Column Geometry - Affine Spaces

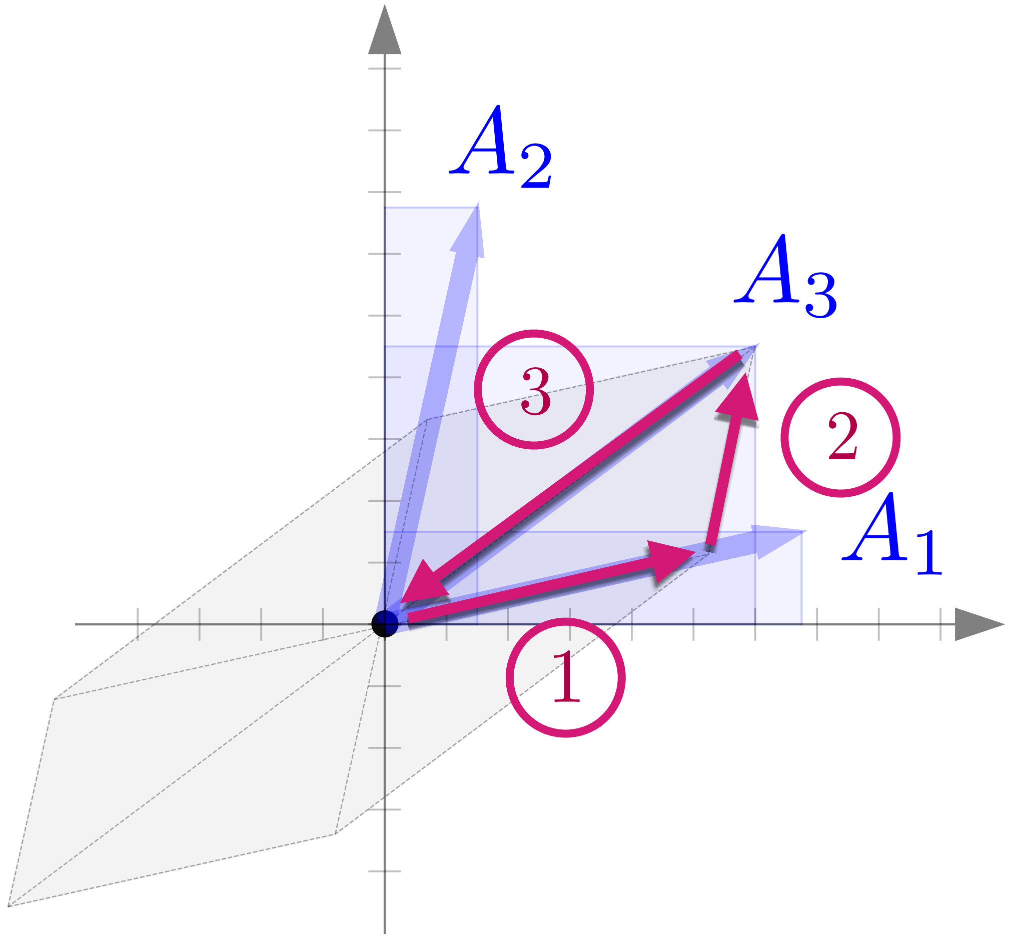
Linear Algebra

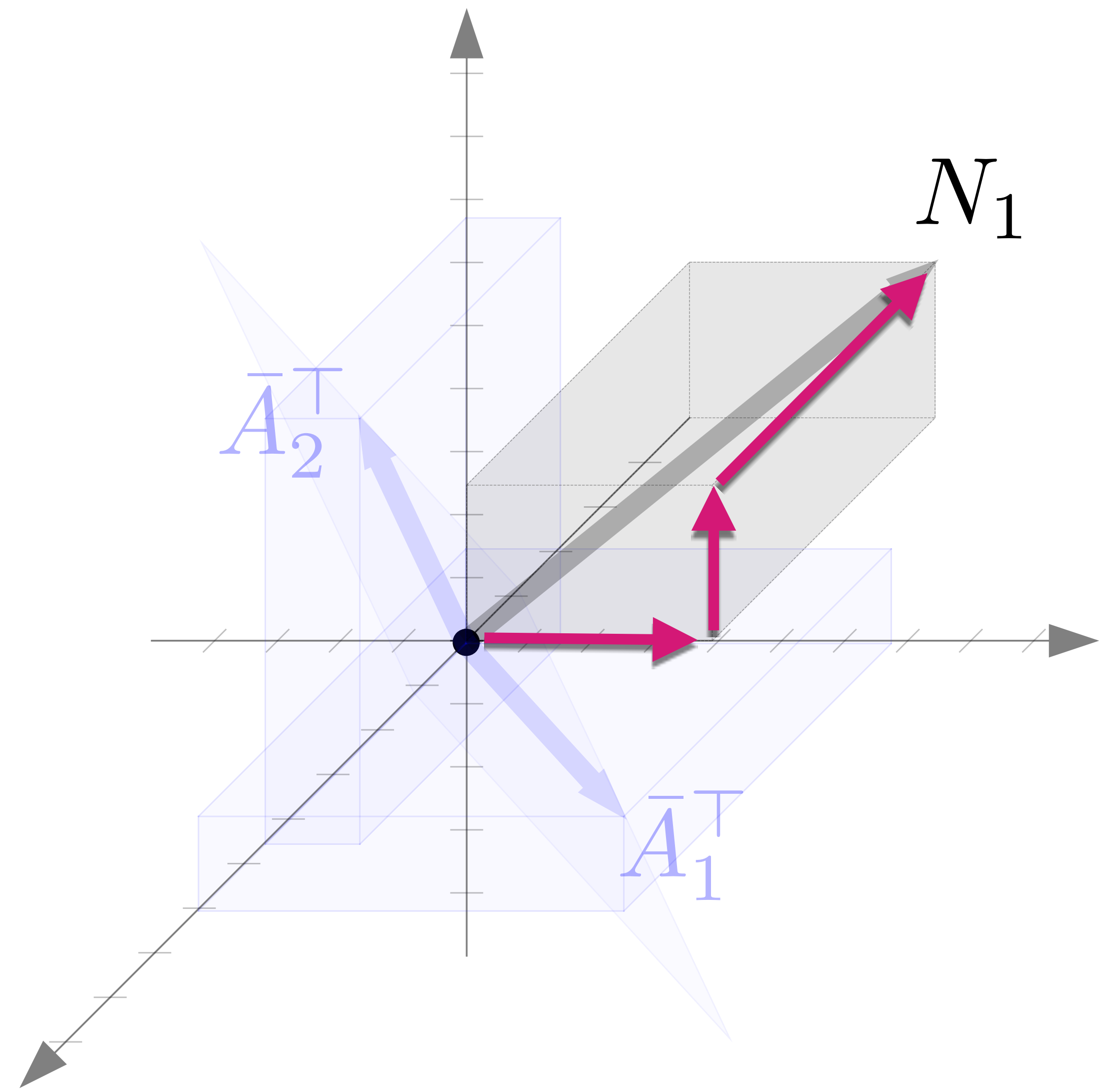
Summer 2023 - Dan Calderone

Nullspace of A

Nullspace of A

"coordinates of 0"

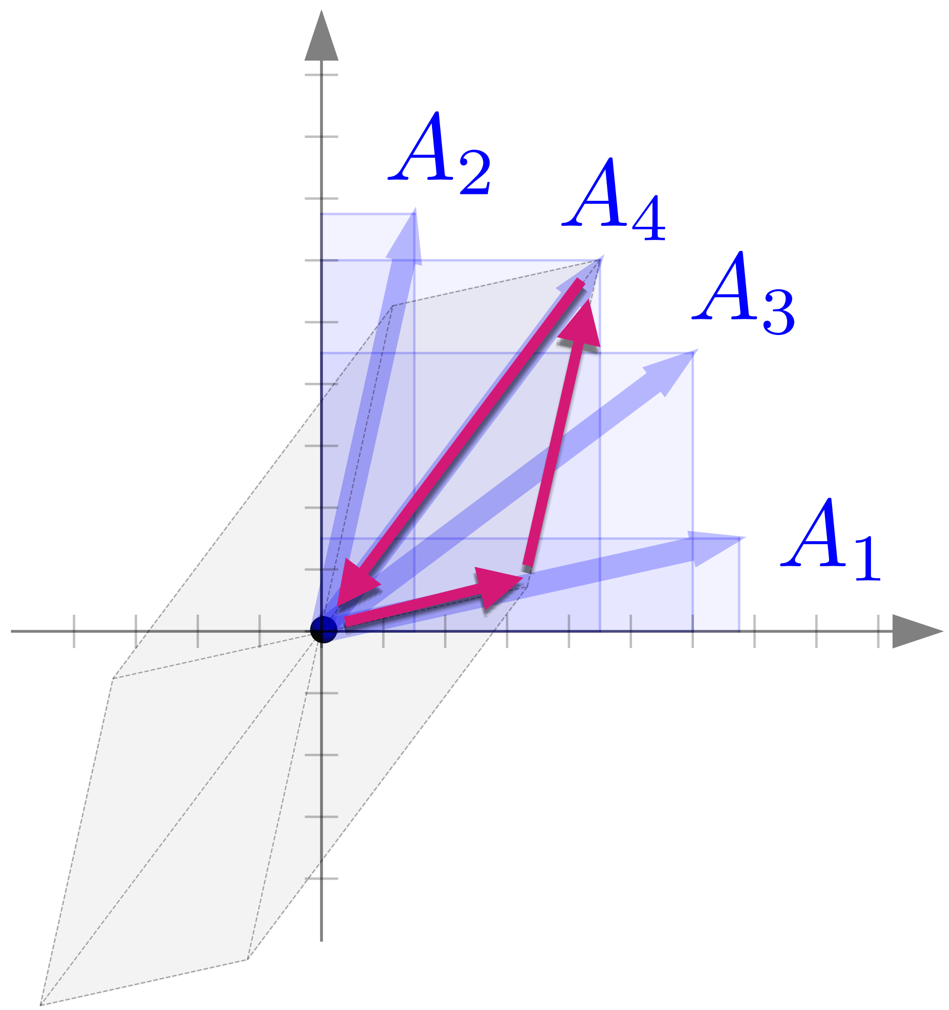


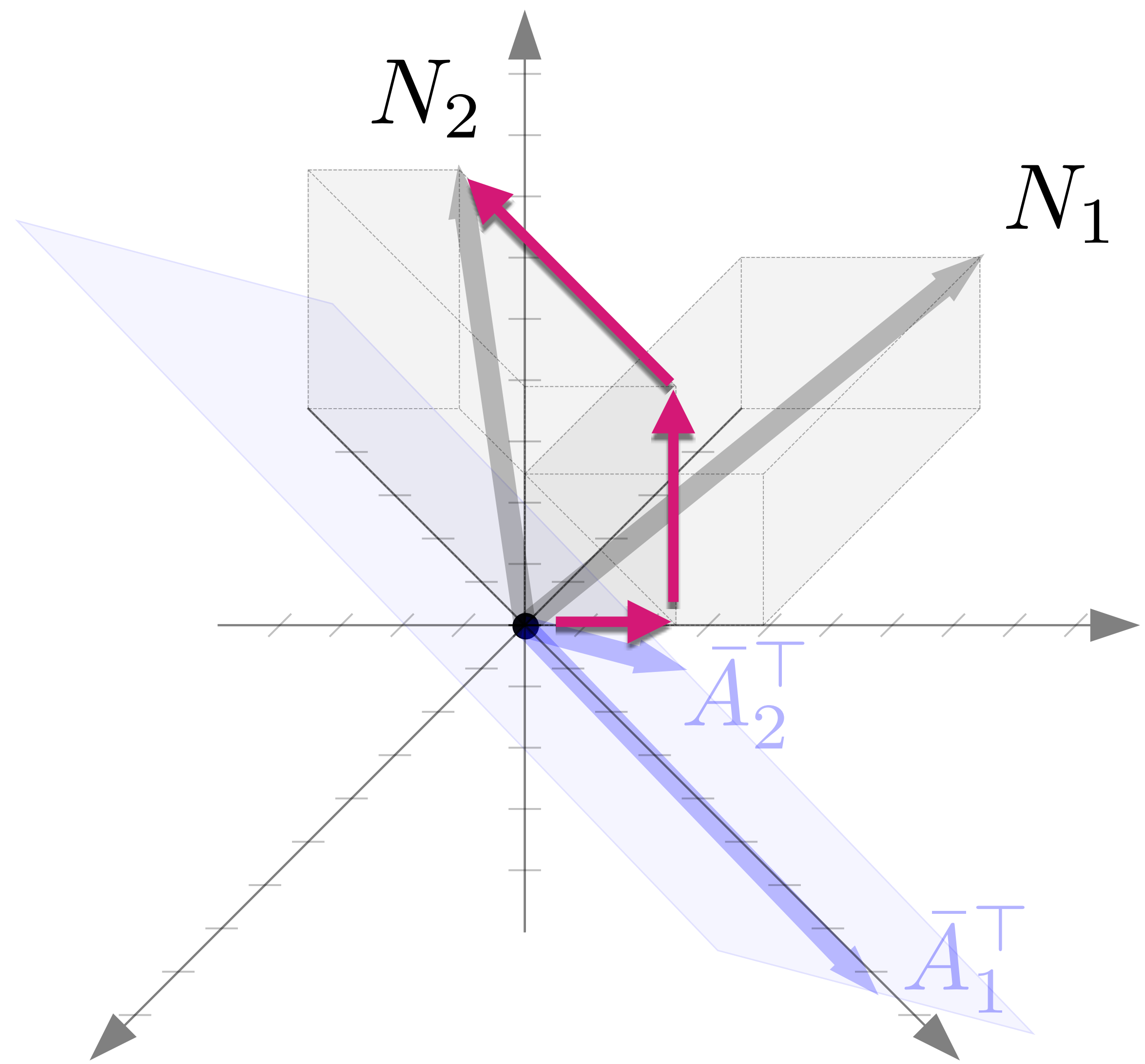


$$\begin{bmatrix} | \\ 0 \\ | \end{bmatrix} = \underbrace{\begin{bmatrix} | & | \\ A_1 & A_2 \\ | & | \end{bmatrix}}_{\text{lin. ind.}} \underbrace{\begin{bmatrix} | \\ A_3 \\ | \end{bmatrix}}_{\text{lin. dep.}} \underbrace{\begin{bmatrix} B_{11} \\ B_{21} \\ -1 \end{bmatrix}}_{N} = \underbrace{\begin{bmatrix} | \\ A_1 \\ | \end{bmatrix} B_{11}}_{①} + \underbrace{\begin{bmatrix} | \\ A_2 \\ | \end{bmatrix} B_{21}}_{②} - \underbrace{\begin{bmatrix} | \\ A_3 \\ | \end{bmatrix}}_{③}$$

Nullspace of A

"coordinates of 0"

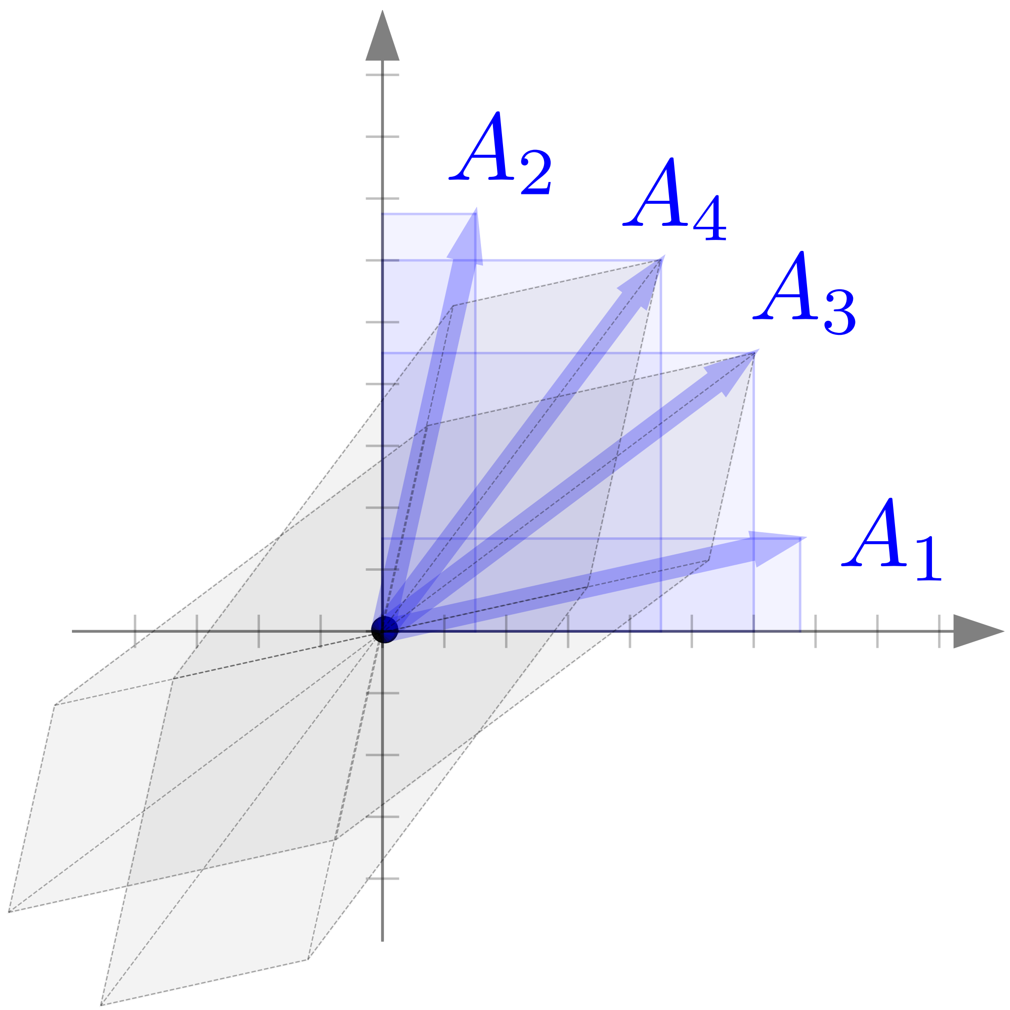


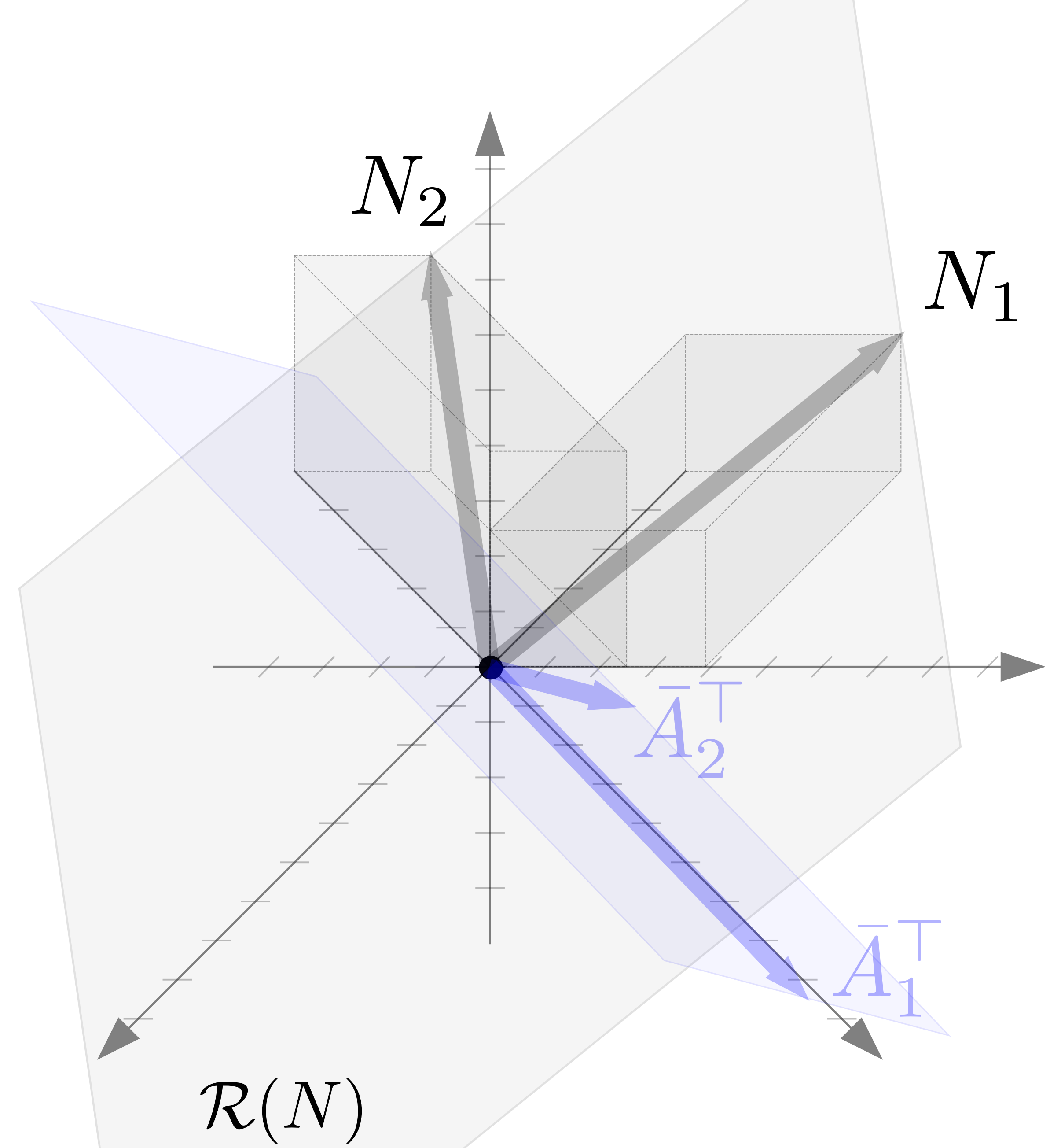


$$\begin{bmatrix} | \\ 0 \\ | \end{bmatrix} = \underbrace{\begin{bmatrix} | & | \\ A_1 & A_2 \\ | & | \end{bmatrix}}_{\text{lin. ind.}} \underbrace{\begin{bmatrix} | & | \\ A_3 & A_4 \\ | & | \end{bmatrix}}_{\text{lin. dep.}} \underbrace{\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ -1 & 0 \\ 0 & -1 \end{bmatrix}}_{N} = \begin{bmatrix} | \\ A_1 \\ | \end{bmatrix} B_{12} + \begin{bmatrix} | \\ A_2 \\ | \end{bmatrix} B_{22} - \begin{bmatrix} | \\ A_4 \\ | \end{bmatrix}$$

Nullspace of A

"coordinates of 0"





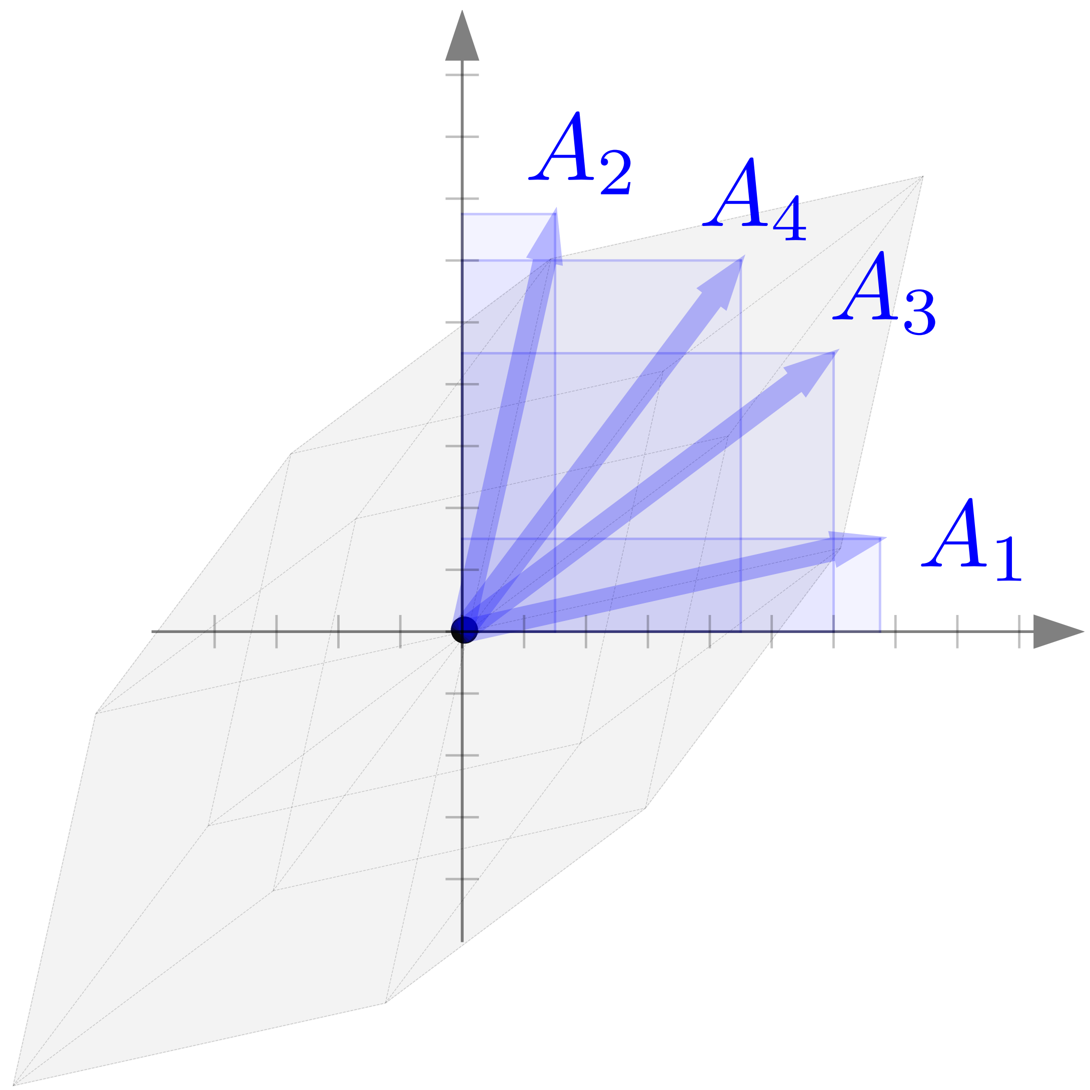
$$\begin{bmatrix} | \\ 0 \\ | \end{bmatrix} = \begin{bmatrix} | & | & | & | \\ A_1 & A_2 & A_3 & A_4 \\ | & | & | & | \end{bmatrix} \underbrace{\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ -1 & 0 \\ 0 & -1 \end{bmatrix}}_{N}$$

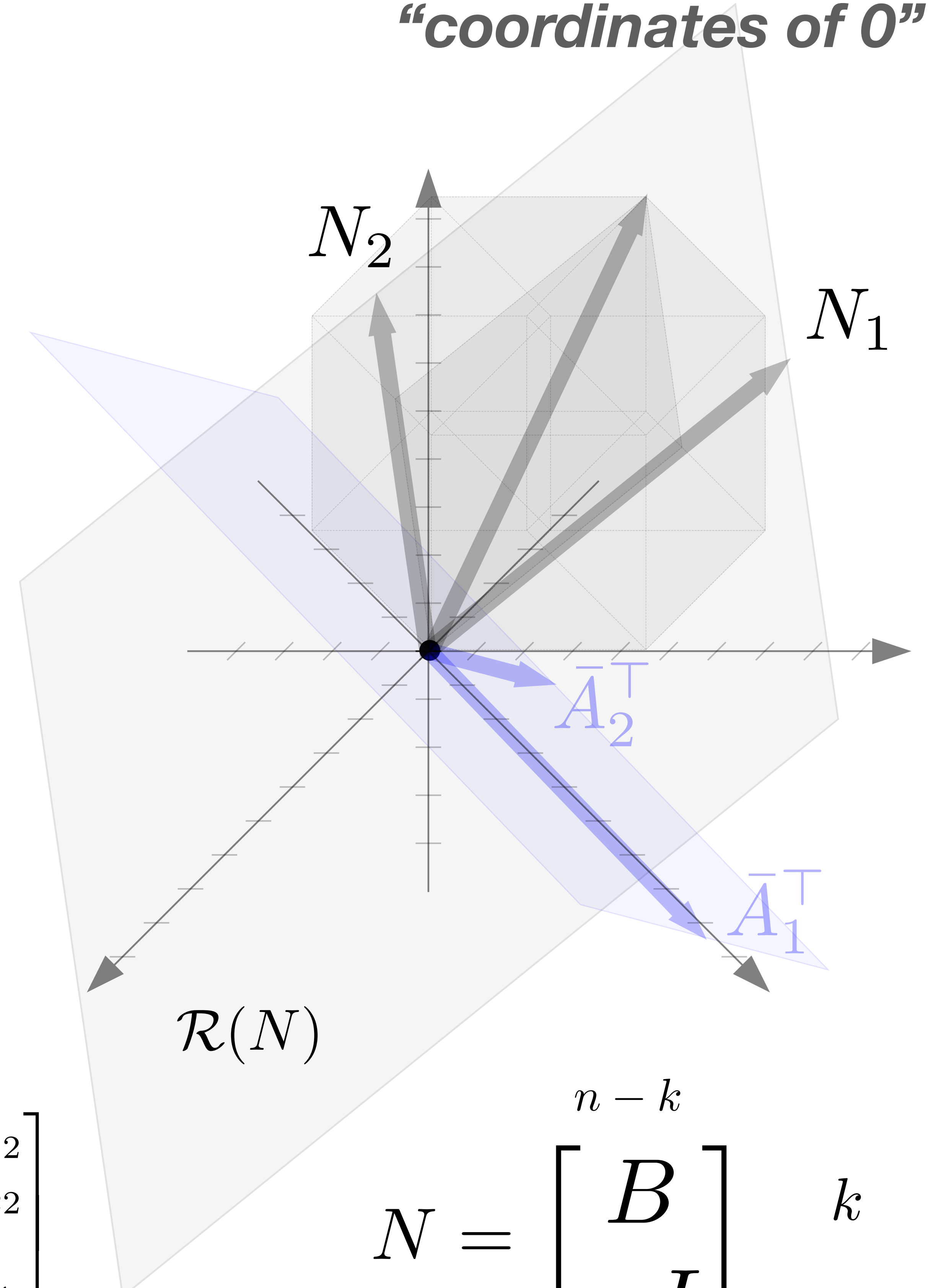
A_1, A_2: lin. ind. A_3, A_4: lin. dep.

$$N = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Nullspace of A

"coordinates of 0"



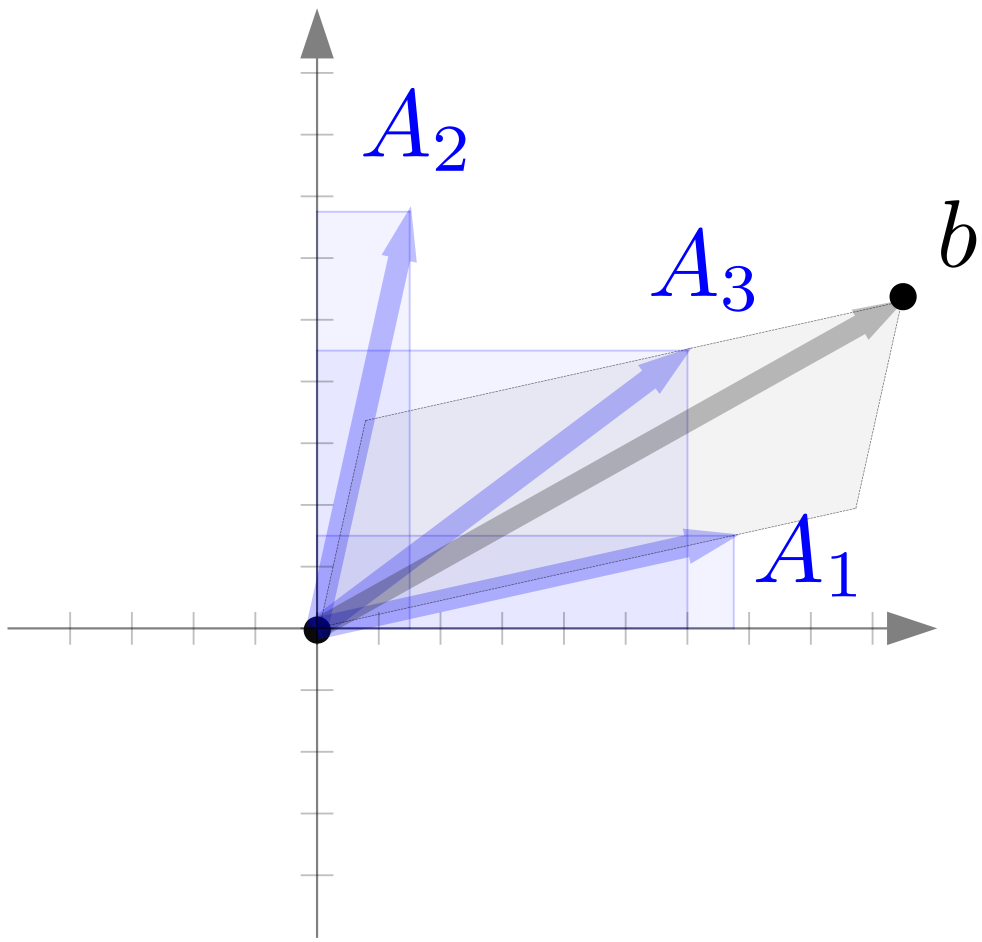


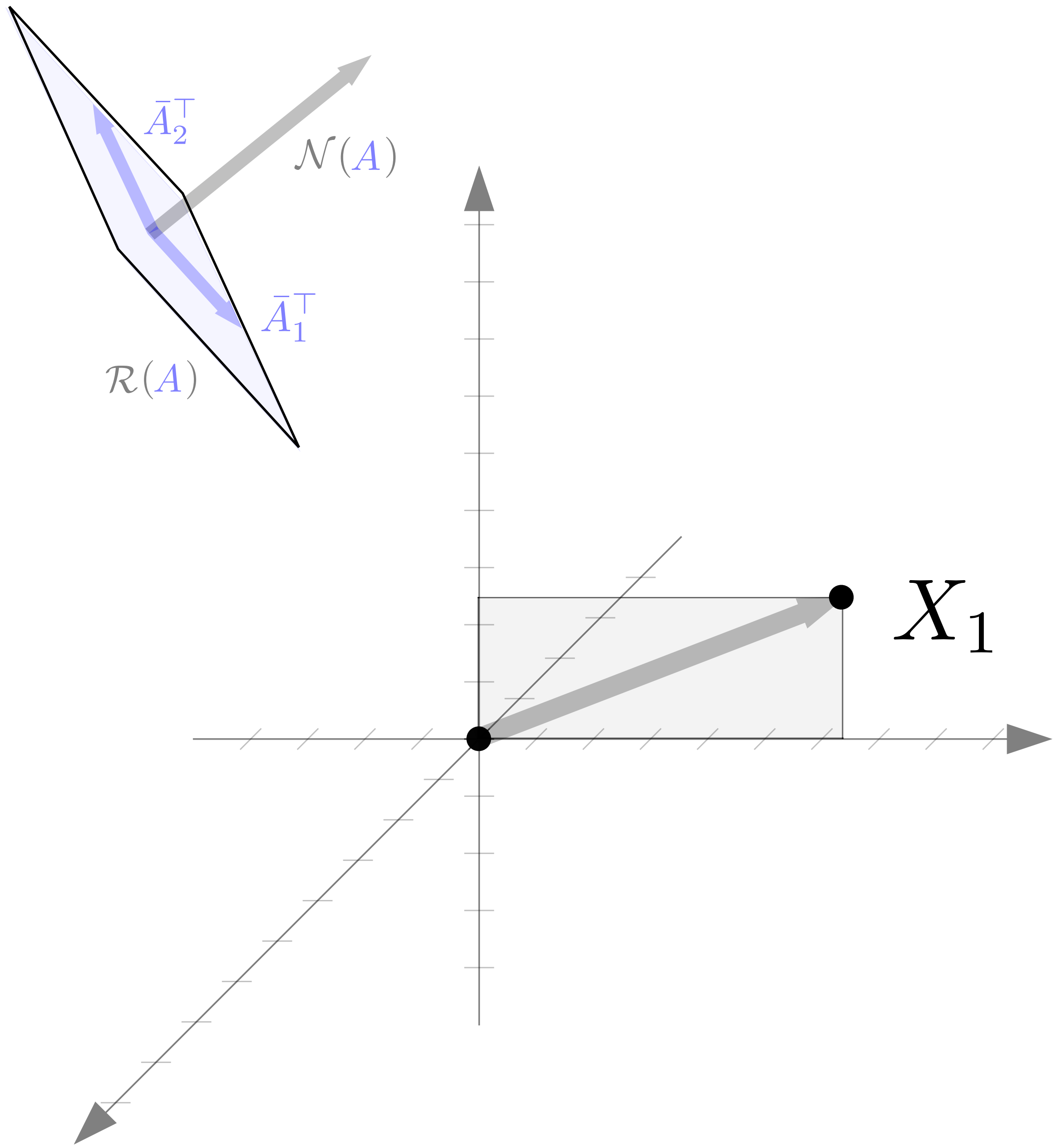
$$\begin{bmatrix} | \\ 0 \\ | \end{bmatrix} = \Big[\underbrace{\overbrace{\begin{matrix} | & | \\ A_1 & A_2 \\ | & | \end{matrix}}^{\operatorname{rank}(A)=k}}_{\text{lin. ind.}} \quad \underbrace{\overbrace{\begin{matrix} | & | \\ A_3 & A_4 \\ | & | \end{matrix}}^{n-k}}_{\text{lin. dep.}}\Big] \underbrace{\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ -1 & 0 \\ 0 & -1 \end{bmatrix}}_{N}$$

$$N = \overbrace{\begin{bmatrix} B \\ -I \end{bmatrix}}^{n-k} \begin{matrix} k \\ n-k \end{matrix}$$

Affine Spaces

Affine Spaces

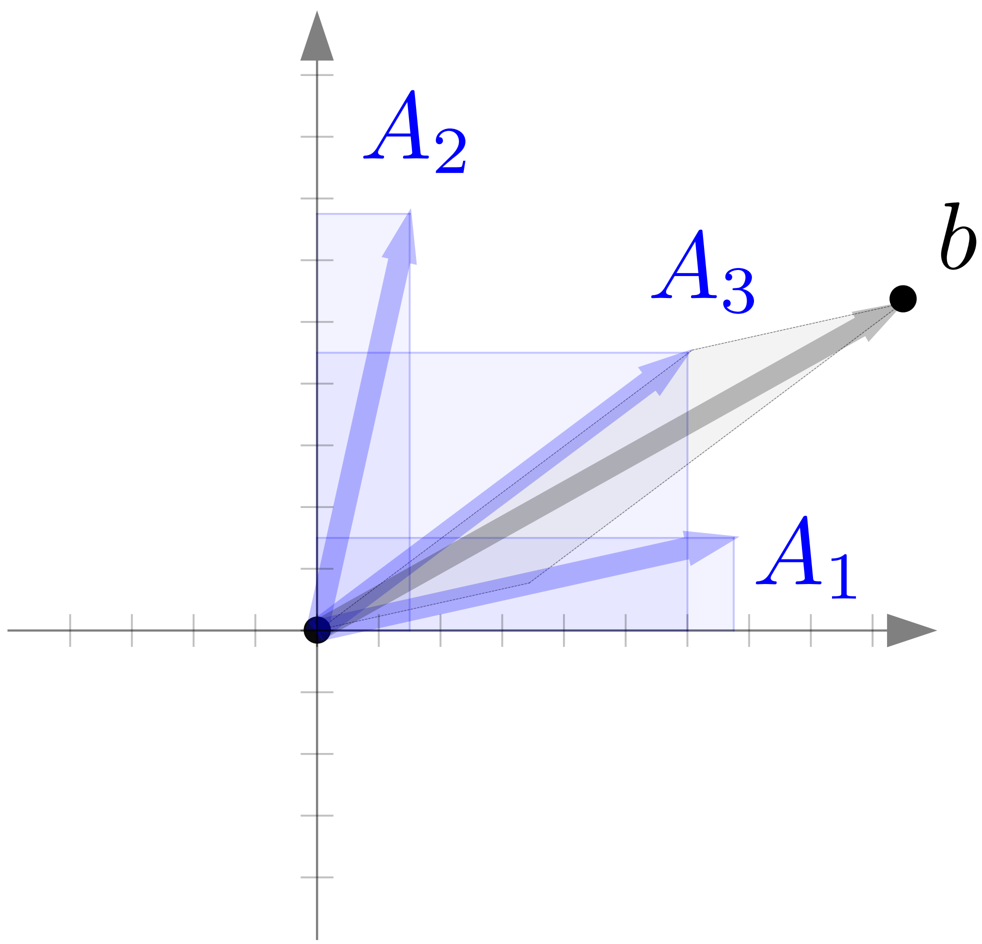


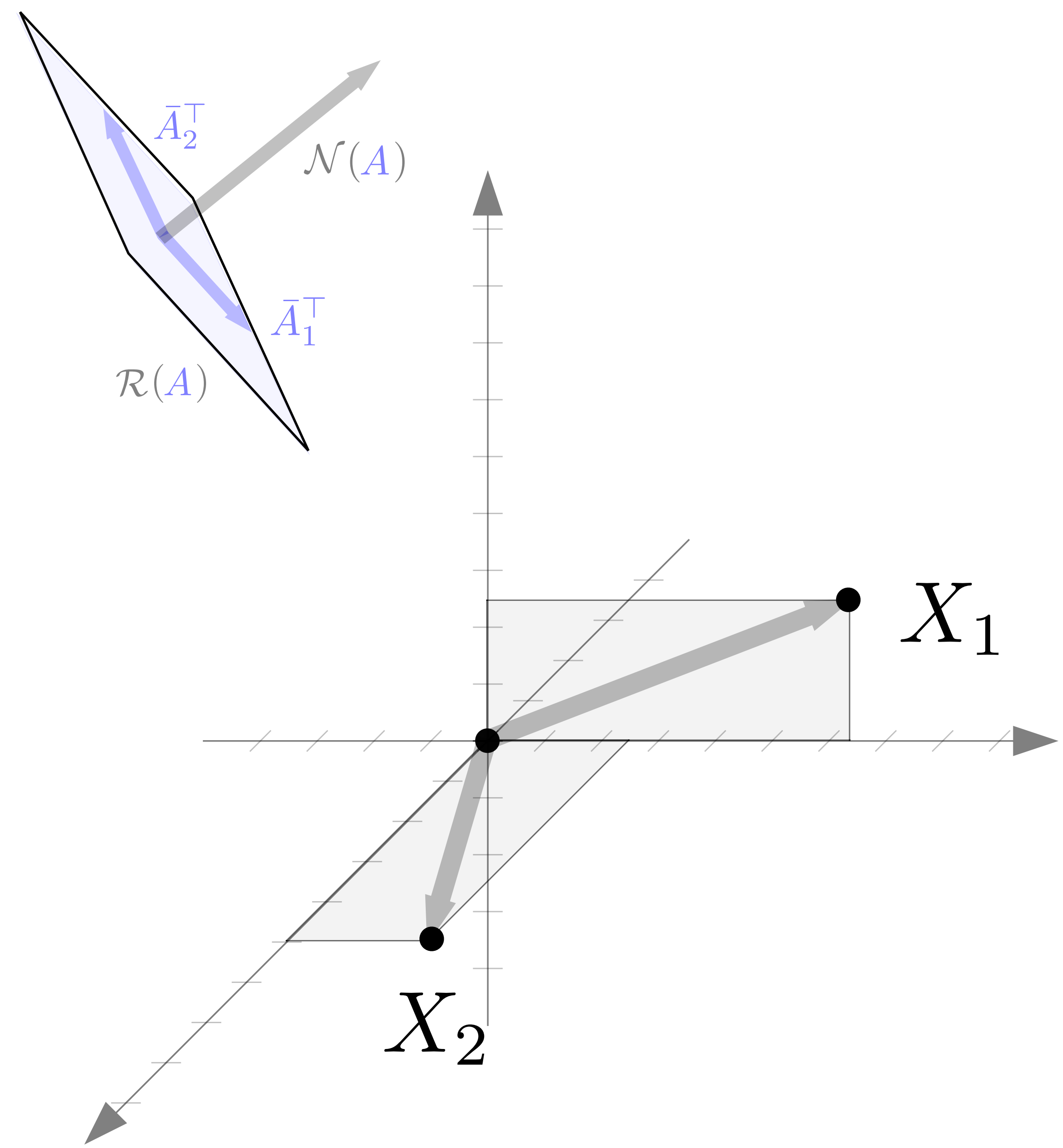


$$\begin{bmatrix} | \\ b \\ | \end{bmatrix} = \begin{bmatrix} | & | & | \\ A_1 & A_2 & A_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} X_{11} \\ X_{21} \\ 0 \end{bmatrix} = \begin{bmatrix} | \\ A_1 \\ | \end{bmatrix} X_{11} + \begin{bmatrix} | \\ A_2 \\ | \end{bmatrix} X_{21}$$

X_1 ① ②

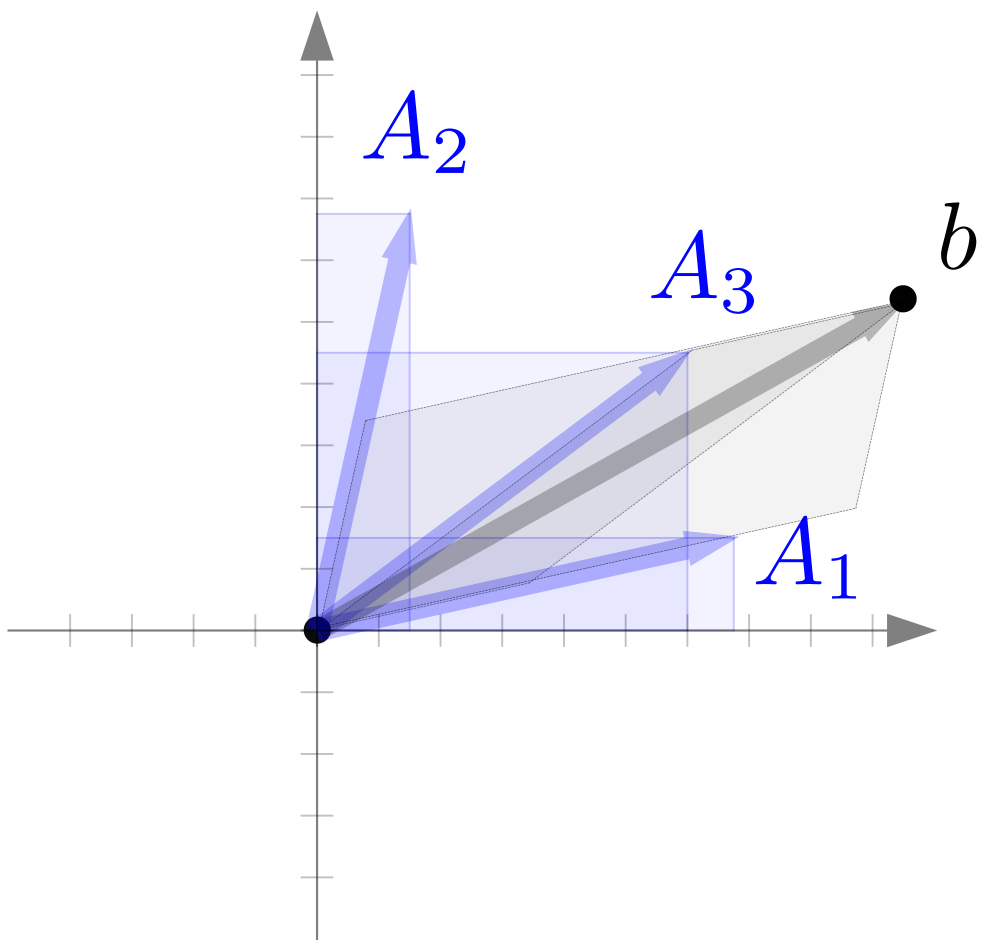
Affine Spaces

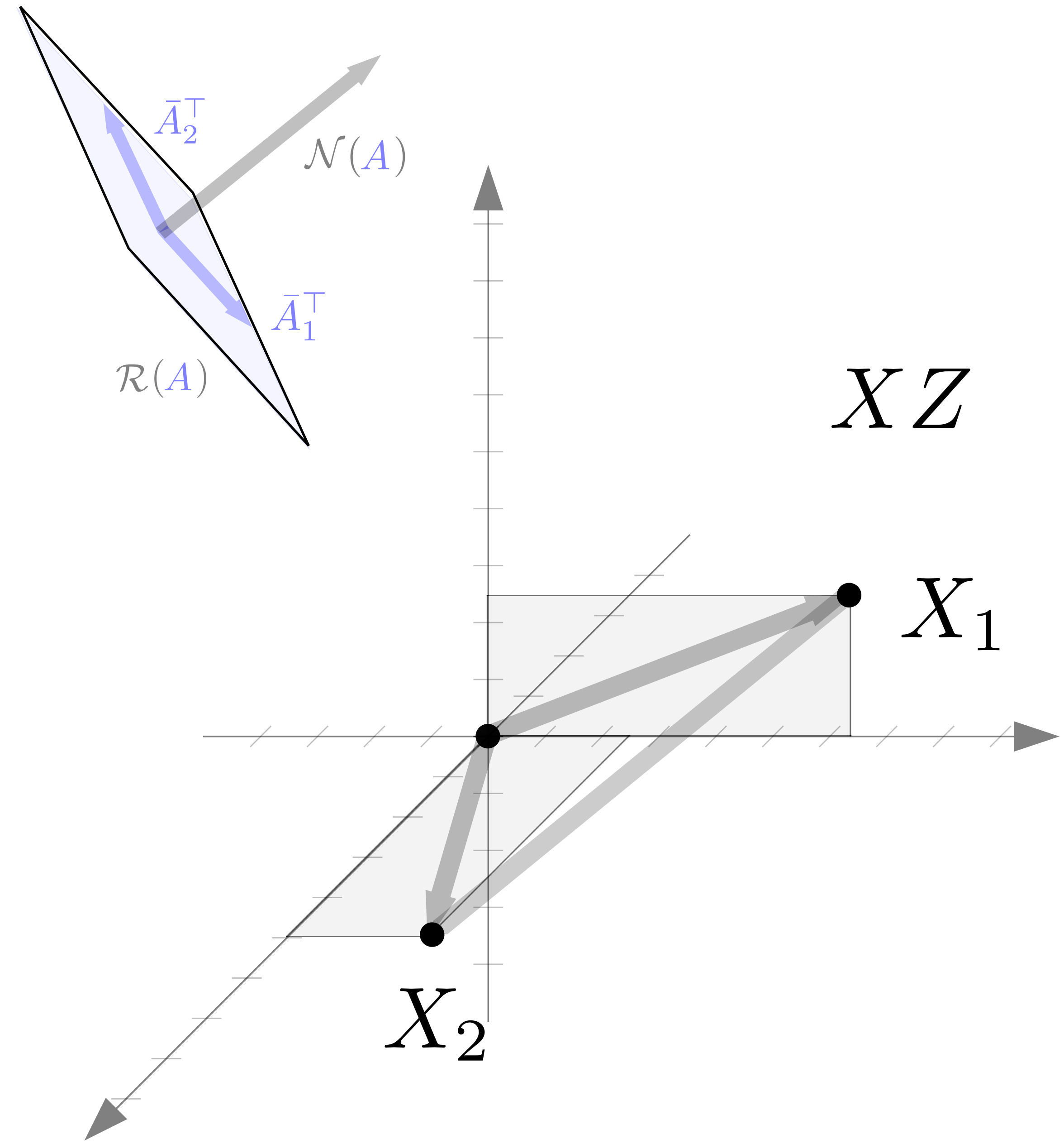




$$\begin{bmatrix} | \\ b \\ | \end{bmatrix} = \begin{bmatrix} | & | & | \\ A_1 & A_2 & A_3 \\ | & | & | \end{bmatrix} \underbrace{\begin{bmatrix} X_{12} \\ 0 \\ X_{32} \end{bmatrix}}_{X_2} = \underbrace{\begin{bmatrix} | \\ A_1 \\ | \end{bmatrix}}_{①} X_{12} + \underbrace{\begin{bmatrix} | \\ A_3 \\ | \end{bmatrix}}_{③} X_{32}$$

Affine Spaces



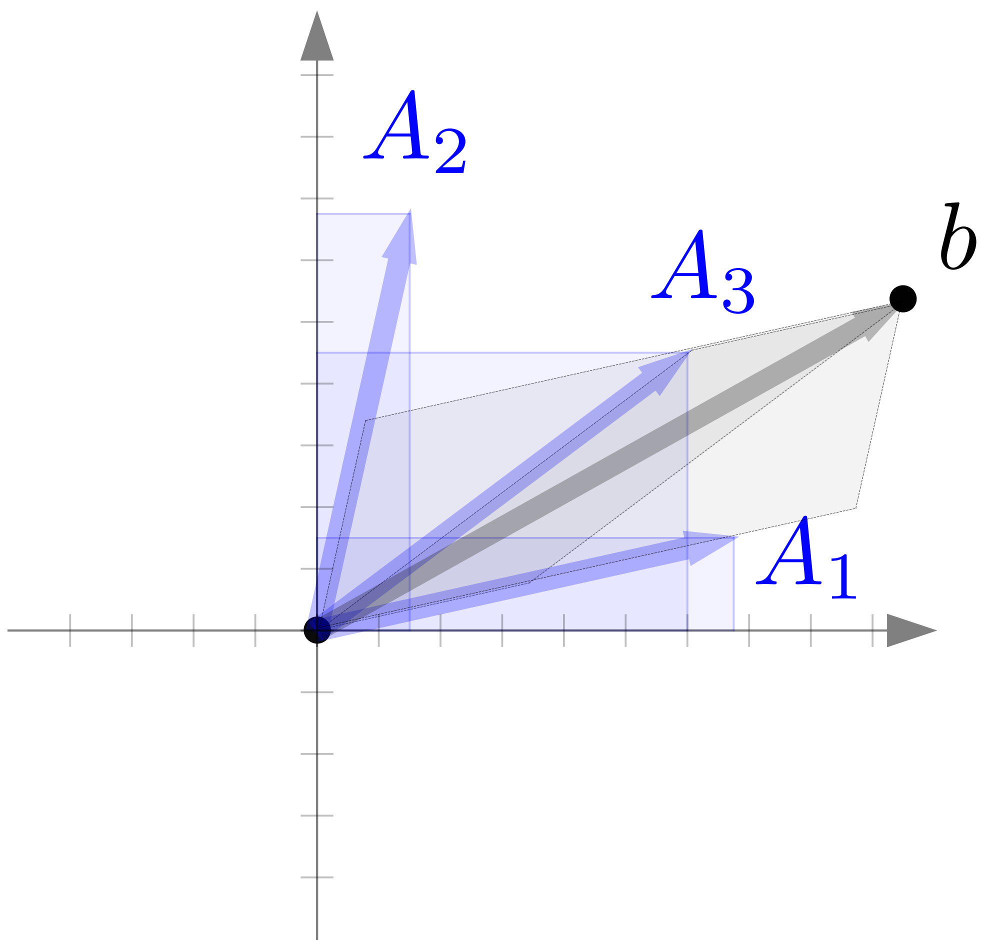


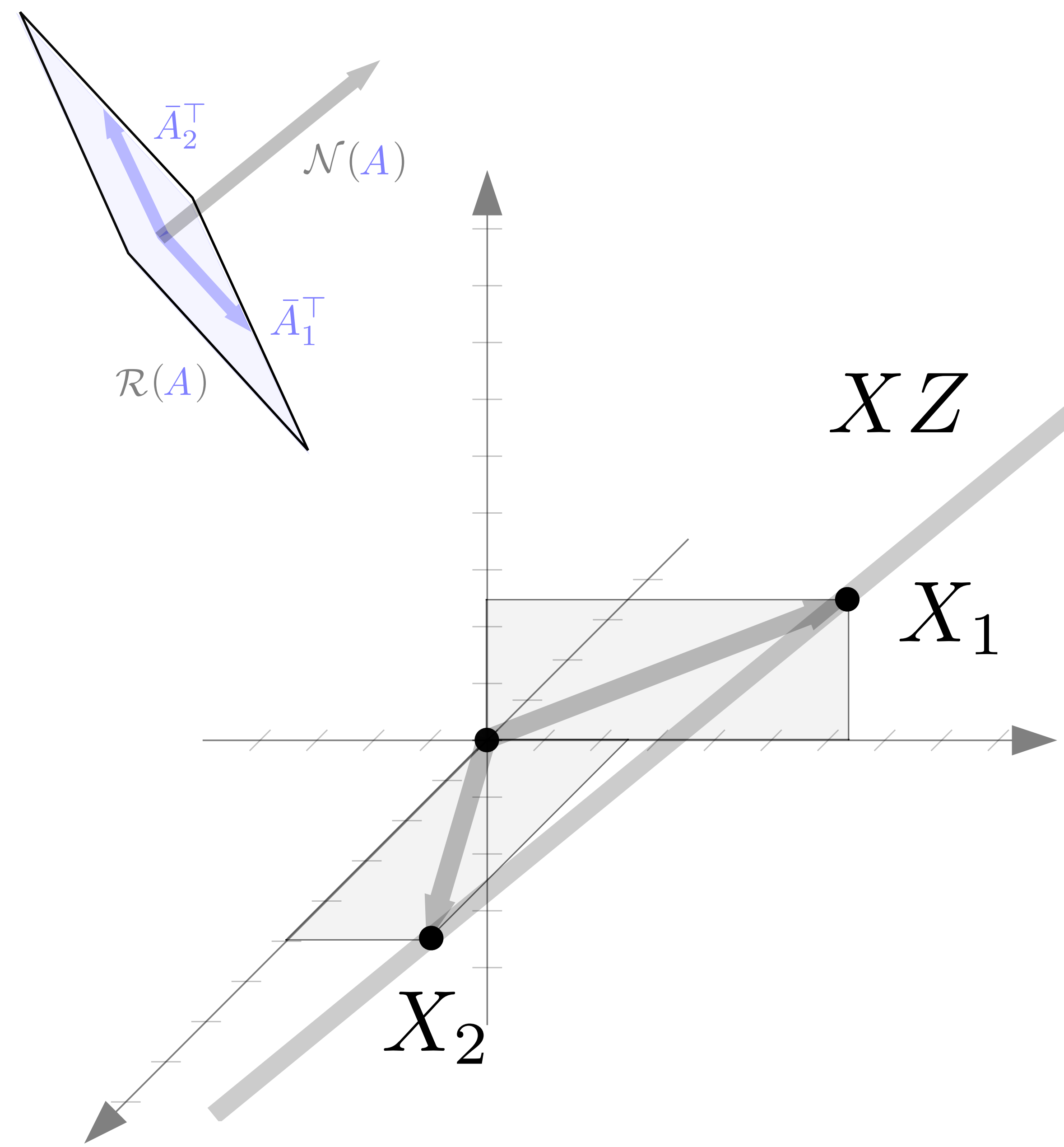
$$\begin{bmatrix} | \\ b \\ | \end{bmatrix} = \begin{bmatrix} | & | & | \\ A_1 & A_2 & A_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} | \\ x \\ | \end{bmatrix}$$

$$x \in XZ \qquad X = \begin{bmatrix} | & | \\ X_1 & X_2 \\ | & | \end{bmatrix}$$

$$\Delta_2 = \left\{ z \in \mathbb{R}^2 \mid \mathbf{1}^\top z = 1, z \geq 0 \right\}$$

Affine Spaces



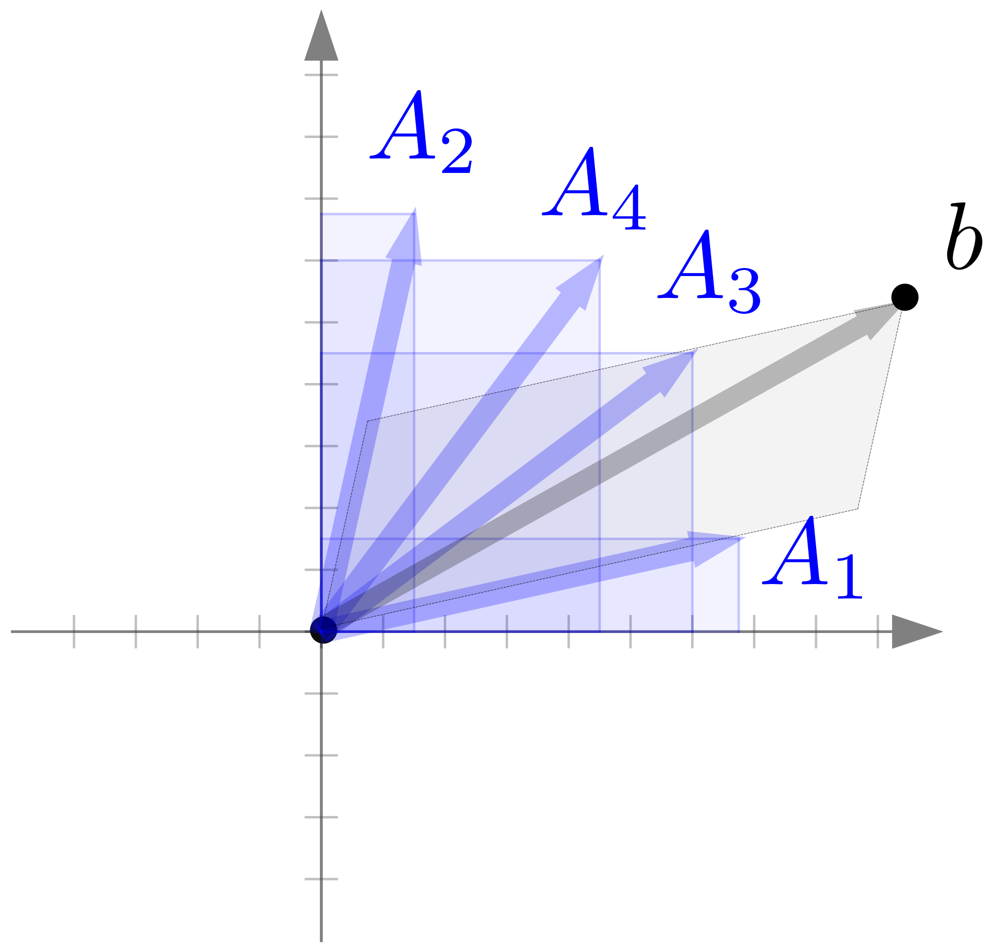


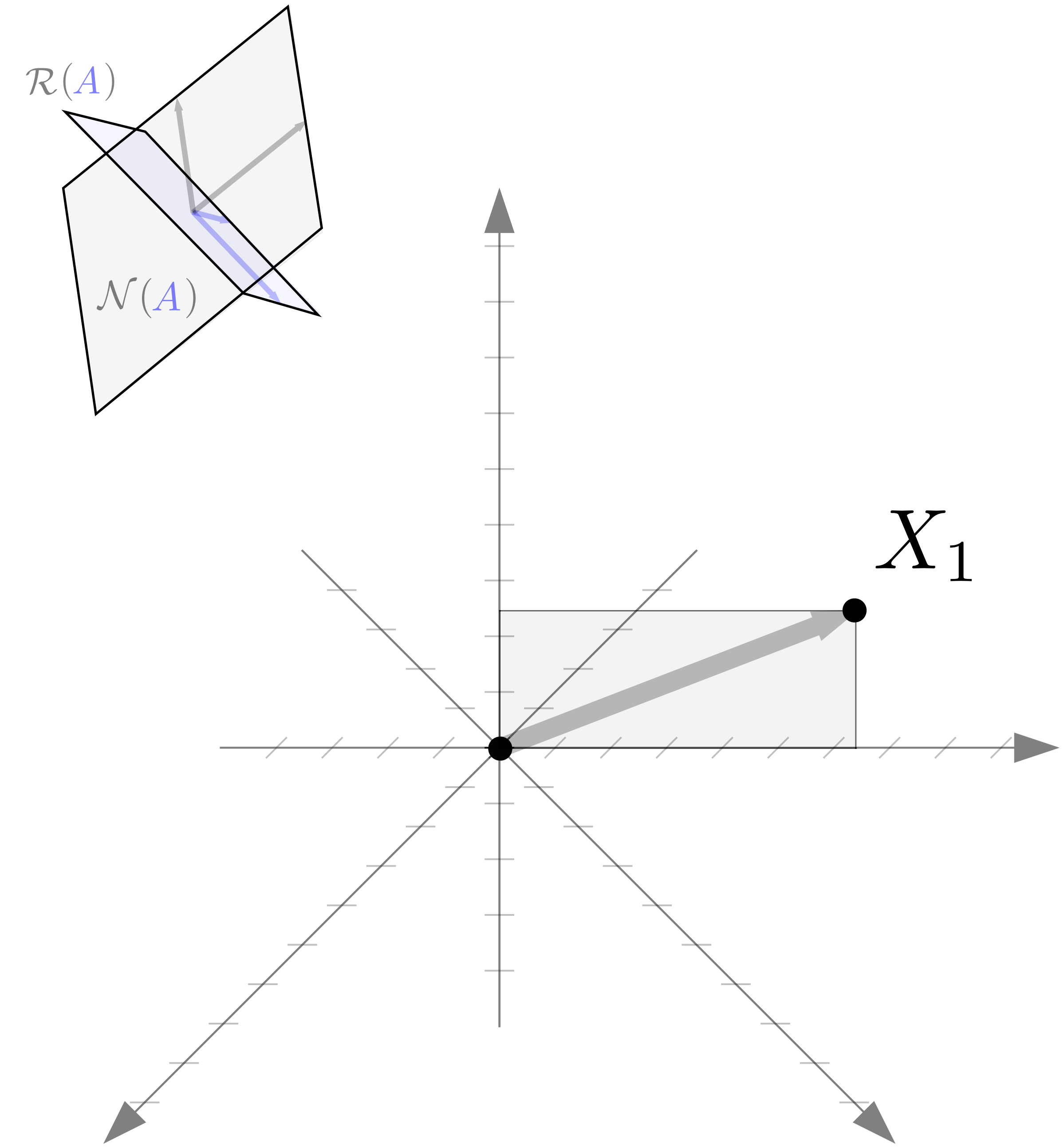
$$\begin{bmatrix} | \\ b \\ | \end{bmatrix} = \begin{bmatrix} | & | & | \\ A_1 & A_2 & A_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} | \\ x \\ | \end{bmatrix}$$

$$x \in XZ \qquad X = \begin{bmatrix} | & | \\ X_1 & X_2 \\ | & | \end{bmatrix}$$

$$\mathcal{L}_2 = \left\{ z \in \mathbb{R}^2 \;\middle|\; \mathbf{1}^\top z = 1 \right\}$$

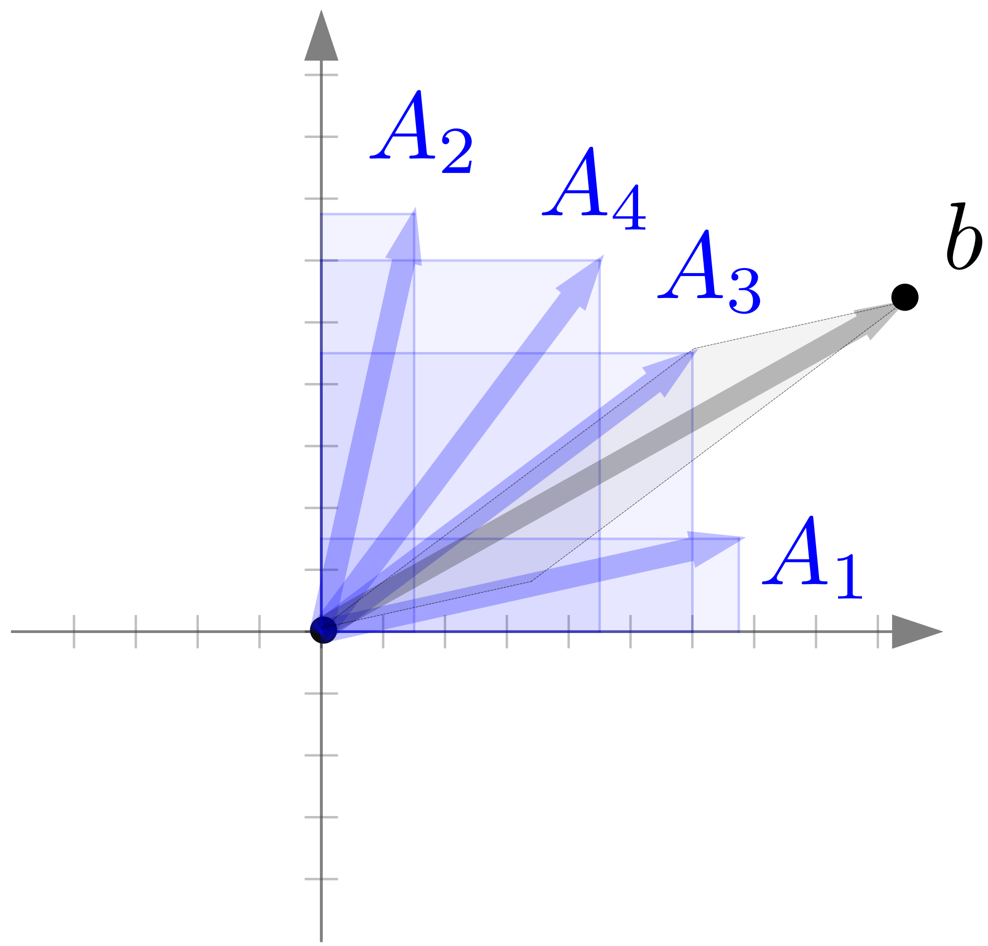
Affine Spaces

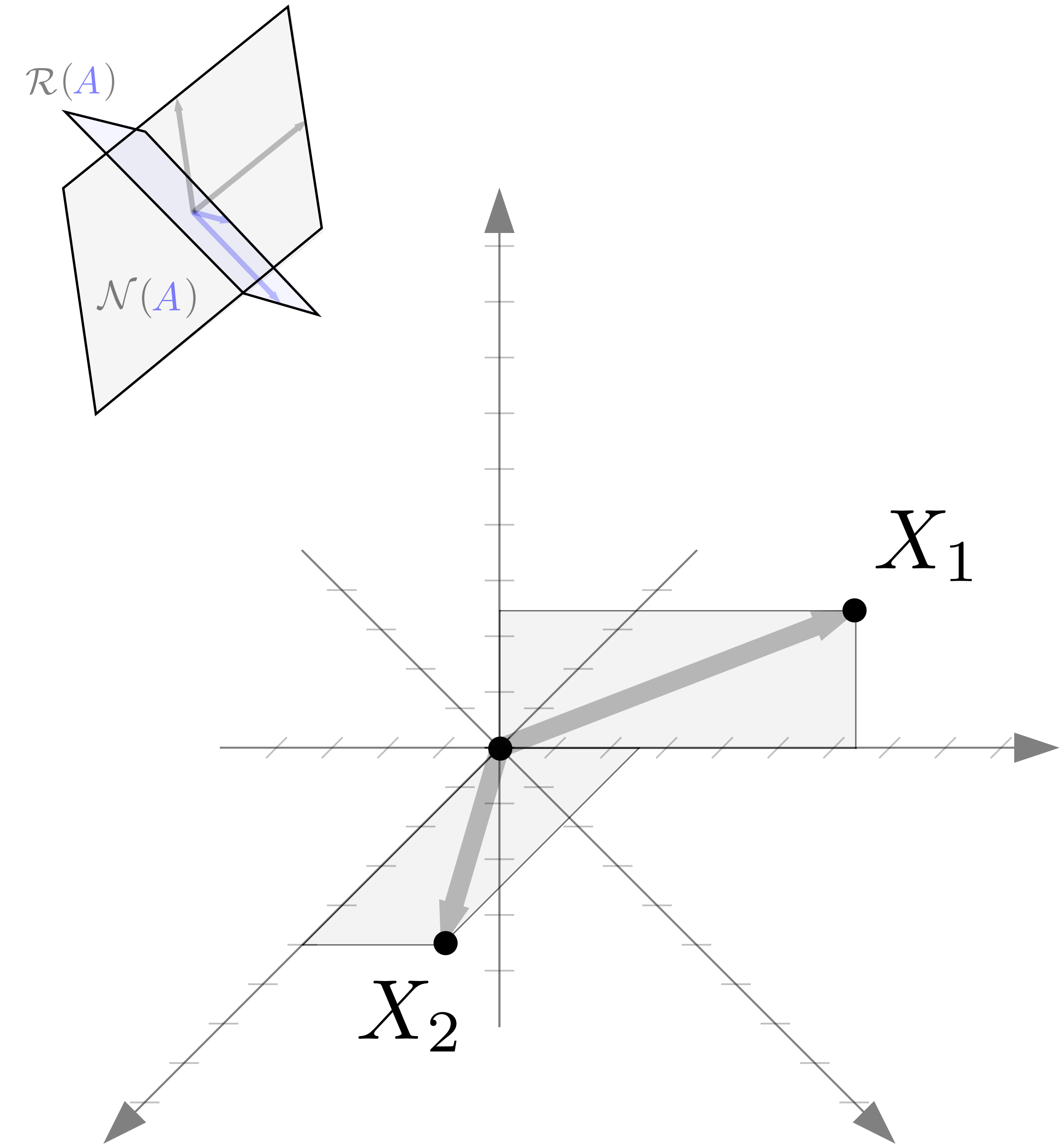




$$\begin{bmatrix} | \\ b \\ | \end{bmatrix} = \begin{bmatrix} | & | & | & | \\ A_1 & A_2 & A_3 & A_4 \\ | & | & | & | \end{bmatrix} \underbrace{\begin{bmatrix} X_{11} \\ X_{21} \\ 0 \\ 0 \end{bmatrix}}_{X_1} = \underbrace{\begin{bmatrix} | \\ A_1 \\ | \end{bmatrix}}_{①} X_{11} + \underbrace{\begin{bmatrix} | \\ A_2 \\ | \end{bmatrix}}_{②} X_{21}$$

Affine Spaces





$$\begin{bmatrix} | \\ b \\ | \end{bmatrix} = \begin{bmatrix} | & | & | & | \\ A_1 & A_2 & A_3 & A_4 \\ | & | & | & | \end{bmatrix} \underbrace{\begin{bmatrix} X_{12} \\ 0 \\ X_{32} \\ 0 \end{bmatrix}}_{X_2} = \underbrace{\begin{bmatrix} | \\ A_1 \\ | \end{bmatrix}}_{①} X_{12} + \underbrace{\begin{bmatrix} | \\ A_3 \\ | \end{bmatrix}}_{③} X_{32}$$

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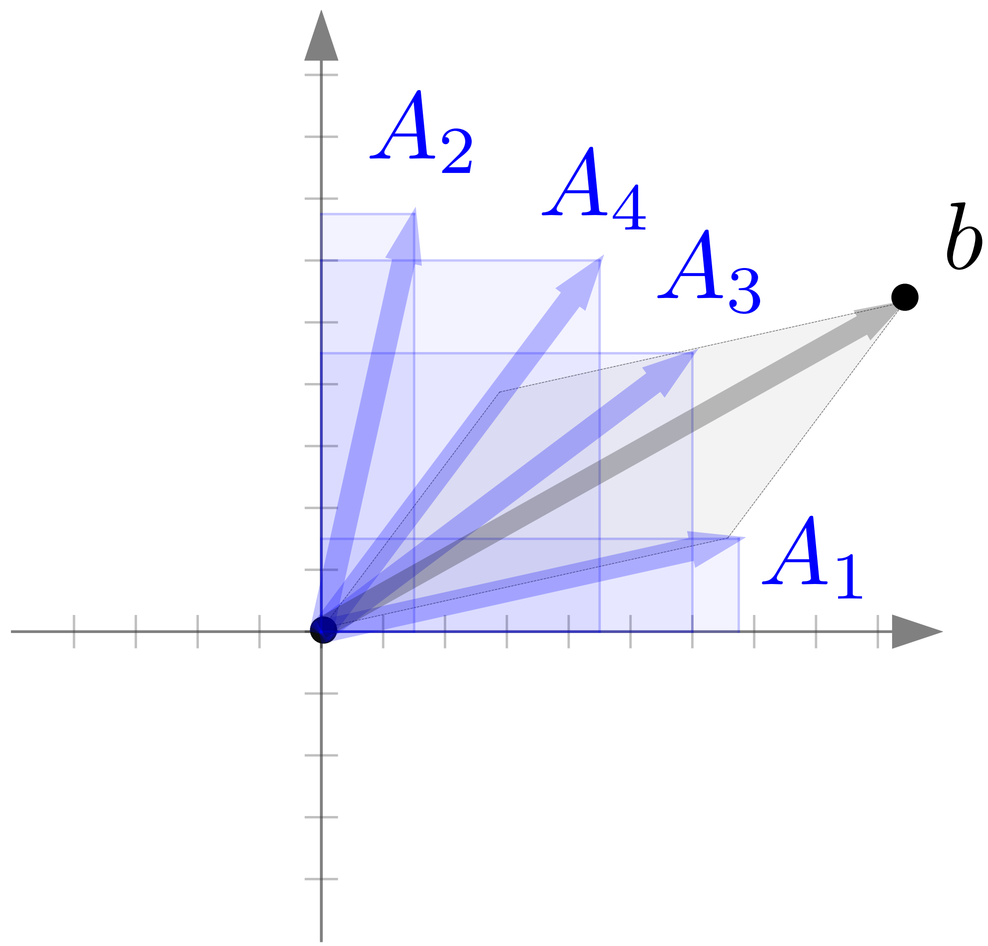
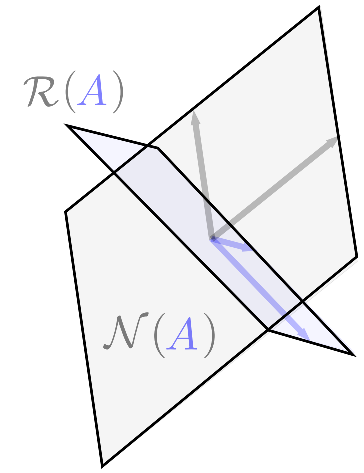
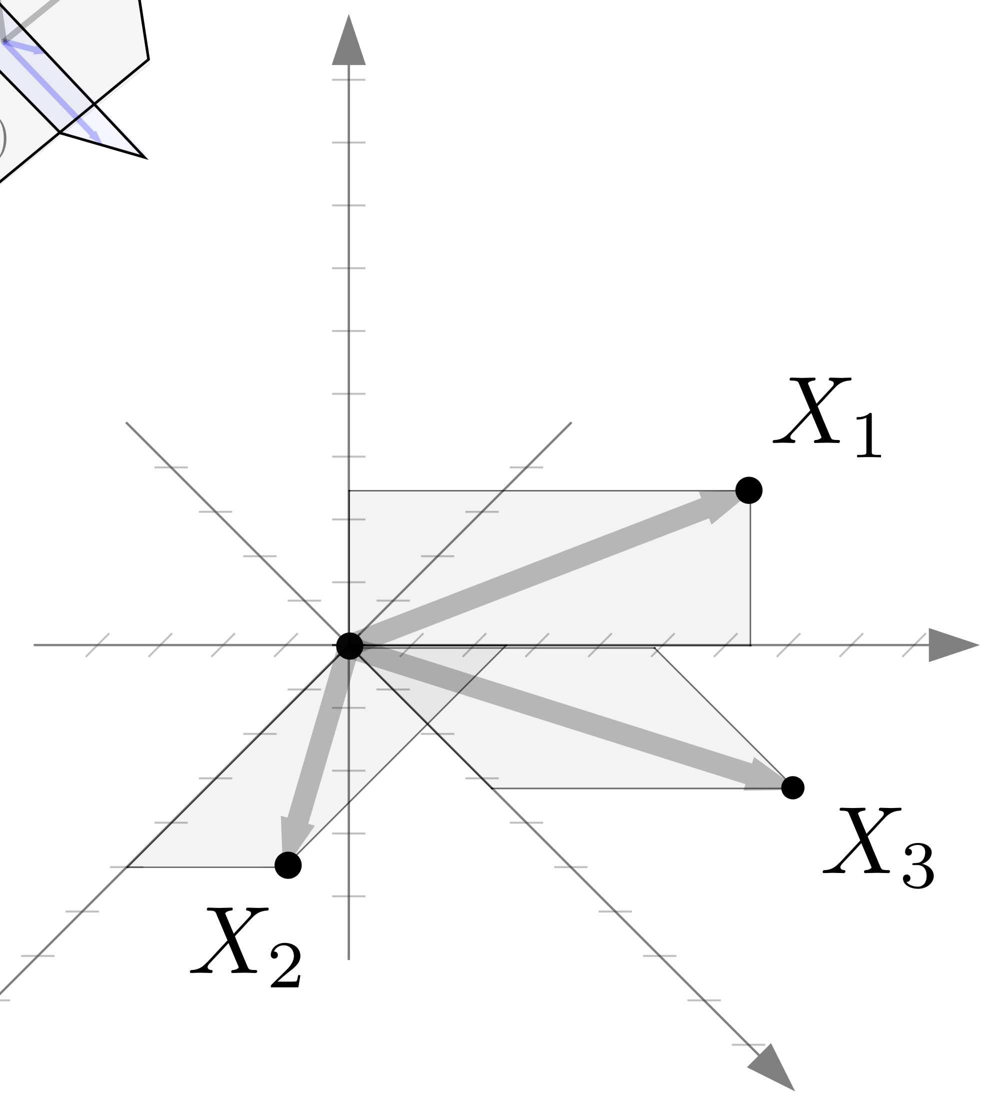
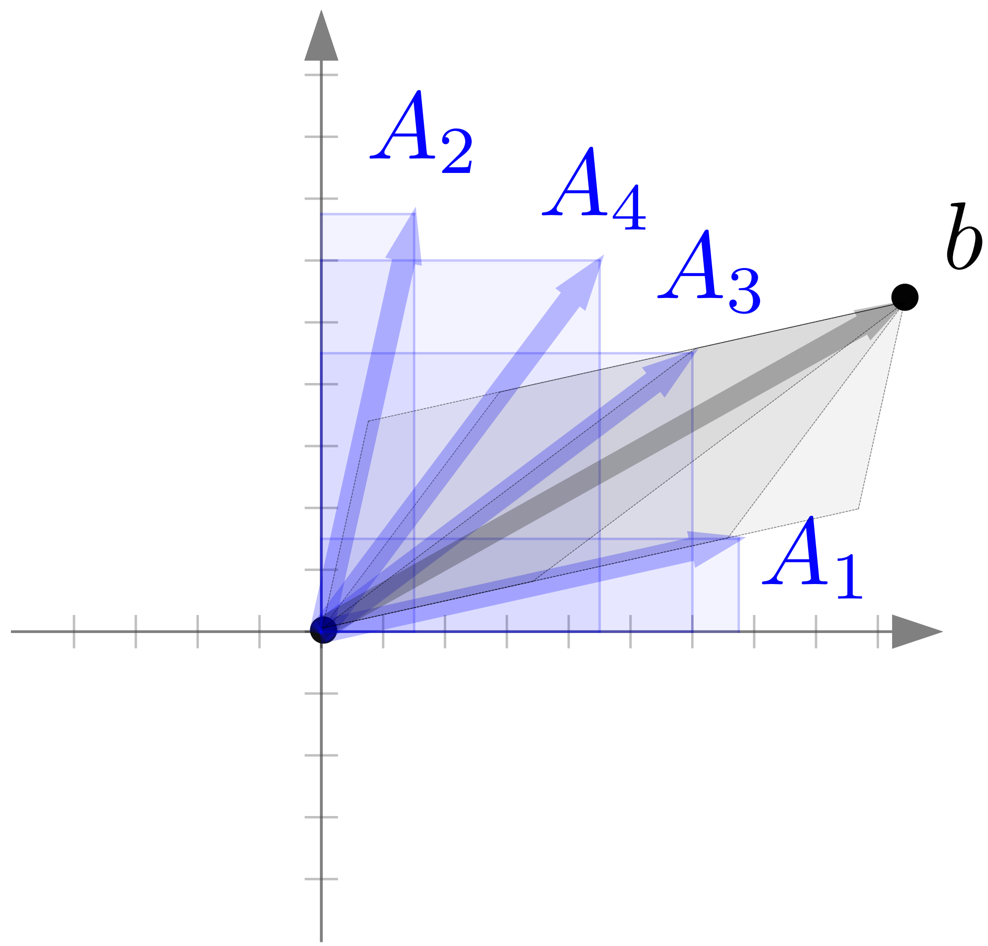


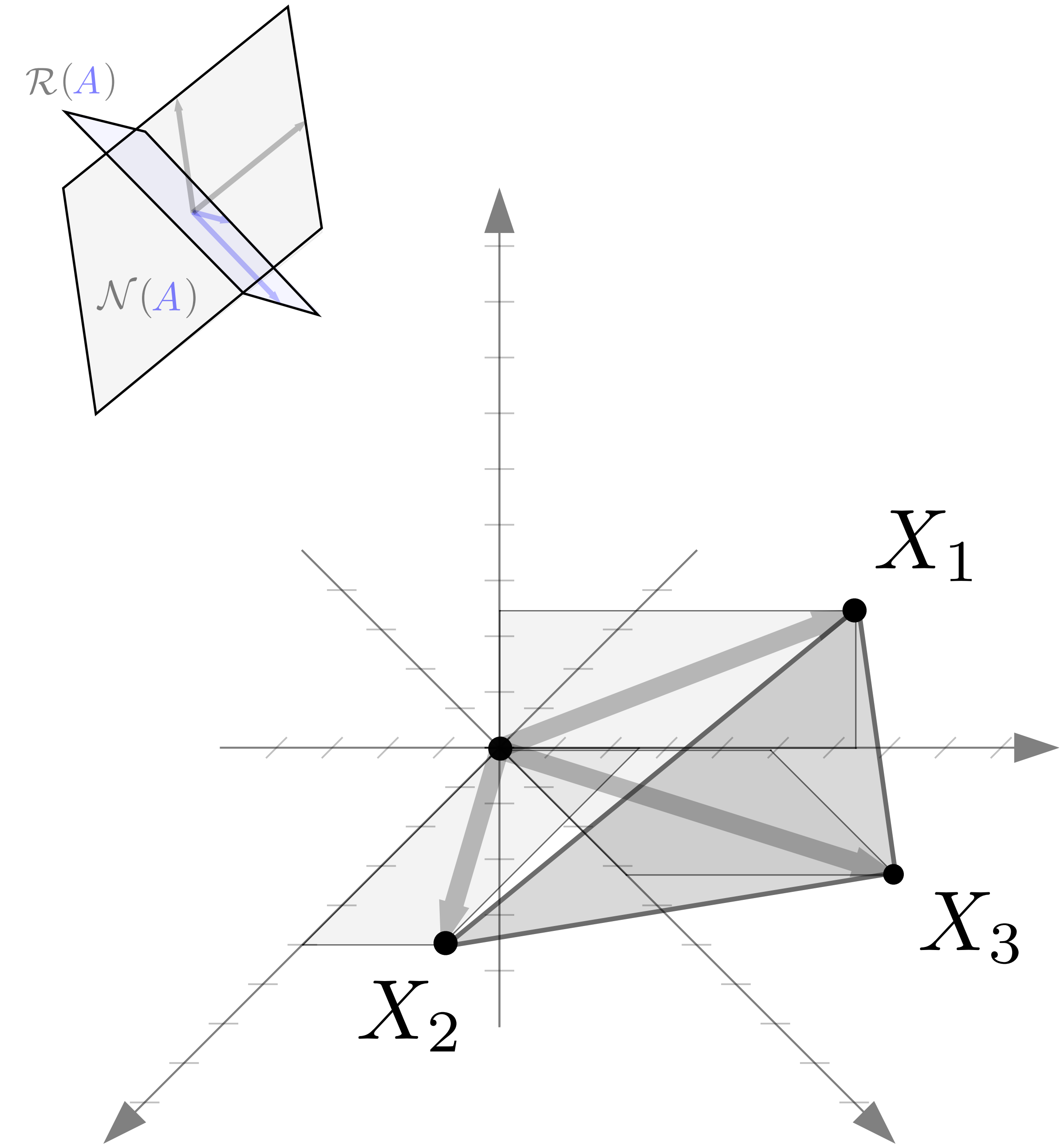




$$\begin{bmatrix} | \\ b \\ | \end{bmatrix} = \begin{bmatrix} | & | & | & | \\ A_1 & A_2 & A_3 & A_4 \\ | & | & | & | \end{bmatrix} \underbrace{\begin{bmatrix} X_{13} \\ 0 \\ 0 \\ X_{43} \end{bmatrix}}_{X_3} = \underbrace{\begin{bmatrix} | \\ A_1 \\ | \end{bmatrix}}_{①} X_{13} + \underbrace{\begin{bmatrix} | \\ A_3 \\ | \end{bmatrix}}_{③} X_{43}$$

Affine Spaces



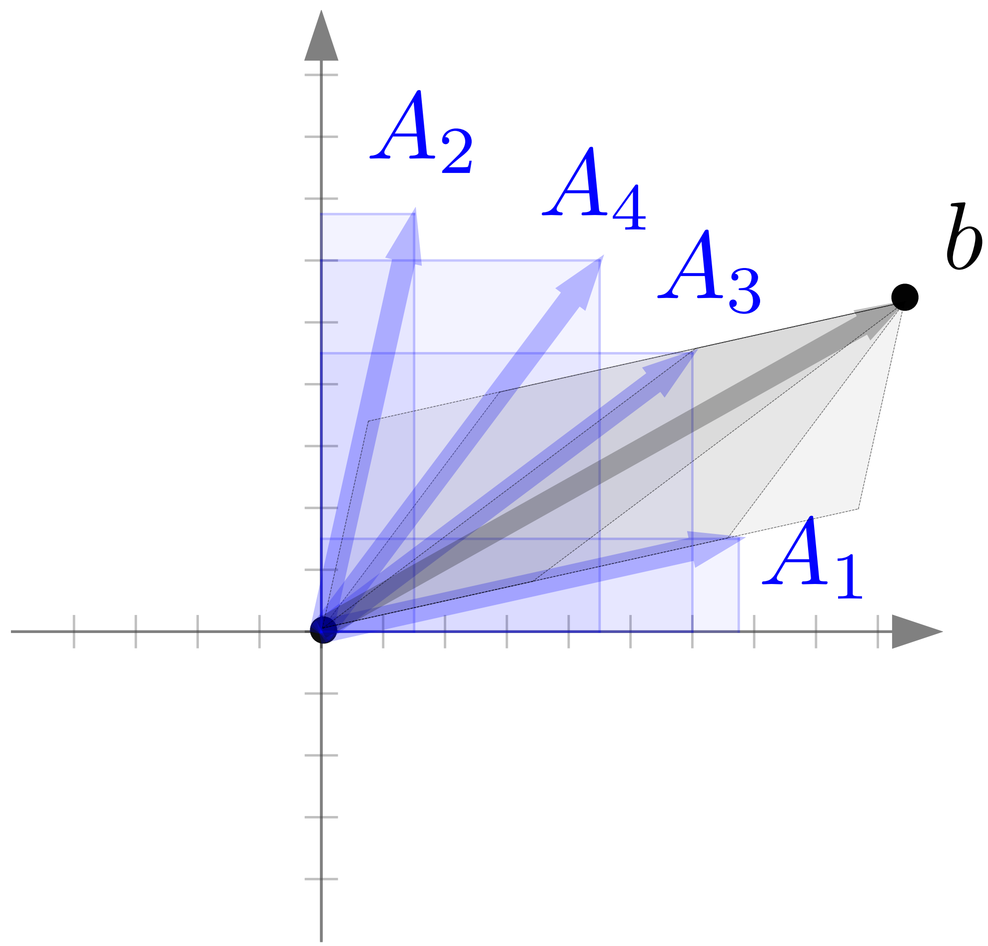


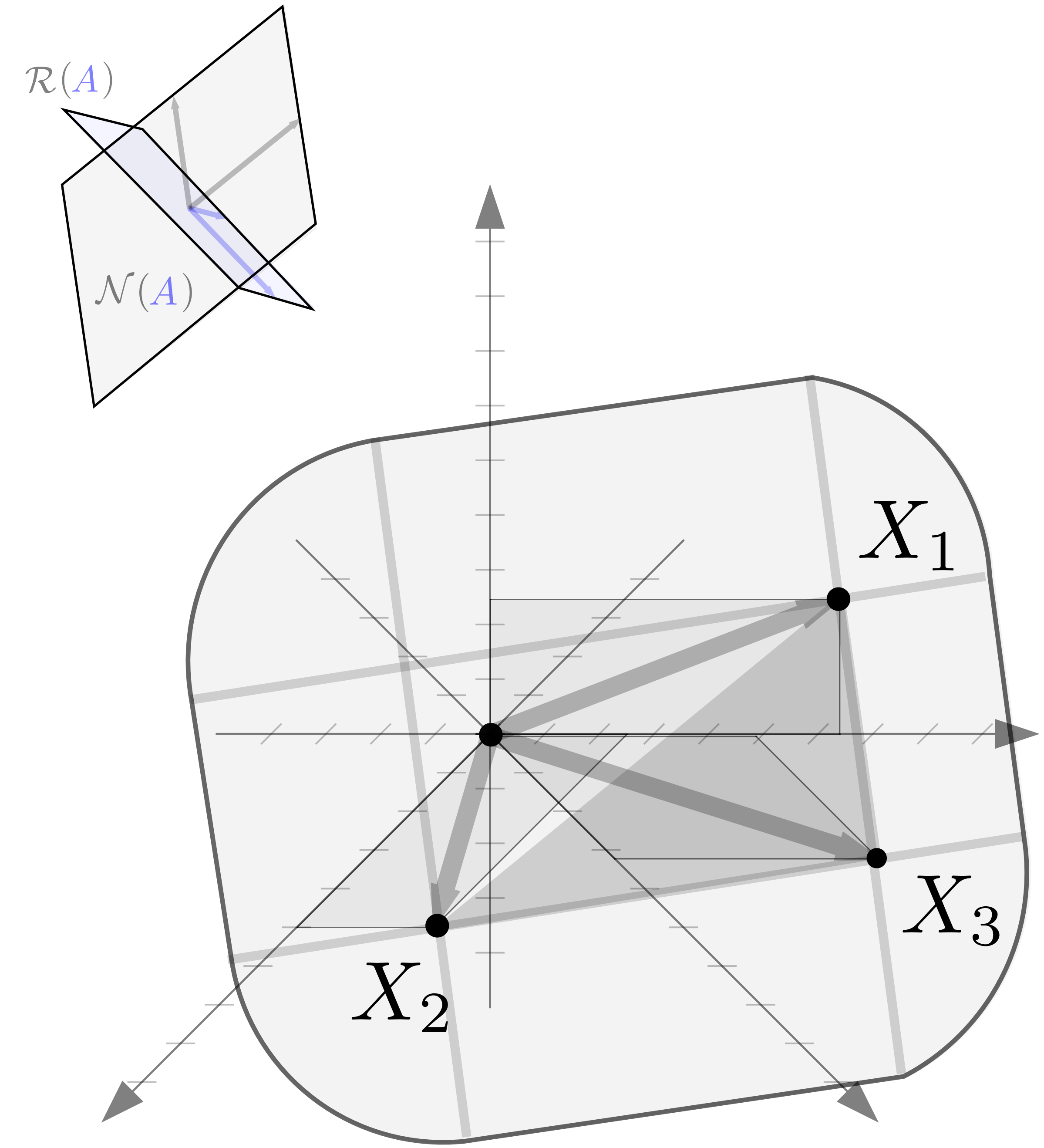
$$\begin{bmatrix} | \\ b \\ | \end{bmatrix} = \begin{bmatrix} | & | & | & | \\ A_1 & A_2 & A_3 & A_4 \\ | & | & | & | \end{bmatrix} \begin{bmatrix} | \\ x \\ | \end{bmatrix}$$

$$x \in XZ \qquad X = \begin{bmatrix} | & | & | \\ X_1 & X_2 & X_3 \\ | & | & | \end{bmatrix}$$

$$\Delta_3 = \{ z \in \mathbb{R}^3 \mid \mathbf{1}^\top z = 1, z \geq 0 \}$$

Affine Spaces





$$\begin{bmatrix} | \\ b \\ | \end{bmatrix} = \begin{bmatrix} | & | & | & | \\ A_1 & A_2 & A_3 & A_4 \\ | & | & | & | \end{bmatrix} \begin{bmatrix} | \\ x \\ | \end{bmatrix}$$

$$x \in XZ \qquad X = \begin{bmatrix} | & | & | \\ X_1 & X_2 & X_3 \\ | & | & | \end{bmatrix}$$

$$\mathcal{L}_3 = \{ z \in \mathbb{R}^3 \mid \mathbf{1}^\top z = 1 \}$$