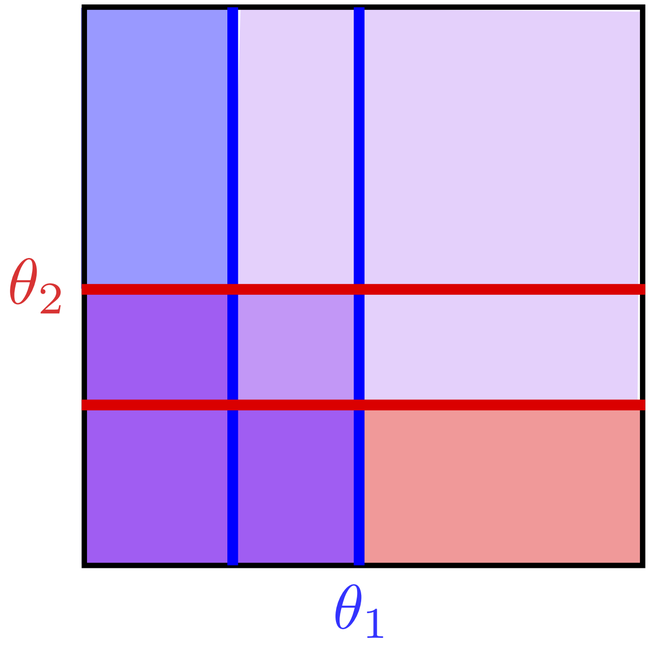
Matrix Game: Prisoner's Dilemma - SVO Nash Structure

"Prisoner's dilemma morphs into other well-known game types as the players' SVO change."





Prisoner's Dilemma
Stag Hunt
Deadlock
Player 1 Optimal
Player 2 Optimal

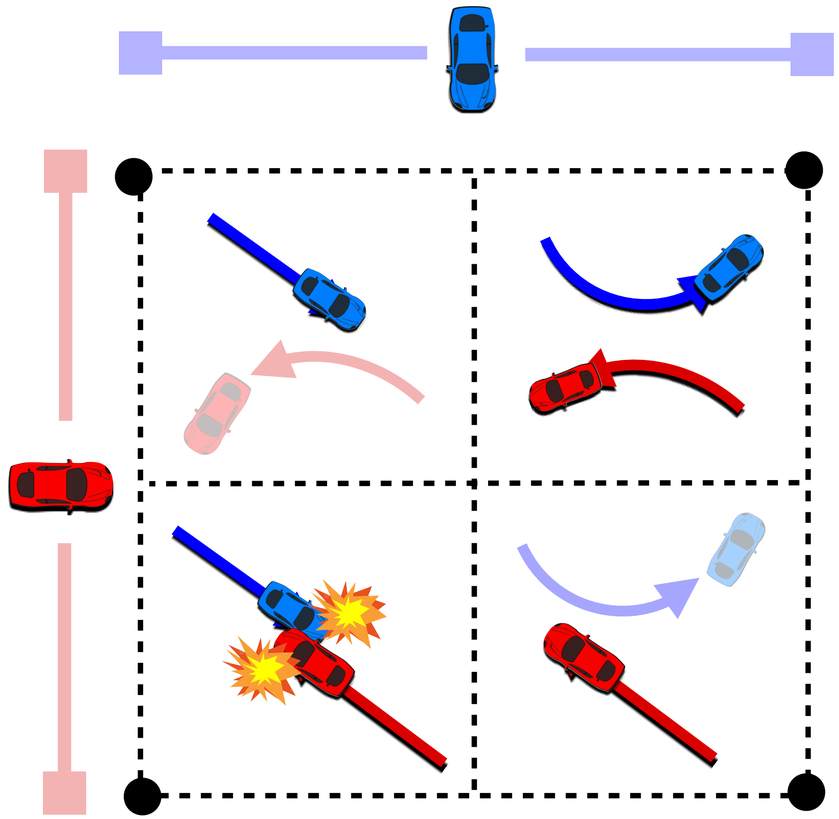


Matrix Game: Chicken

Matrix Game: Chicken

$+1$	0
-3	-1

-1	0
-3	$+1$

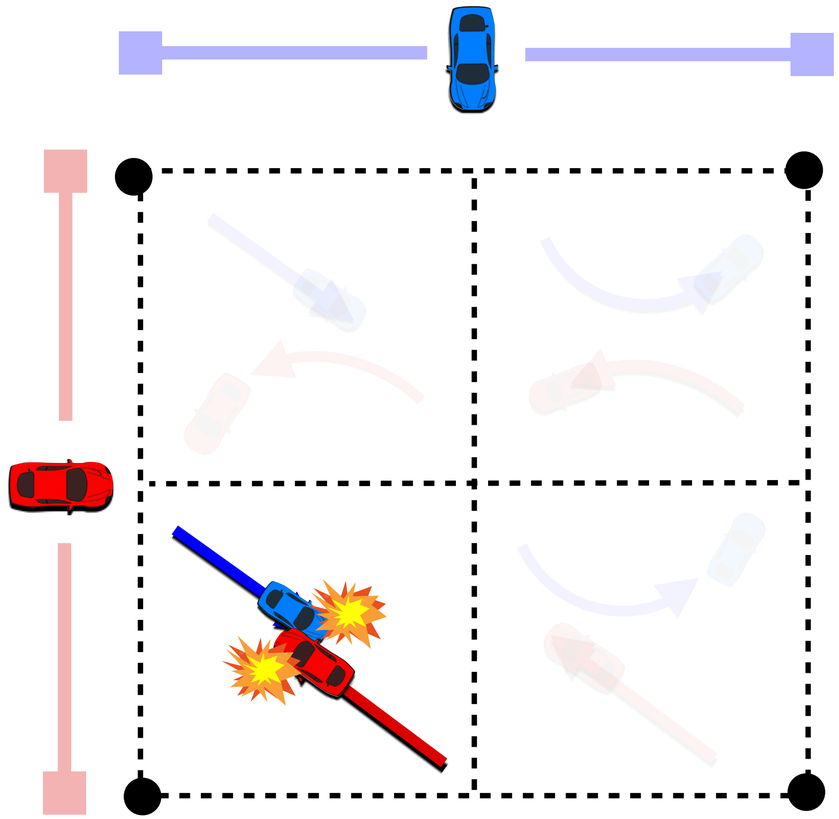


- Two drivers dare each other to flinch to avoid a head on collision

Matrix Game: Chicken

$+1$	0
-3	-1

-1	0
-3	$+1$

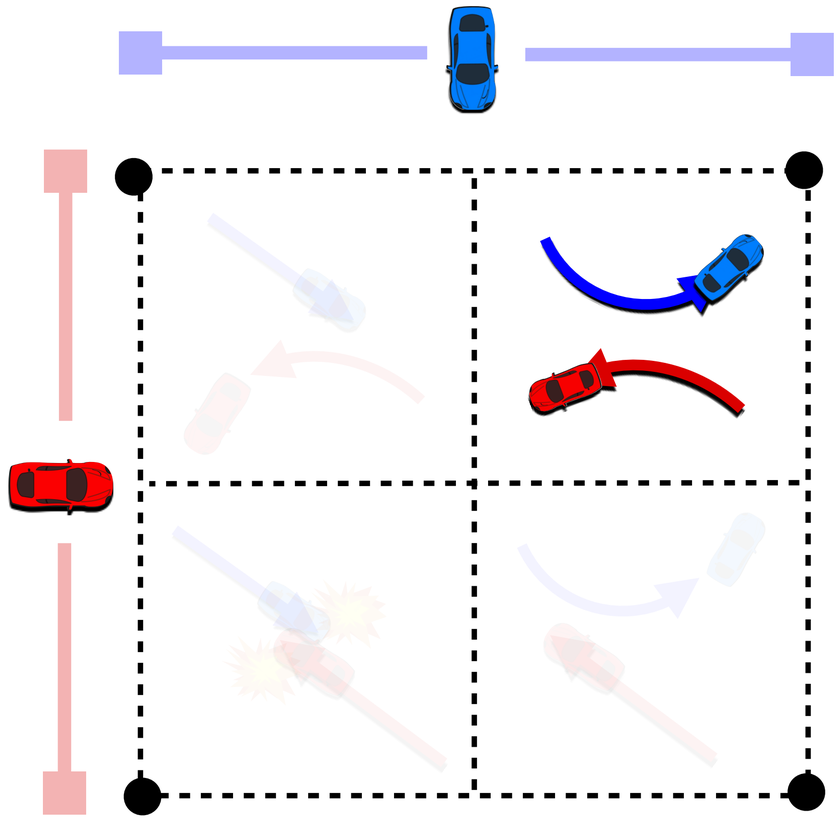


- **Two drivers dare each other to flinch to avoid a head on collision**
- If neither flinches, they crash into each other. reward = (-3,-3)

Matrix Game: Chicken

$+1$	0
-3	-1

-1	0
-3	$+1$

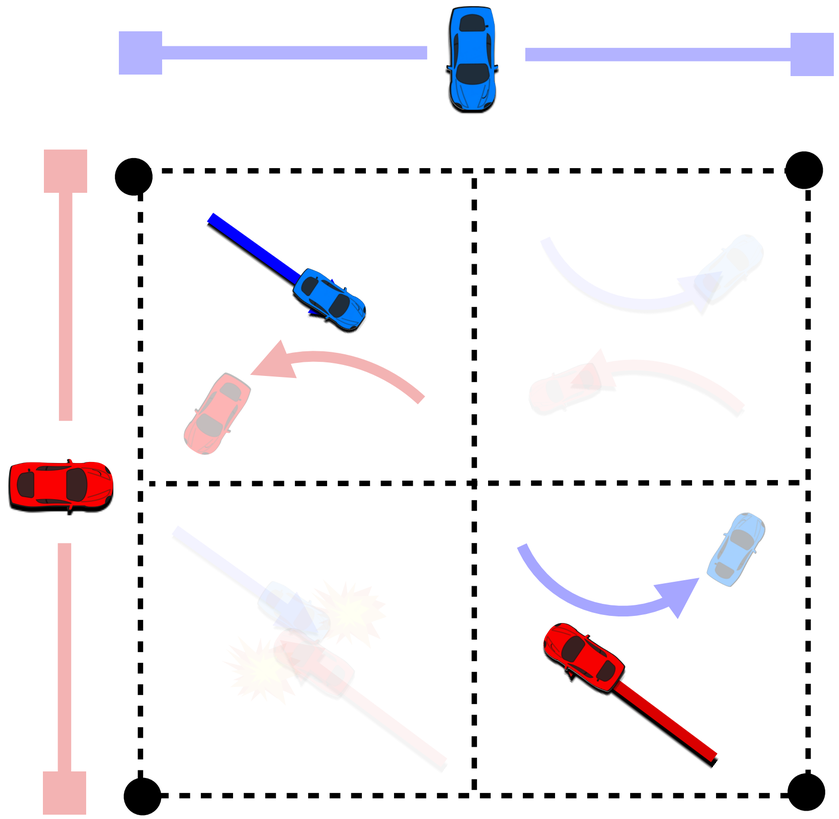


- **Two drivers dare each other to flinch to avoid a head on collision**
- If neither flinches, they crash into each other. reward = (-3,-3)
- If they both flinch, no one wins. reward = (0,0)

Matrix Game: Chicken

$+1$	0
-3	-1

-1	0
-3	$+1$

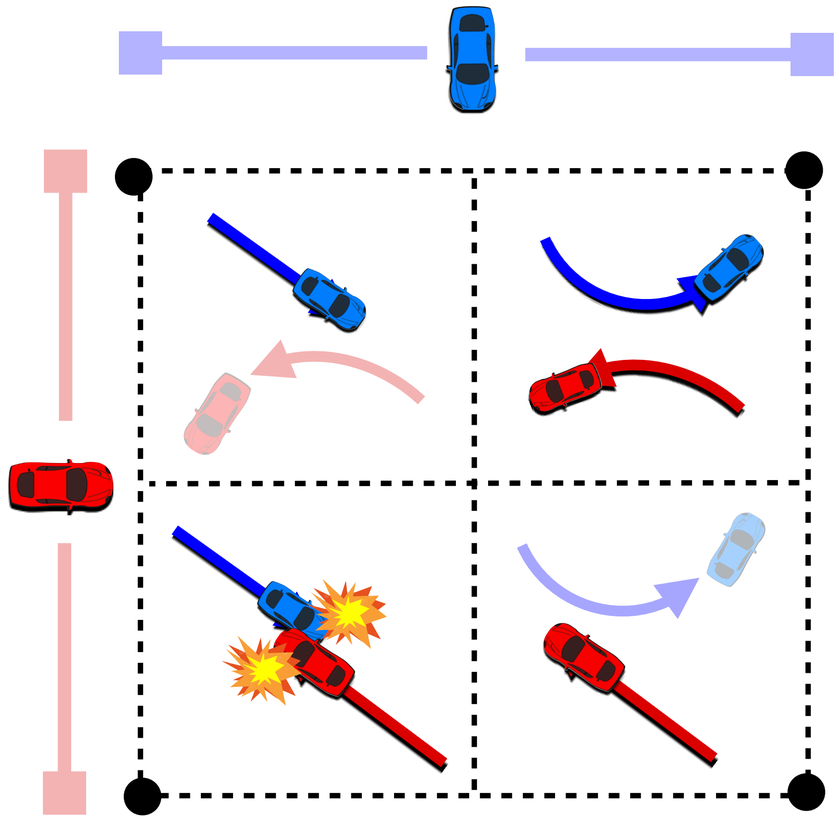


- **Two drivers dare each other to flinch to avoid a head on collision**
- If neither flinches, they crash into each other. reward = (-3,-3)
- If they both flinch, no one wins. reward = (0,0)
- If only one flinches, the other wins the dare. ex. reward = (+1,-1)

Matrix Game: Chicken

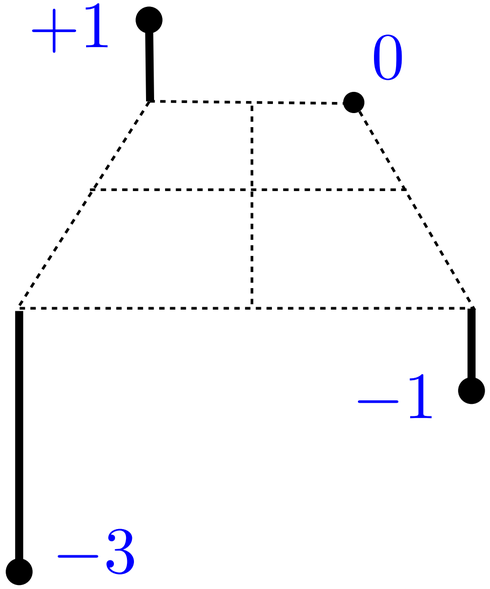
$+1$	0
-3	-1

-1	0
-3	$+1$

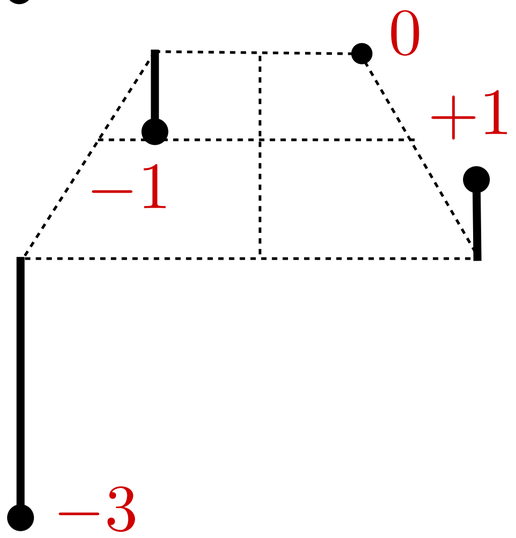


- **Two drivers dare each other to flinch to avoid a head on collision**
- If neither flinches, they crash into each other. reward = (-3,-3)
- If they both flinch, no one wins. reward = (0,0)
- If only one flinches, the other wins the dare. ex. reward = (+1,-1)
- **What will they do?**

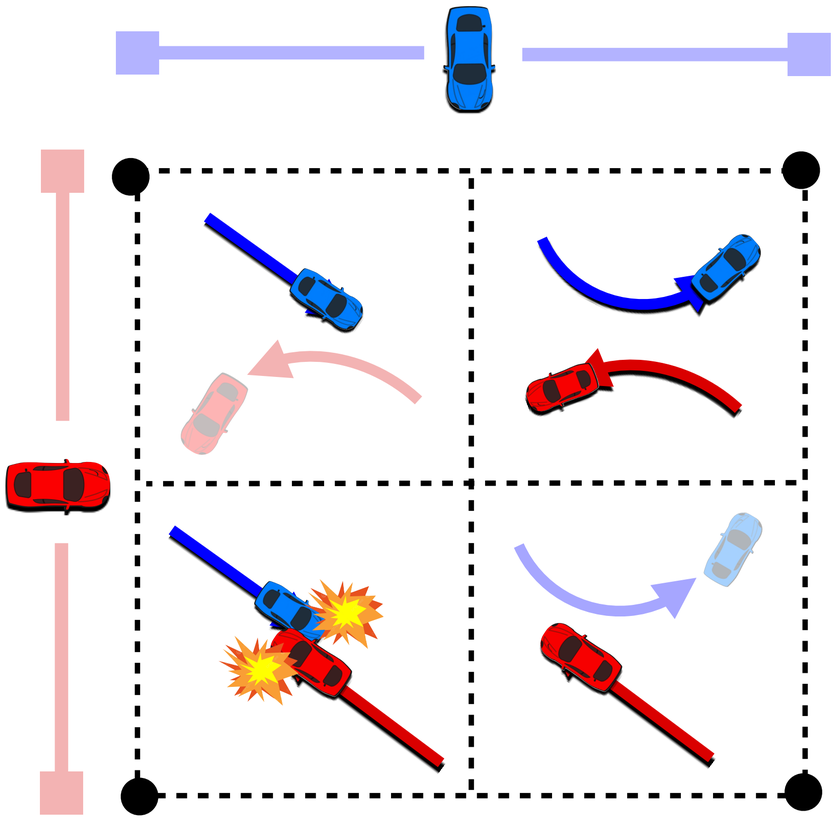
Matrix Game: Chicken



+1	0
−3	−1

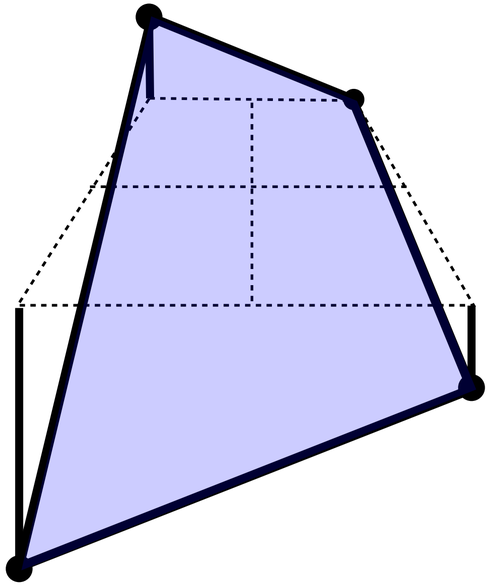


−1	0
−3	+1

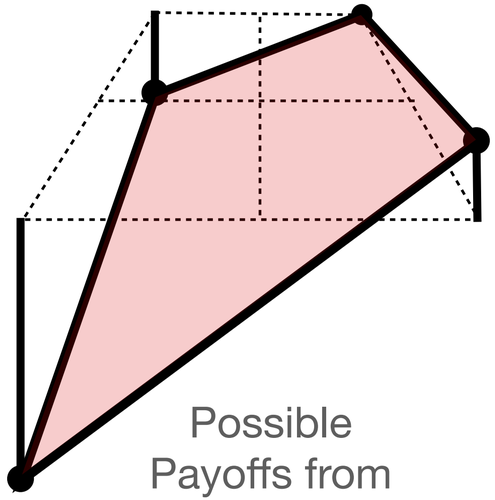


- **Two drivers dare each other to flinch to avoid a head on collision**
- If neither flinches, they crash into each other. reward = (-3,-3)
- If they both flinch, no one wins. reward = (0,0)
- If only one flinches, the other wins the dare. ex. reward = (+1,-1)
- **What will they do?**

Matrix Game: Chicken

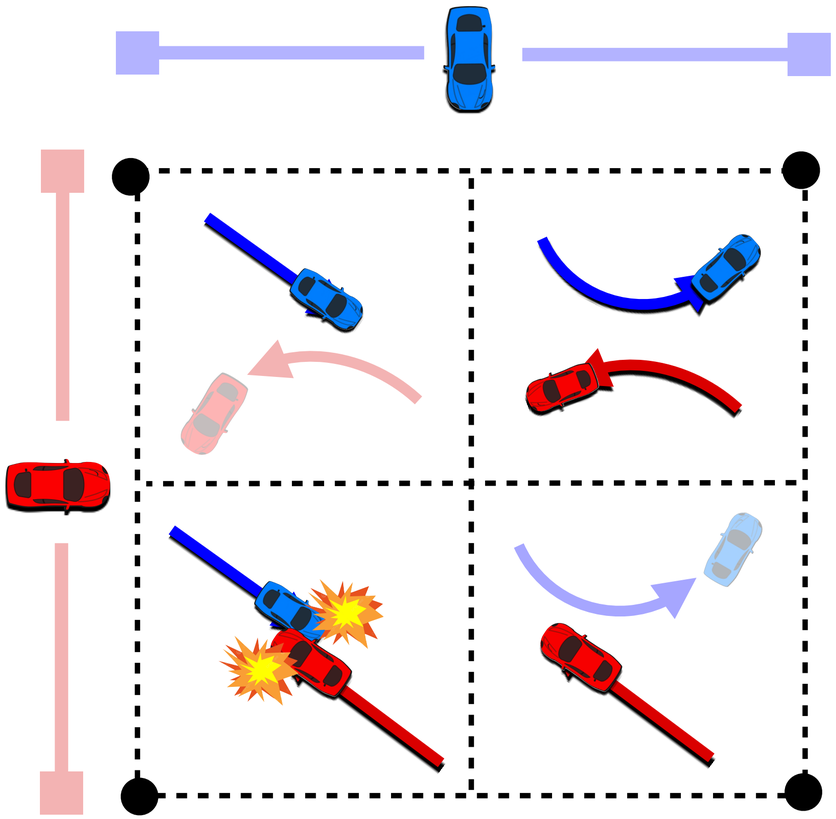


$+1$	0
-3	-1



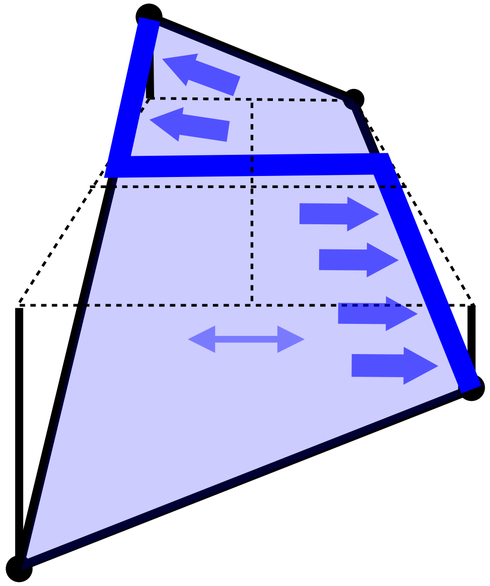
-1	0
-3	$+1$

Possible Payoffs from Mixed Strategies

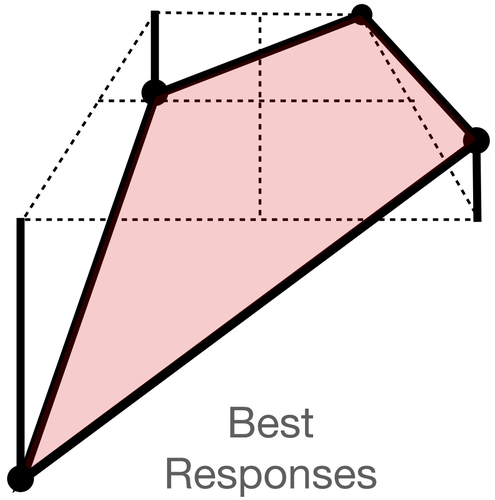


- **Two drivers dare each other to flinch to avoid a head on collision**
- If neither flinches, they crash into each other. reward = (-3,-3)
- If they both flinch, no one wins. reward = (0,0)
- If only one flinches, the other wins the dare. ex. reward = (+1,-1)
- **What will they do?**

Matrix Game: Chicken - Best Responses

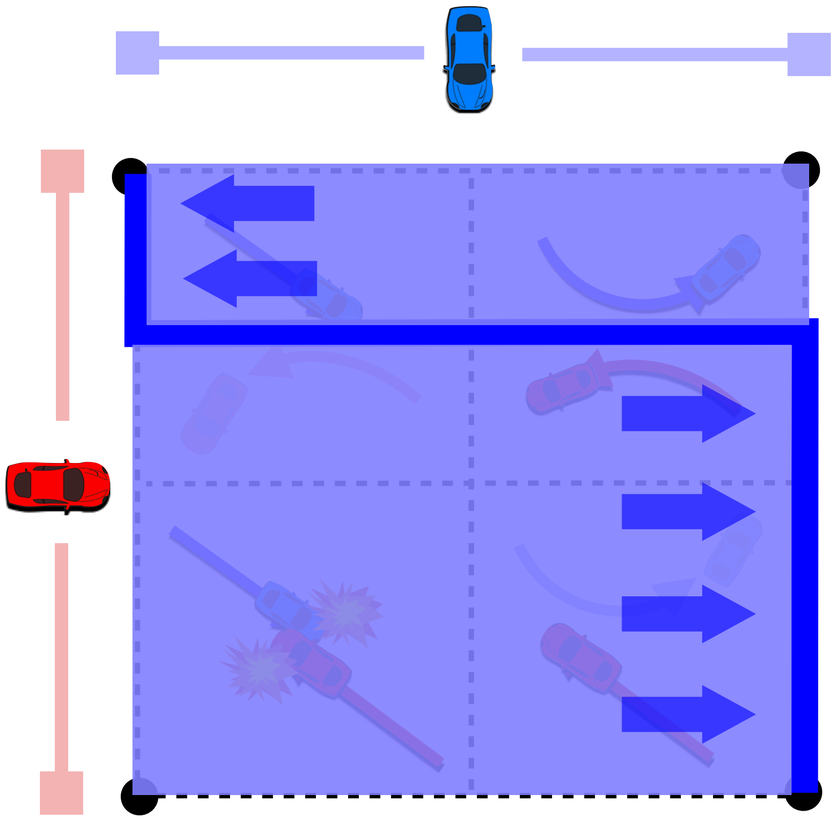


$+1$	0
-3	-1



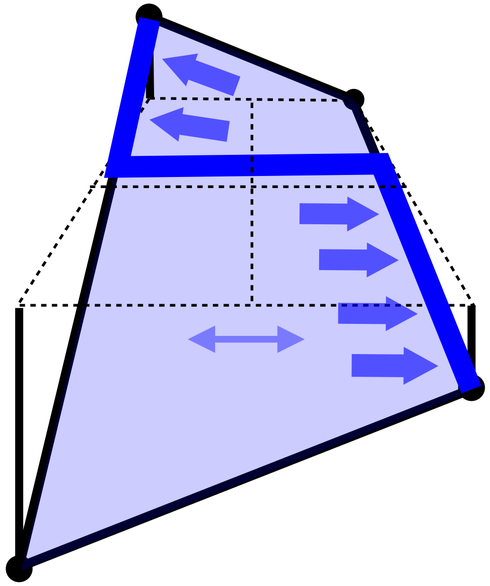
-1	0
-3	$+1$

Best Responses for Blue Player…

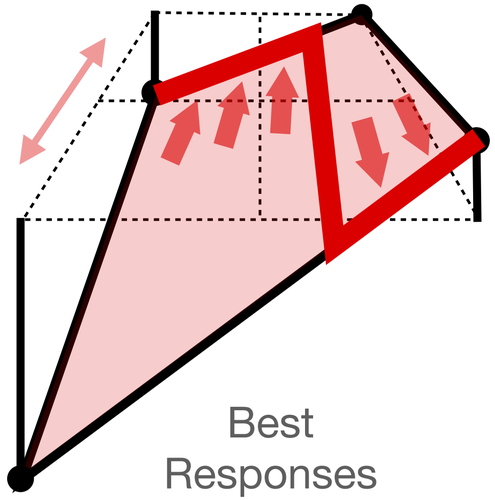


- **Two drivers dare each other to flinch to avoid a head on collision**
- If neither flinches, they crash into each other. reward = (-3,-3)
- If they both flinch, no one wins. reward = (0,0)
- If only one flinches, the other wins the dare. ex. reward = (+1,-1)
- **What will they do?**

Matrix Game: Chicken - Best Responses

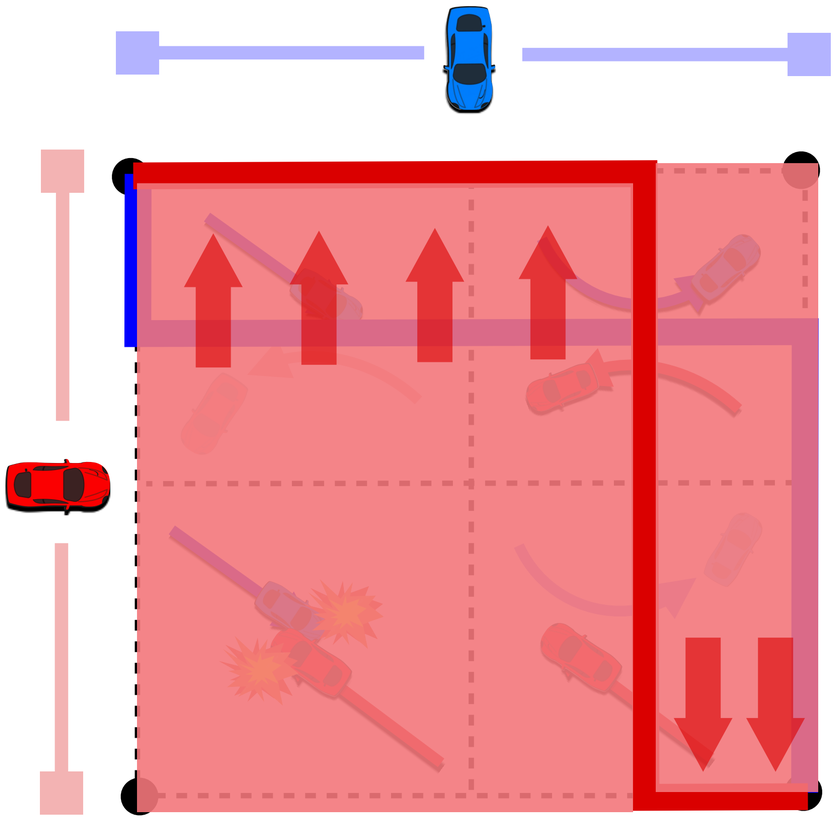


$+1$	0
-3	-1



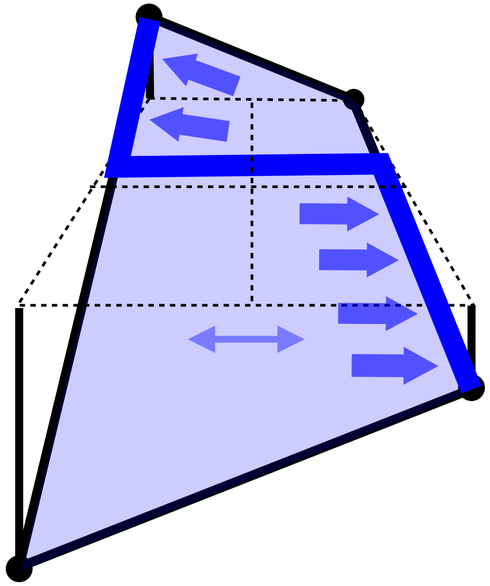
-1	0
-3	$+1$

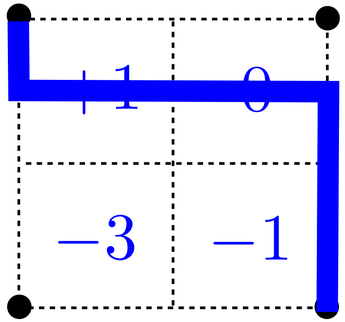
Best Responses for Red Player…

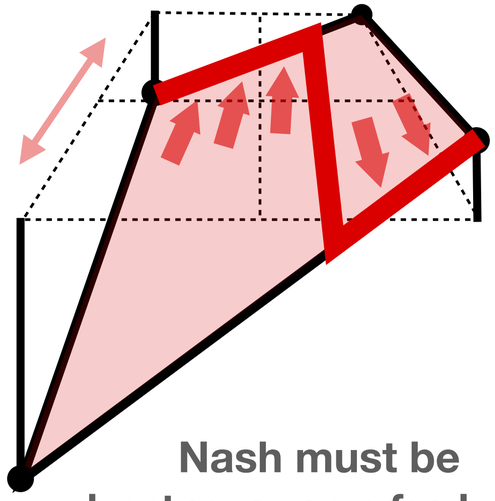


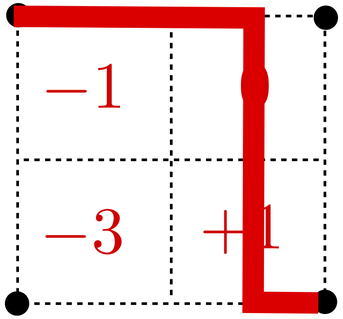
- **Two drivers dare each other to flinch to avoid a head on collision**
- If neither flinches, they crash into each other. reward = (-3,-3)
- If they both flinch, no one wins. reward = (0,0)
- If only one flinches, the other wins the dare. ex. reward = (+1,-1)
- **What will they do?**

Matrix Game: Chicken - SVO Nash

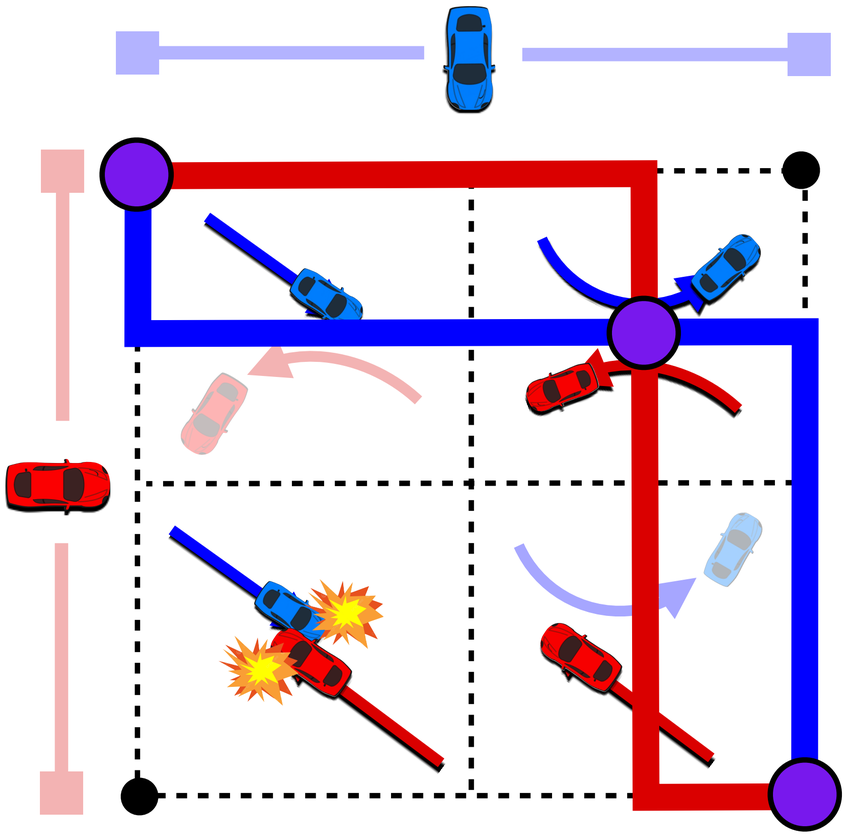


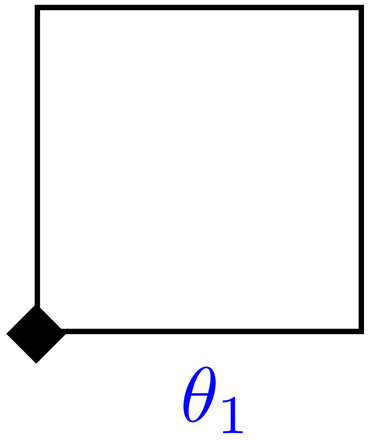






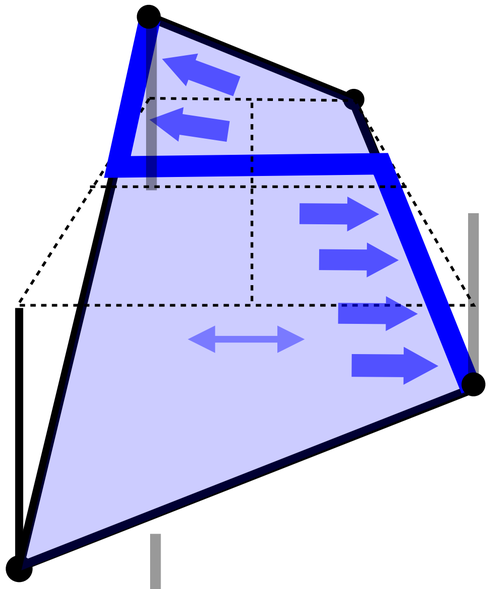
Nash must be
best response for both

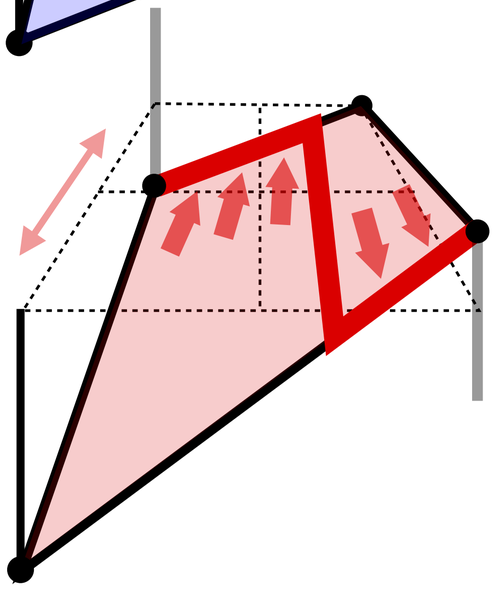


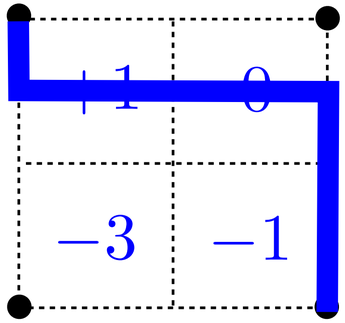


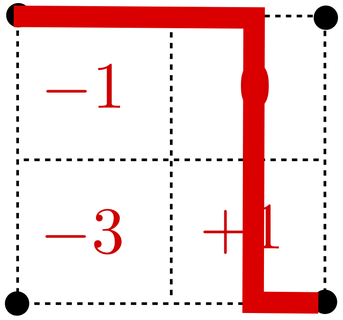
Nash 1 = Red always swerves
Nash 2 = Blue always swerves
Nash 3 = both swerve 75% of the time

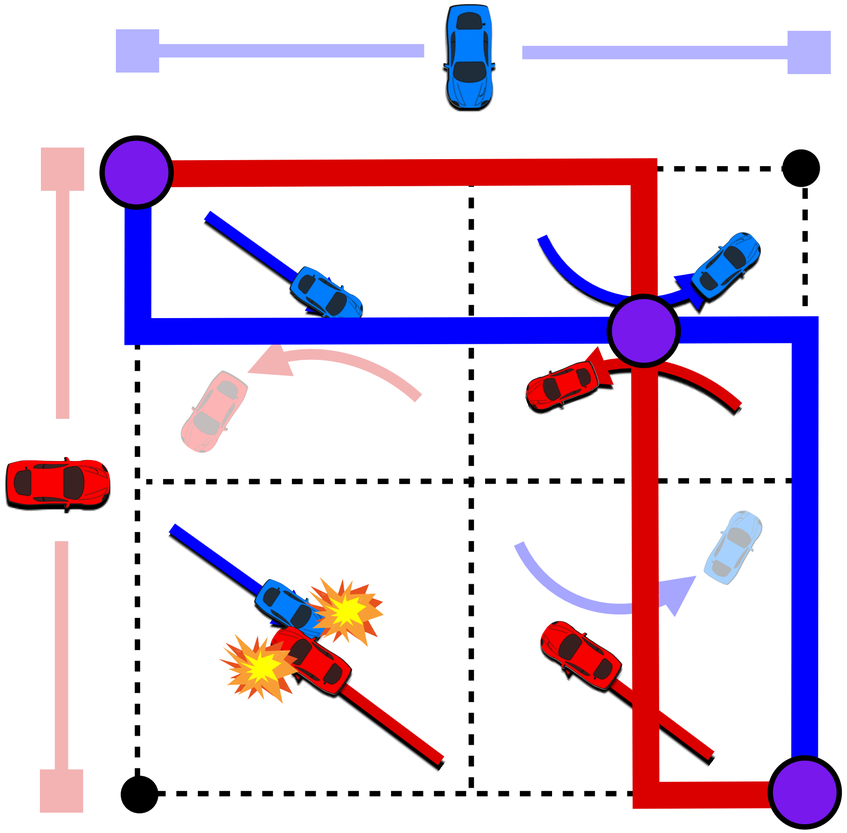
Matrix Game: Chicken - SVO Nash

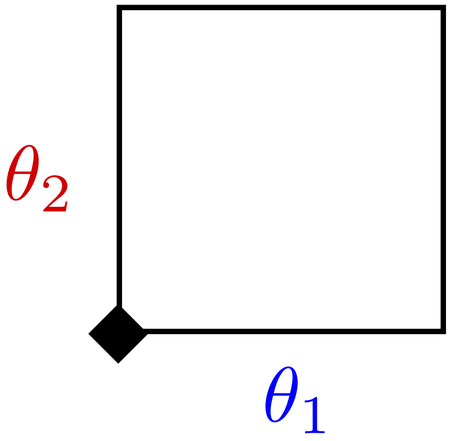




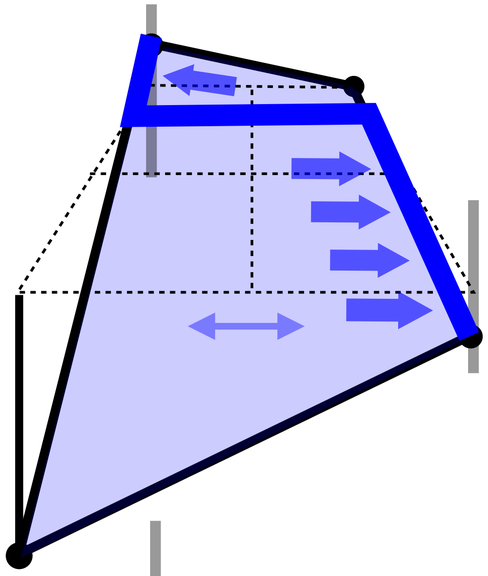


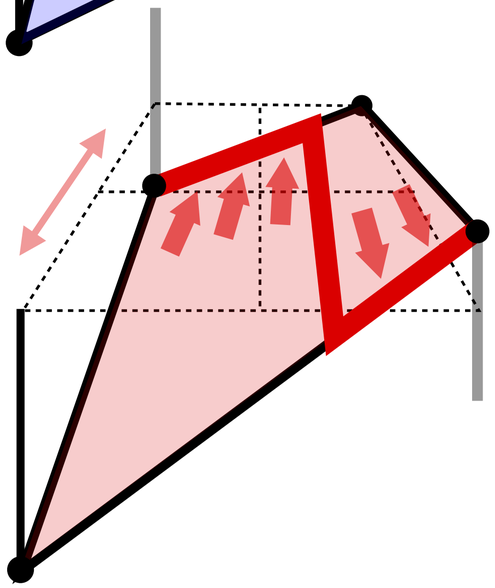


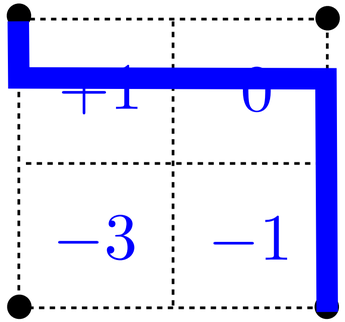


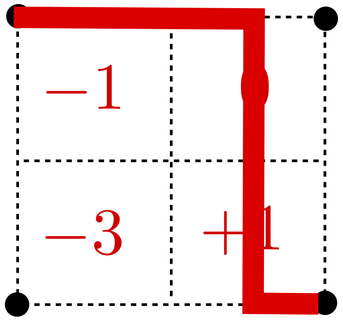


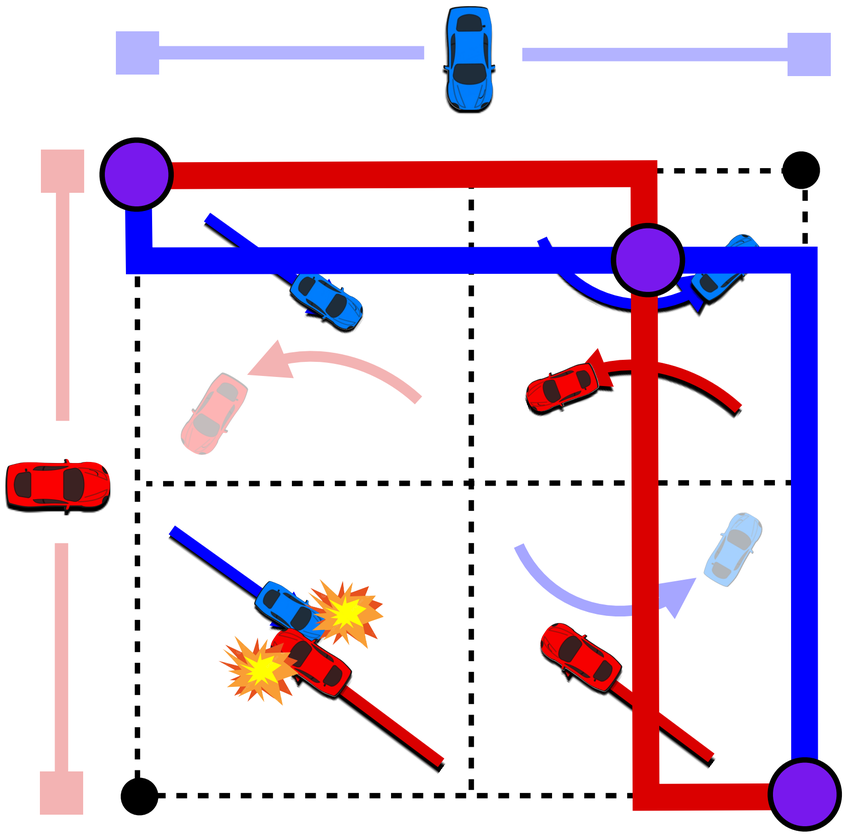
Matrix Game: Chicken - SVO Nash

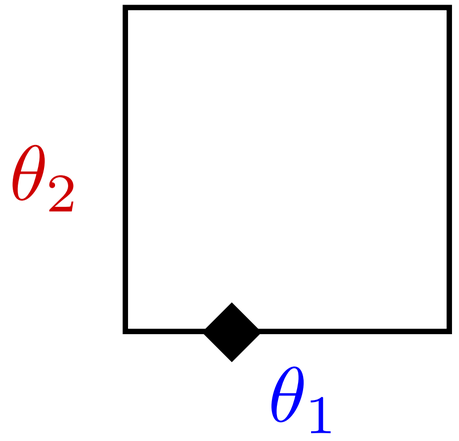




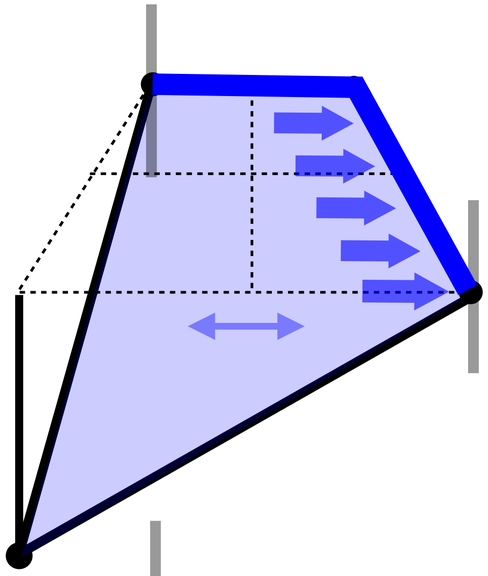


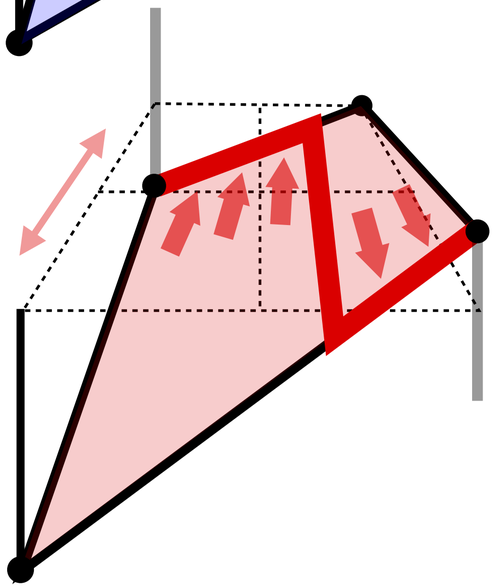


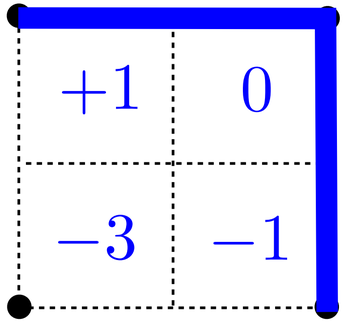


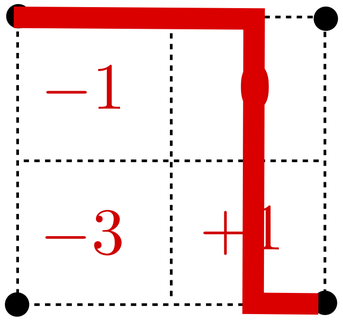


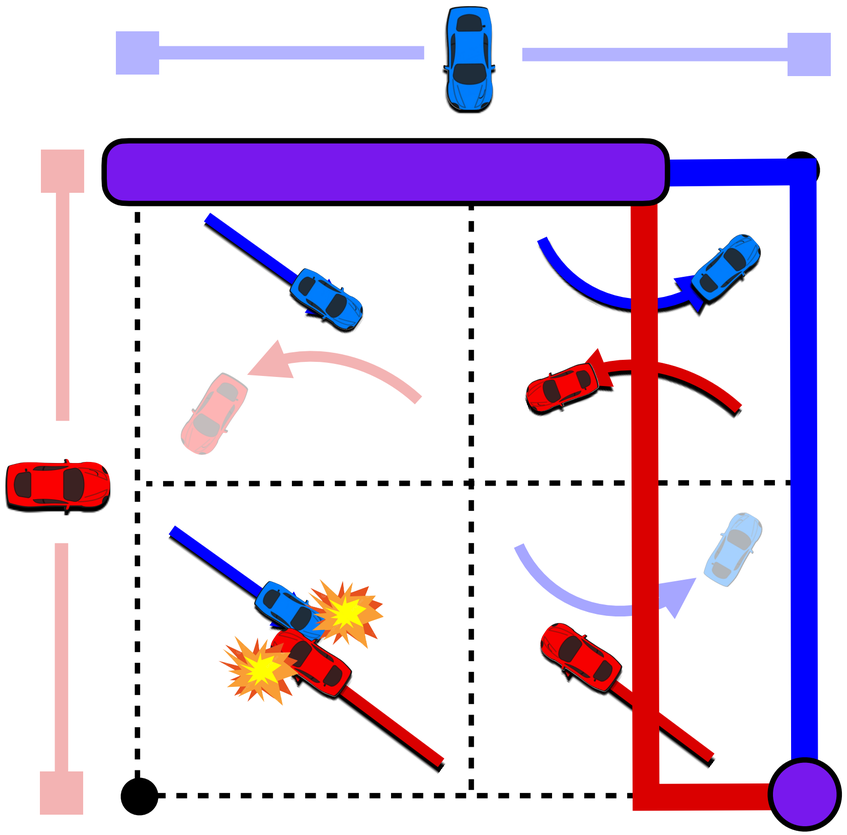
Matrix Game: Chicken - SVO Nash

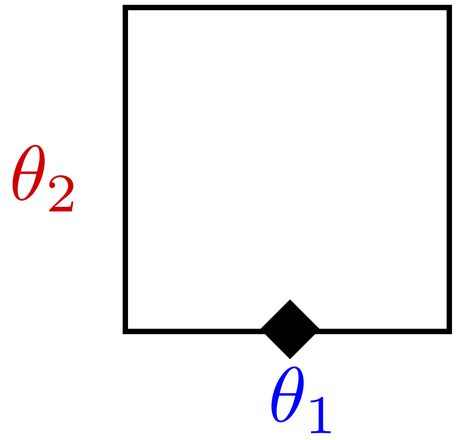




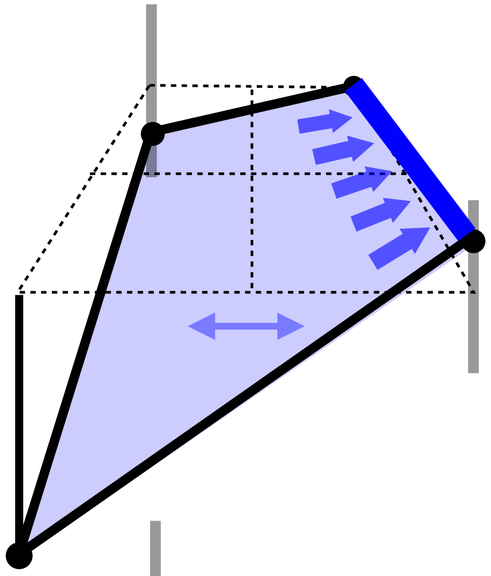


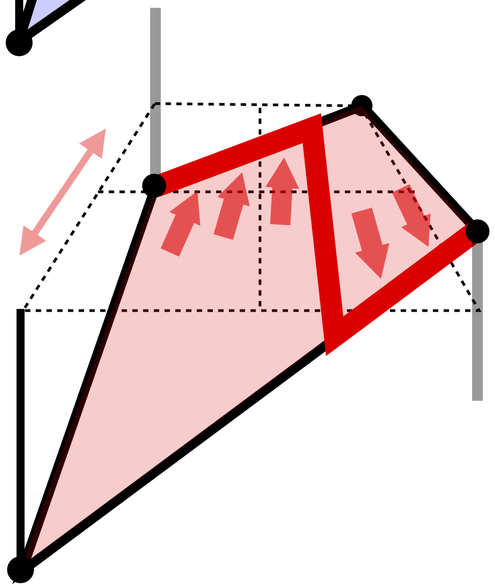






Matrix Game: Chicken - SVO Nash





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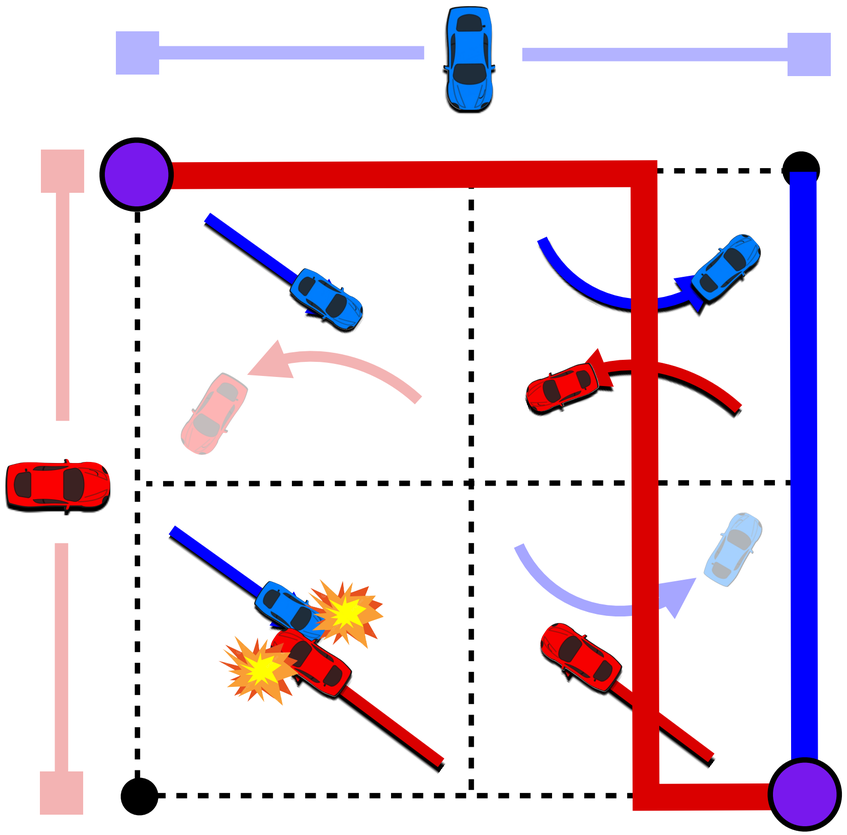
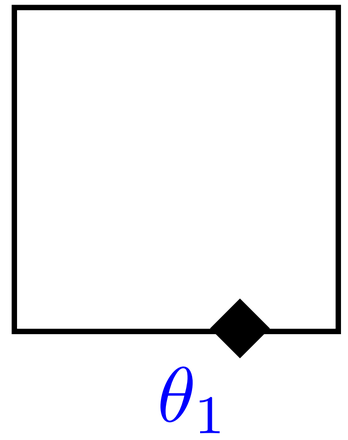
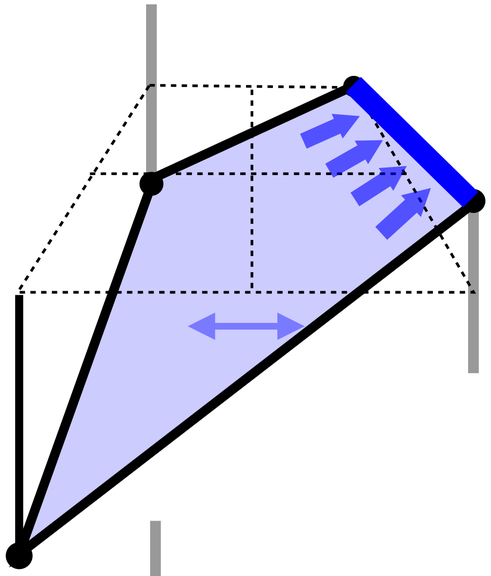
-3	-1

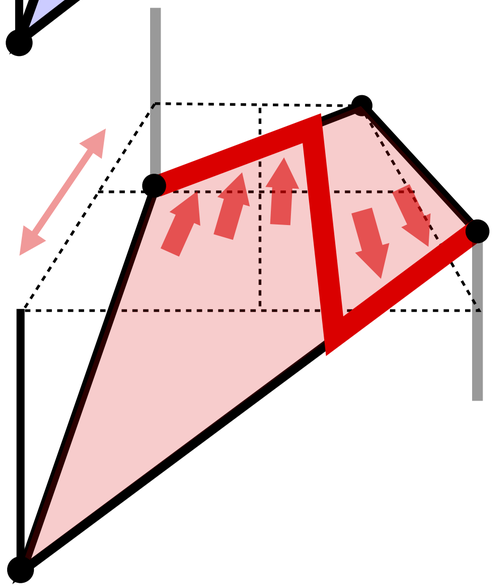
-1	0
-3	$+1$





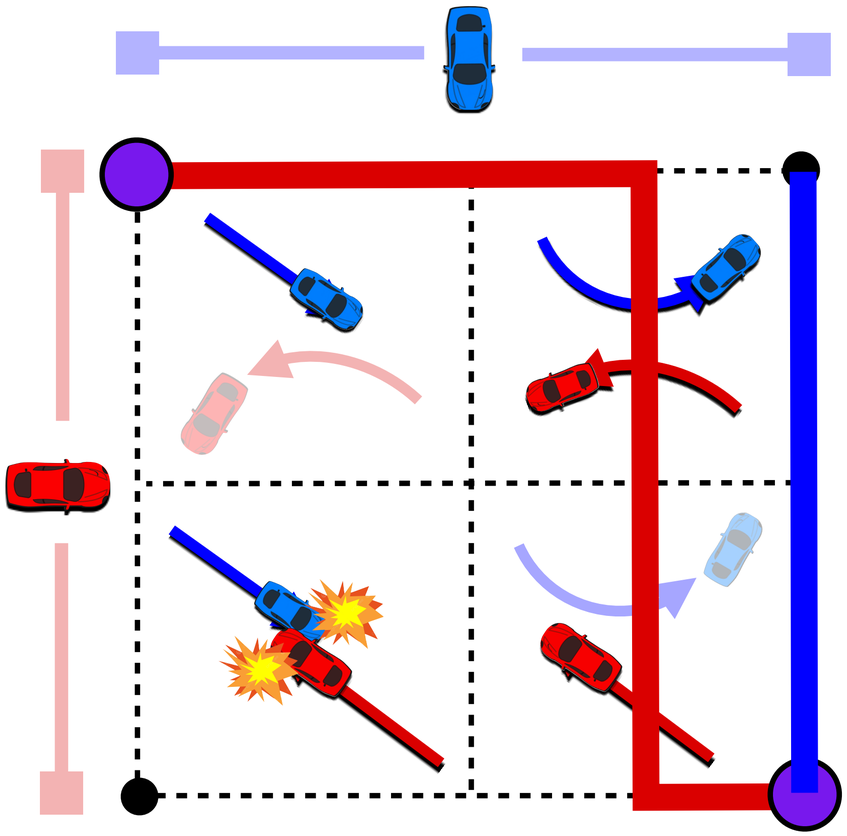
Matrix Game: Chicken - SVO Nash

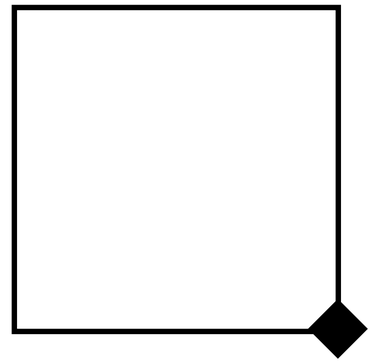




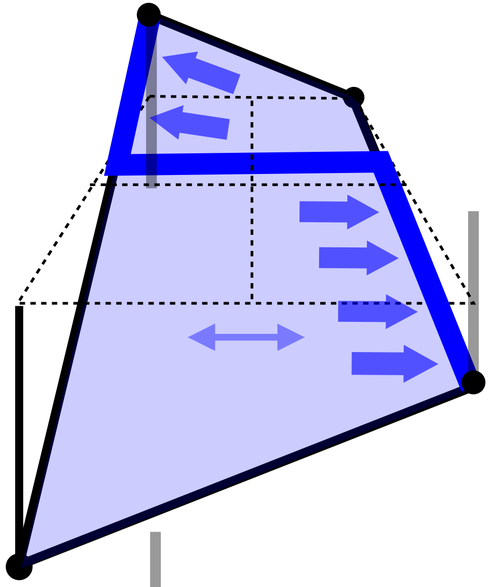
$+1$	0
-3	-1

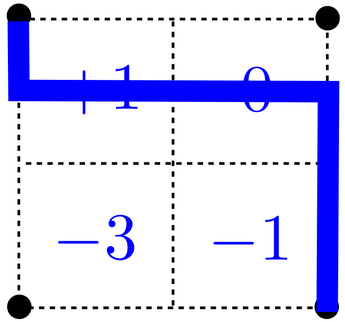
-1	0
-3	$+1$

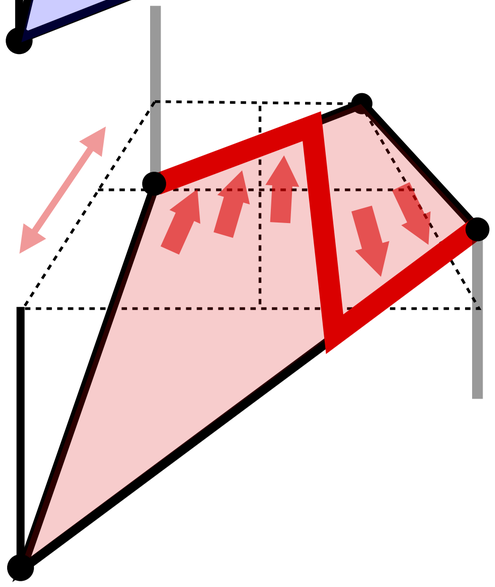


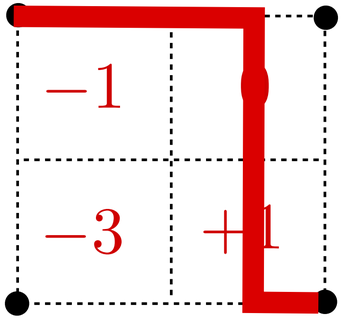


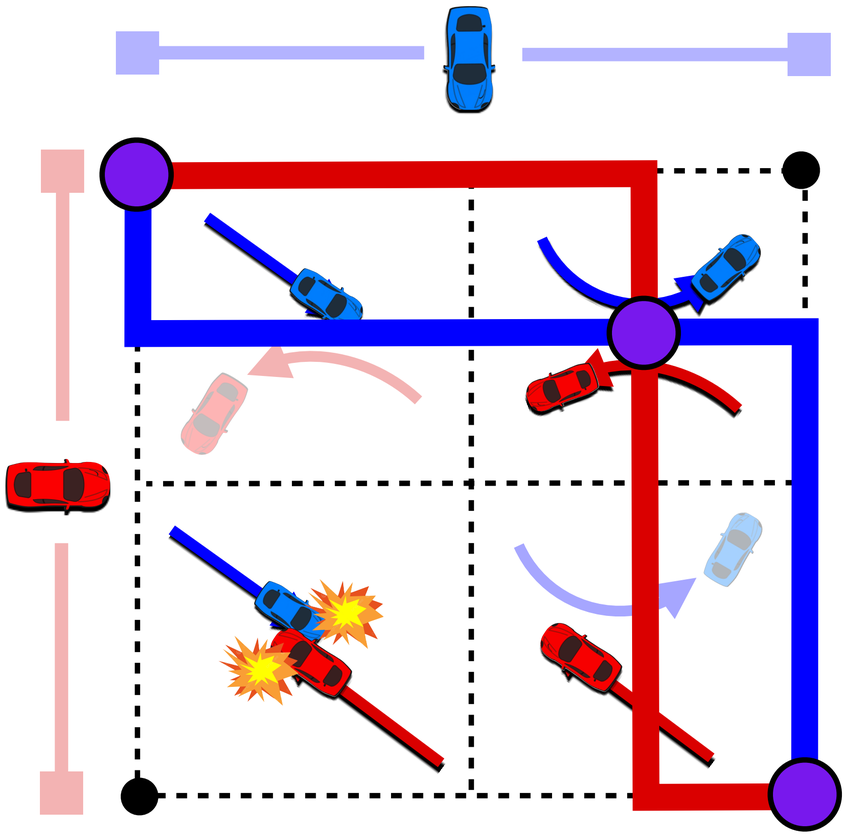
Matrix Game: Chicken - SVO Nash

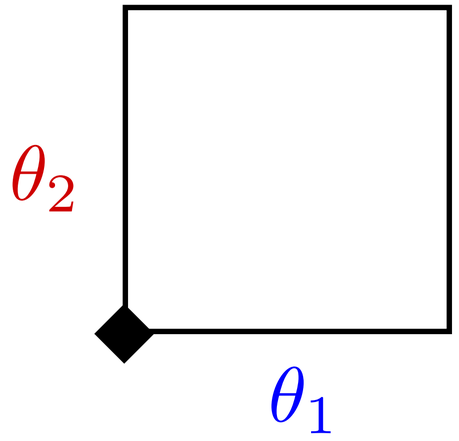




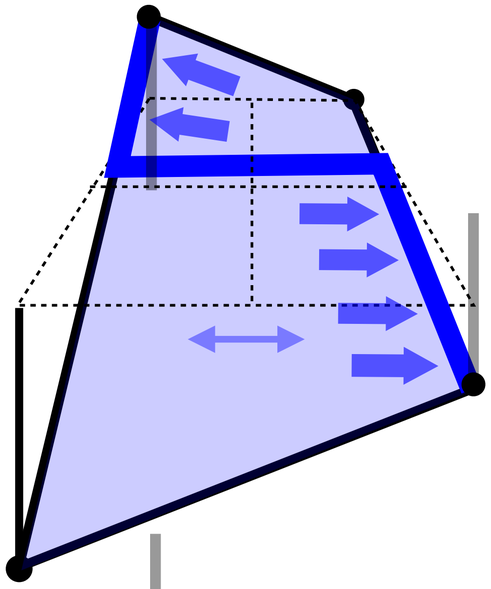


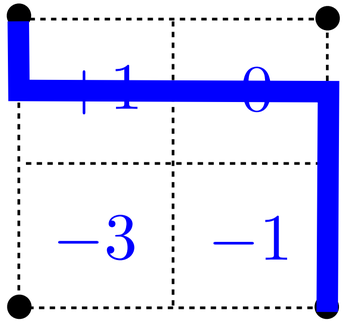


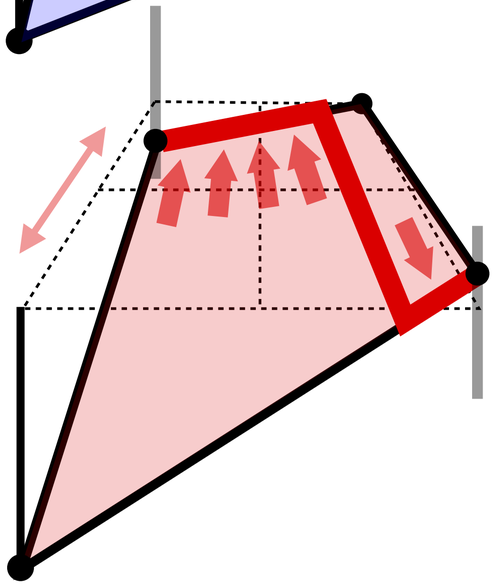


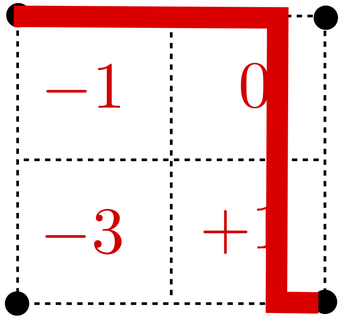


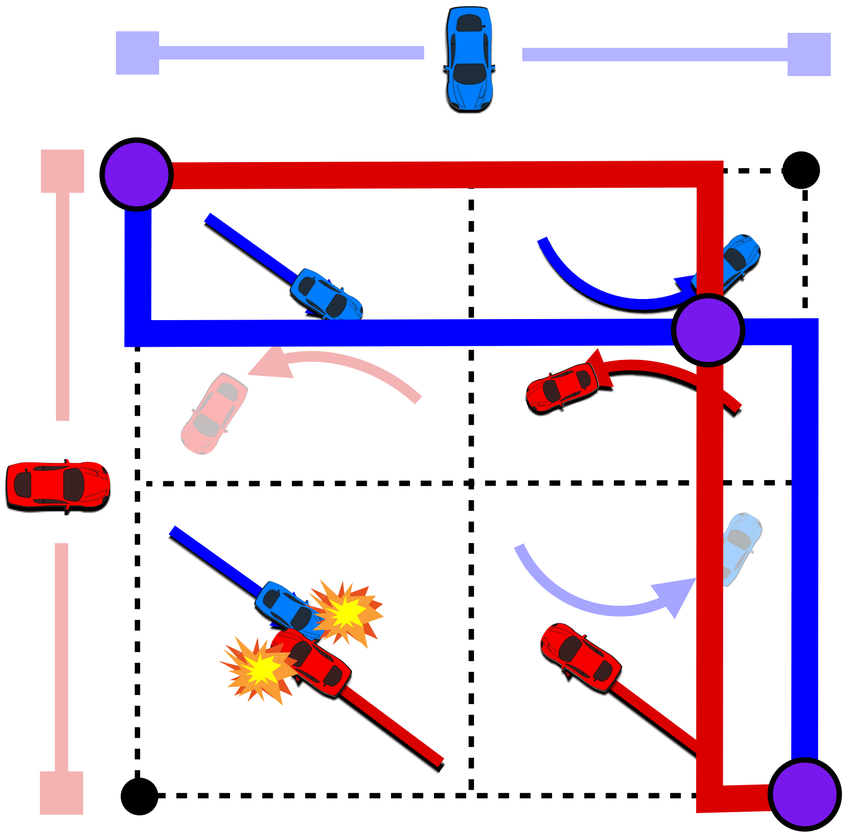
Matrix Game: Chicken - SVO Nash

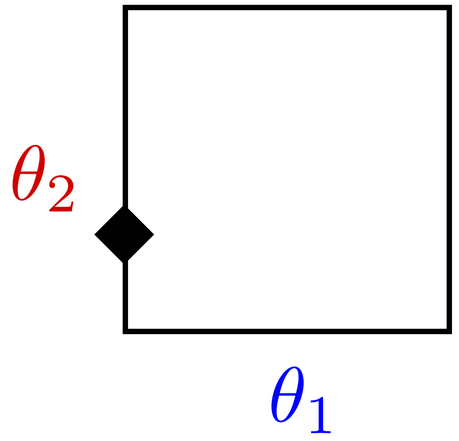




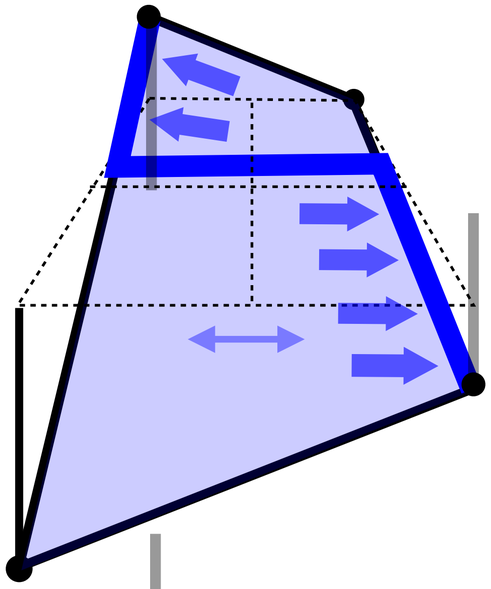


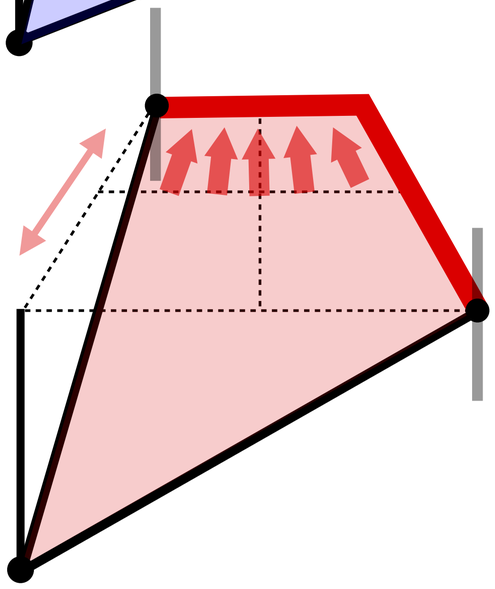






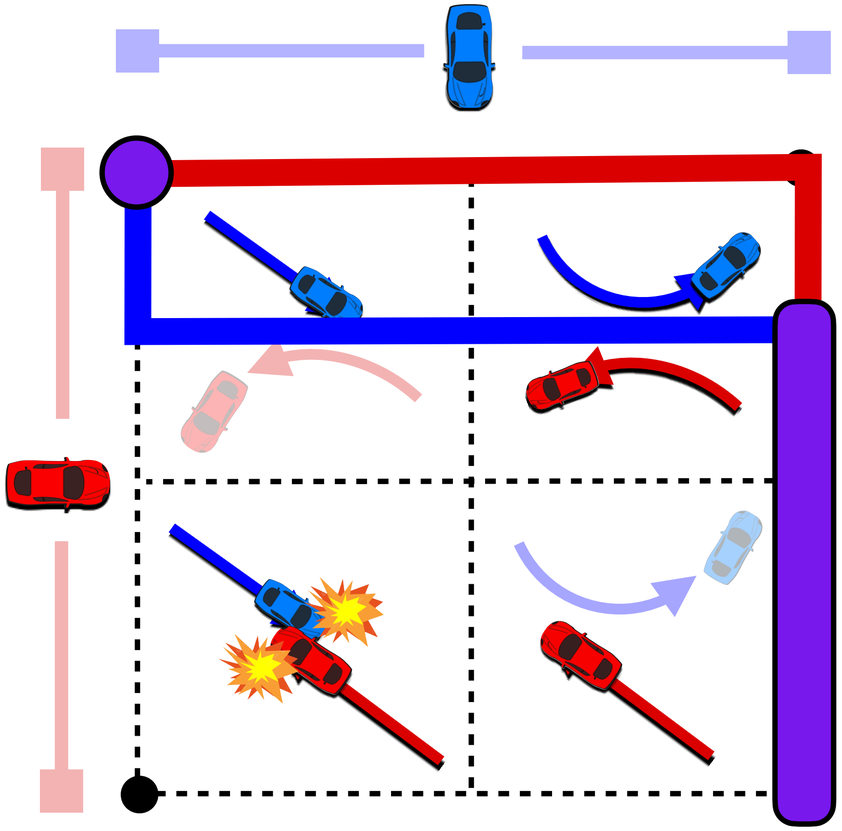
Matrix Game: Chicken - SVO Nash

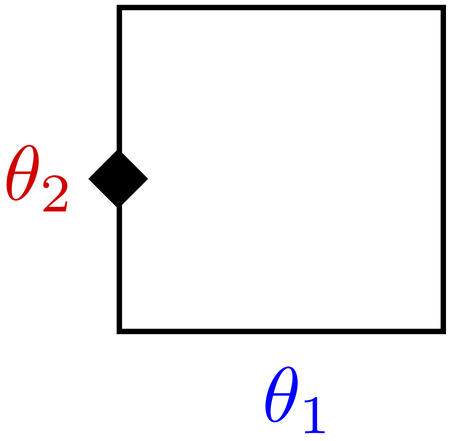




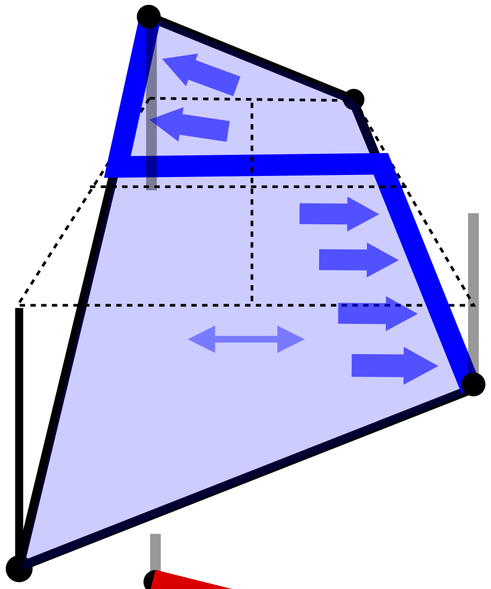
$+1$	0
-3	-1

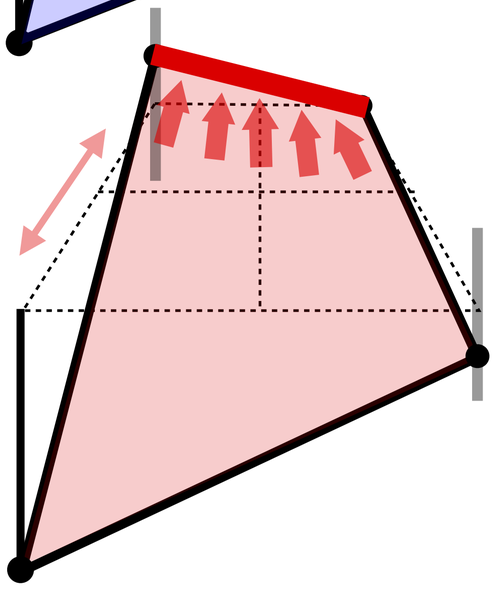
-1	0
-3	$+1$

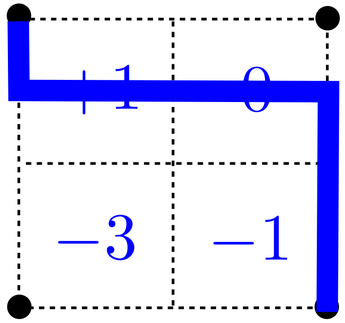


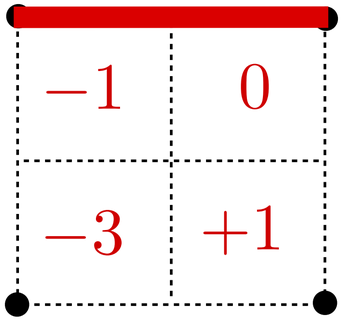


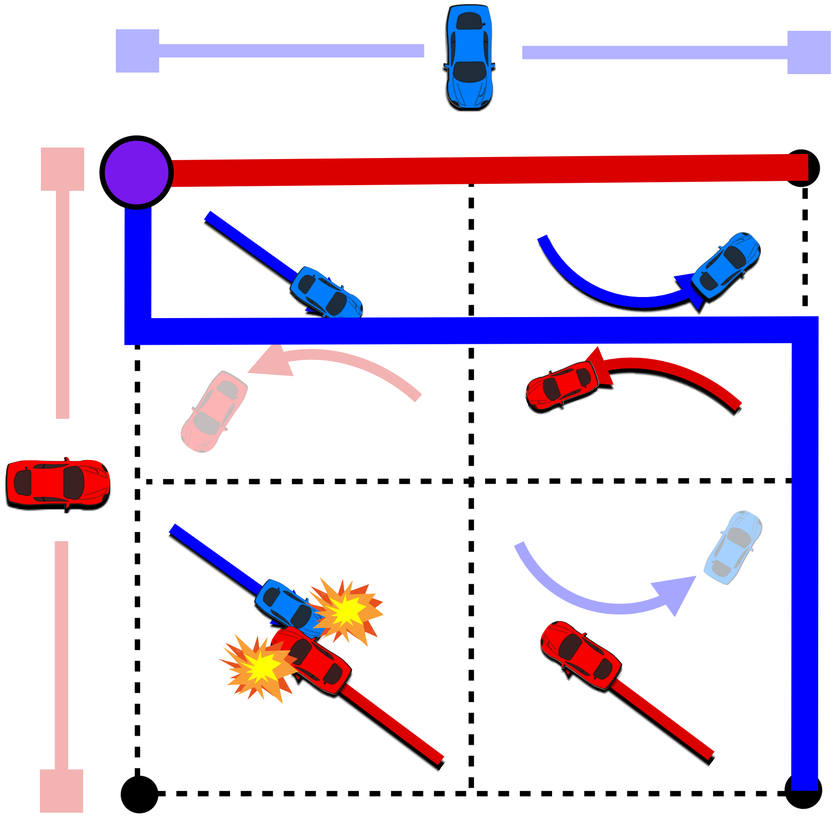
Matrix Game: Chicken - SVO Nash

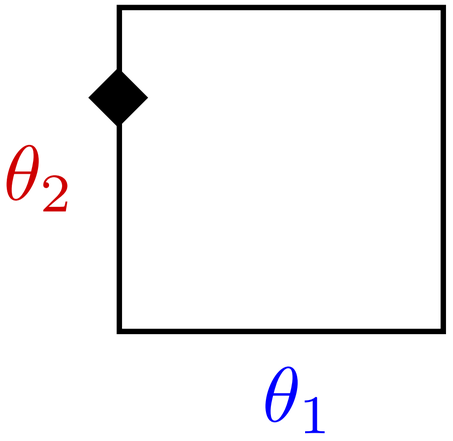




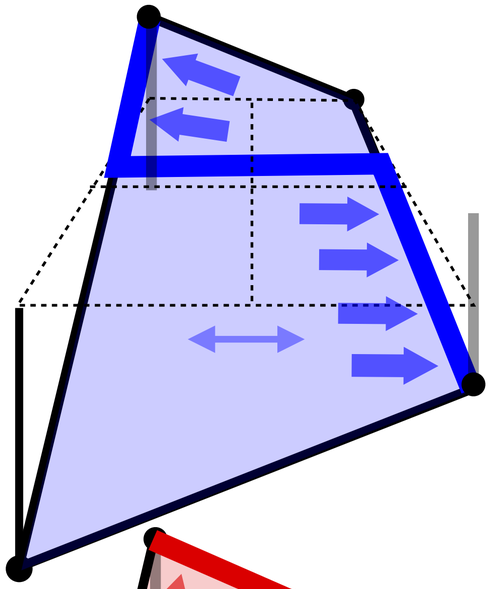


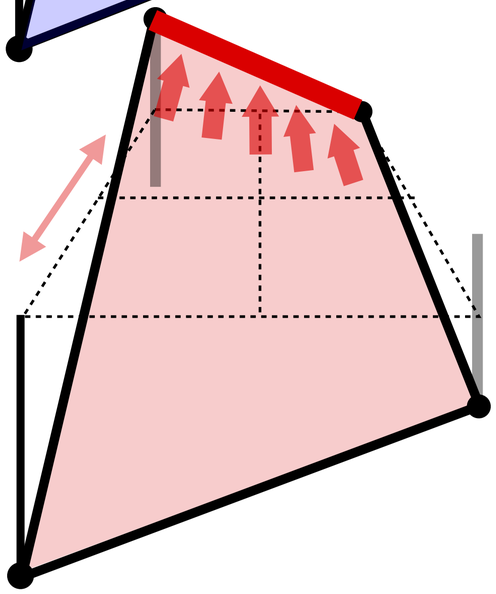


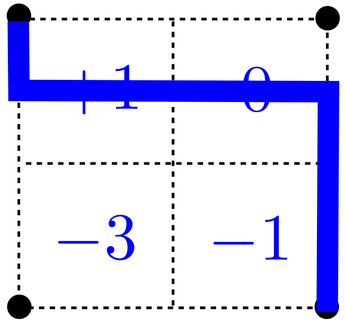


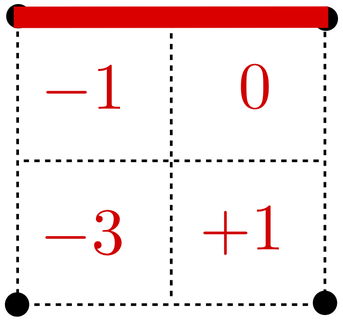


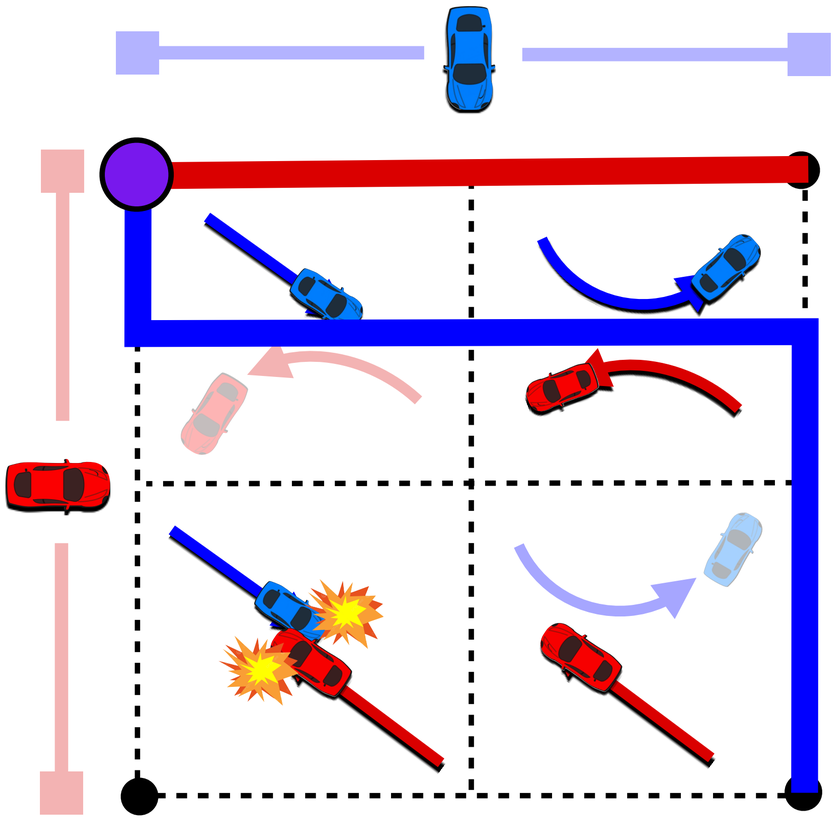
Matrix Game: Chicken - SVO Nash

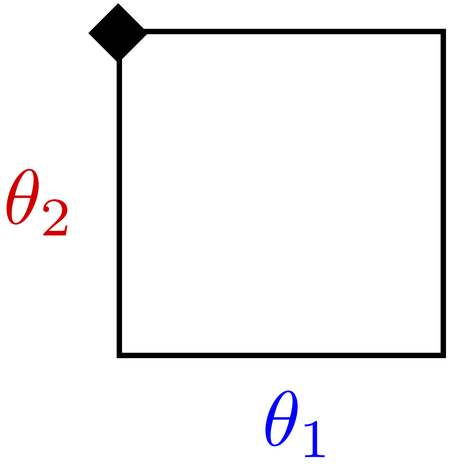




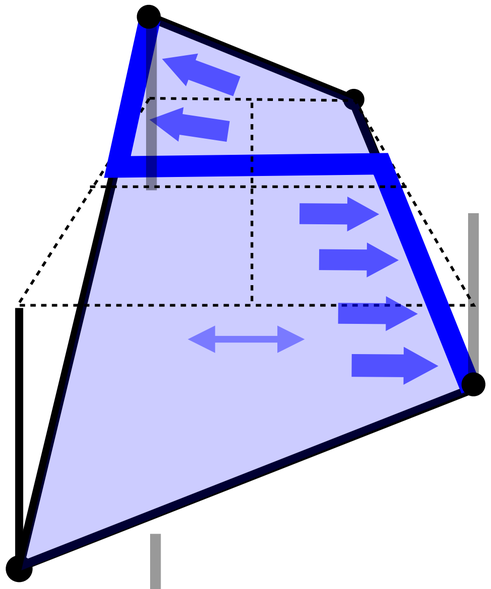


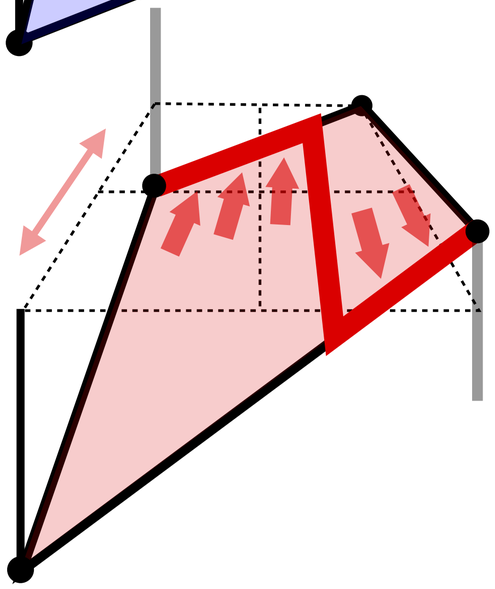


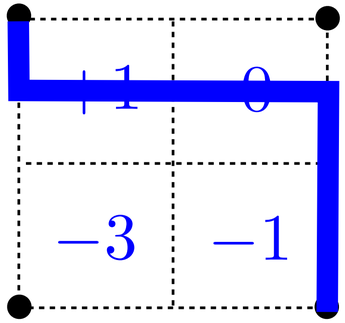


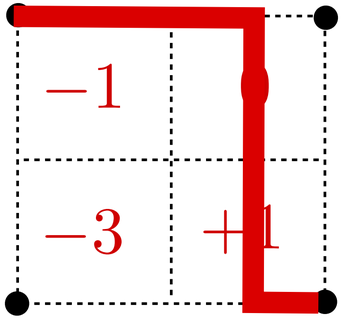


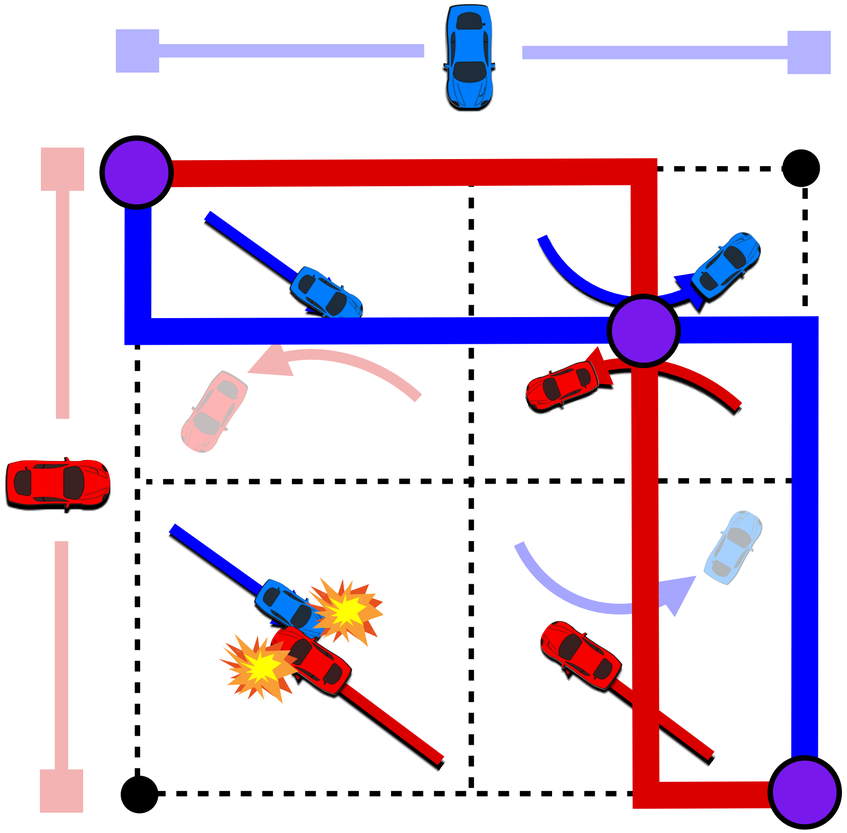
Matrix Game: Chicken - SVO Nash

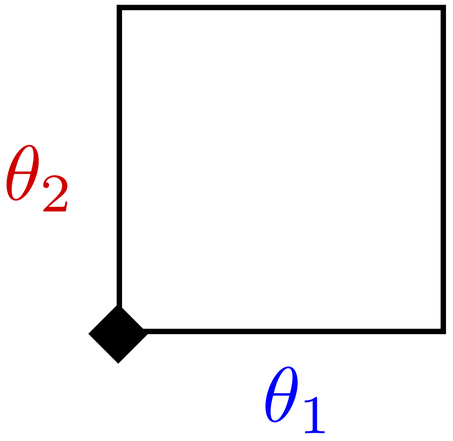




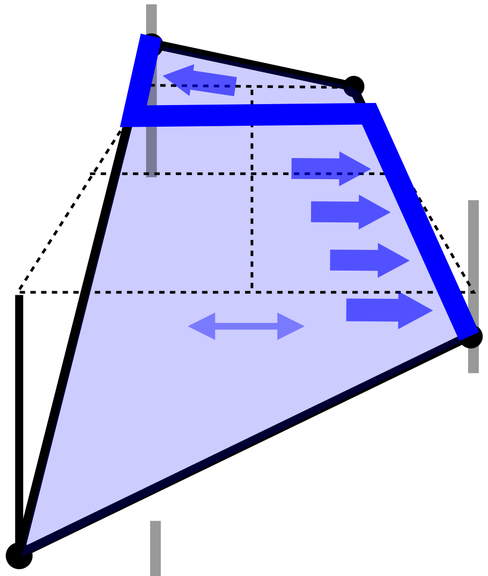


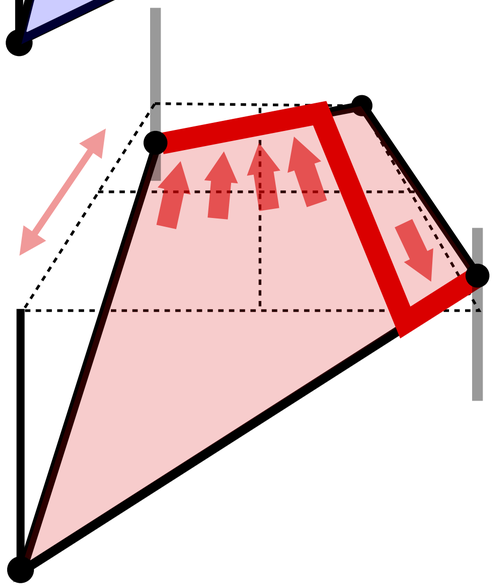


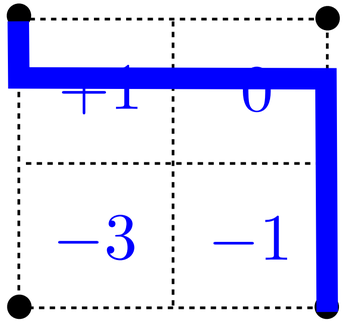


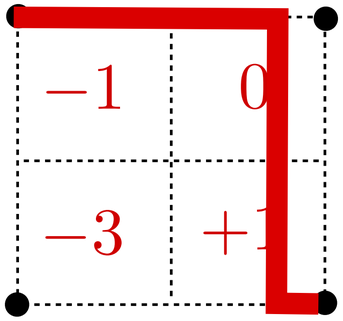


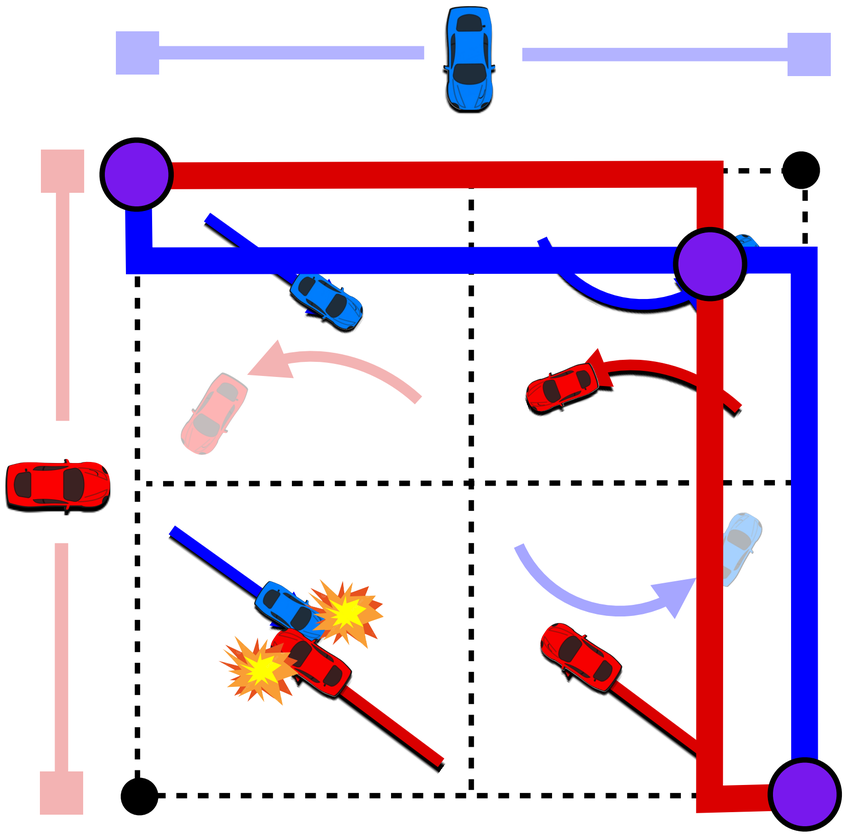
Matrix Game: Chicken - SVO Nash

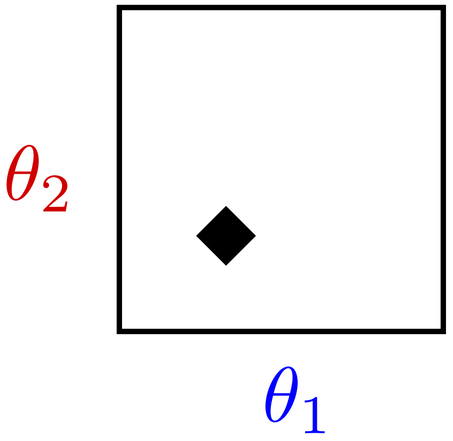




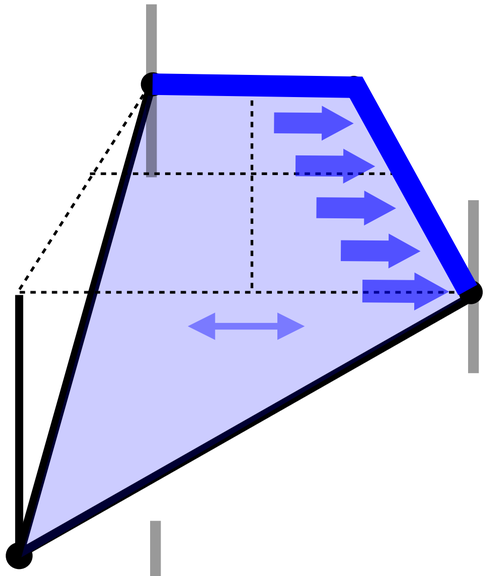




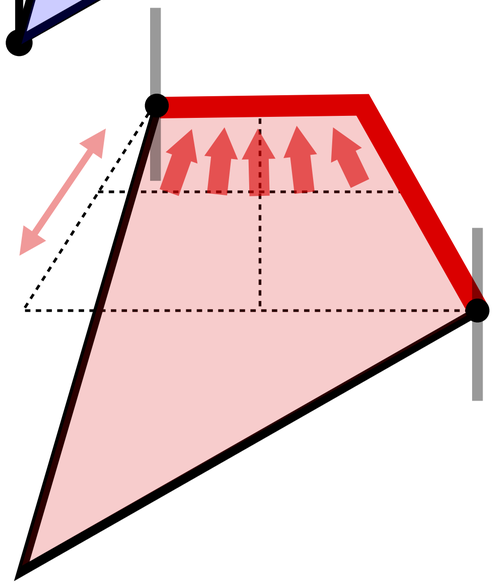




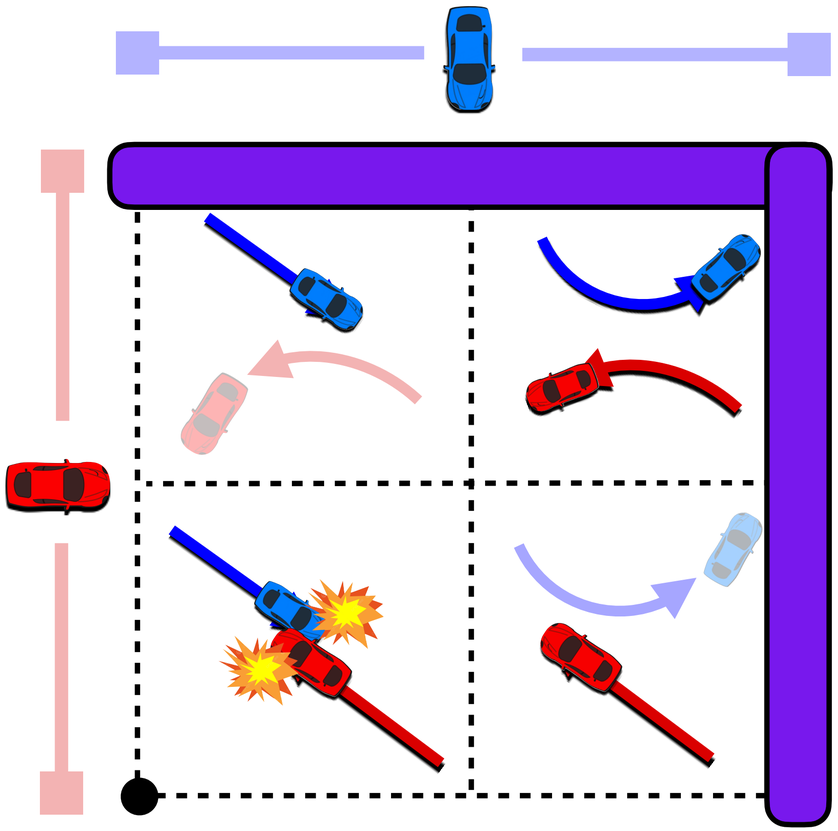
Matrix Game: Chicken - SVO Nash

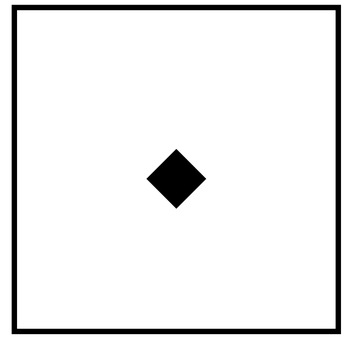


$+1$	0
-3	-1

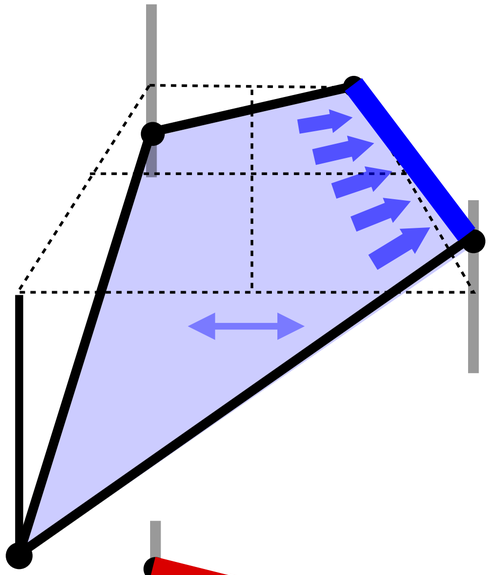


-1	0
-3	$+1$

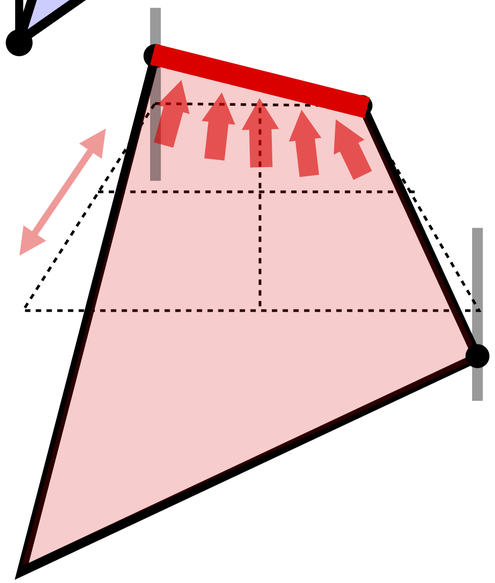




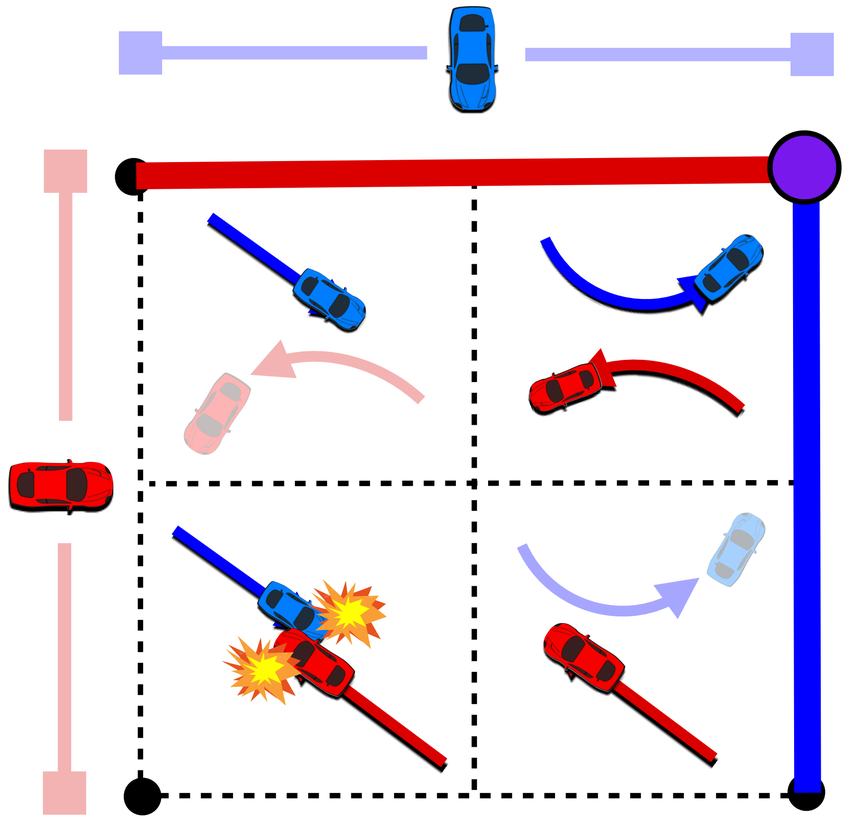
Matrix Game: Chicken - SVO Nash

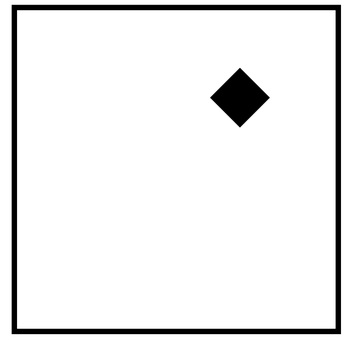


$+1$	0
-3	-1



-1	0
-3	$+1$

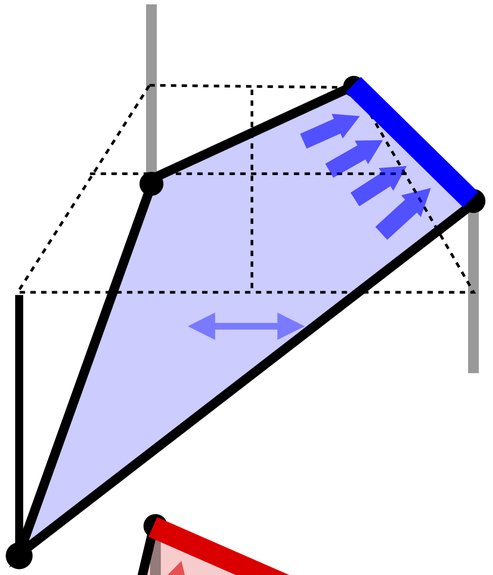




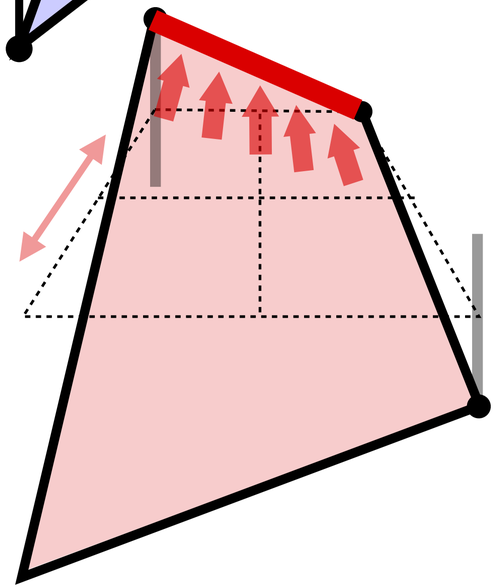
θ_2

θ_1

Matrix Game: Chicken - SVO Nash



$+1$	0
-3	-1



-1	0
-3	$+1$