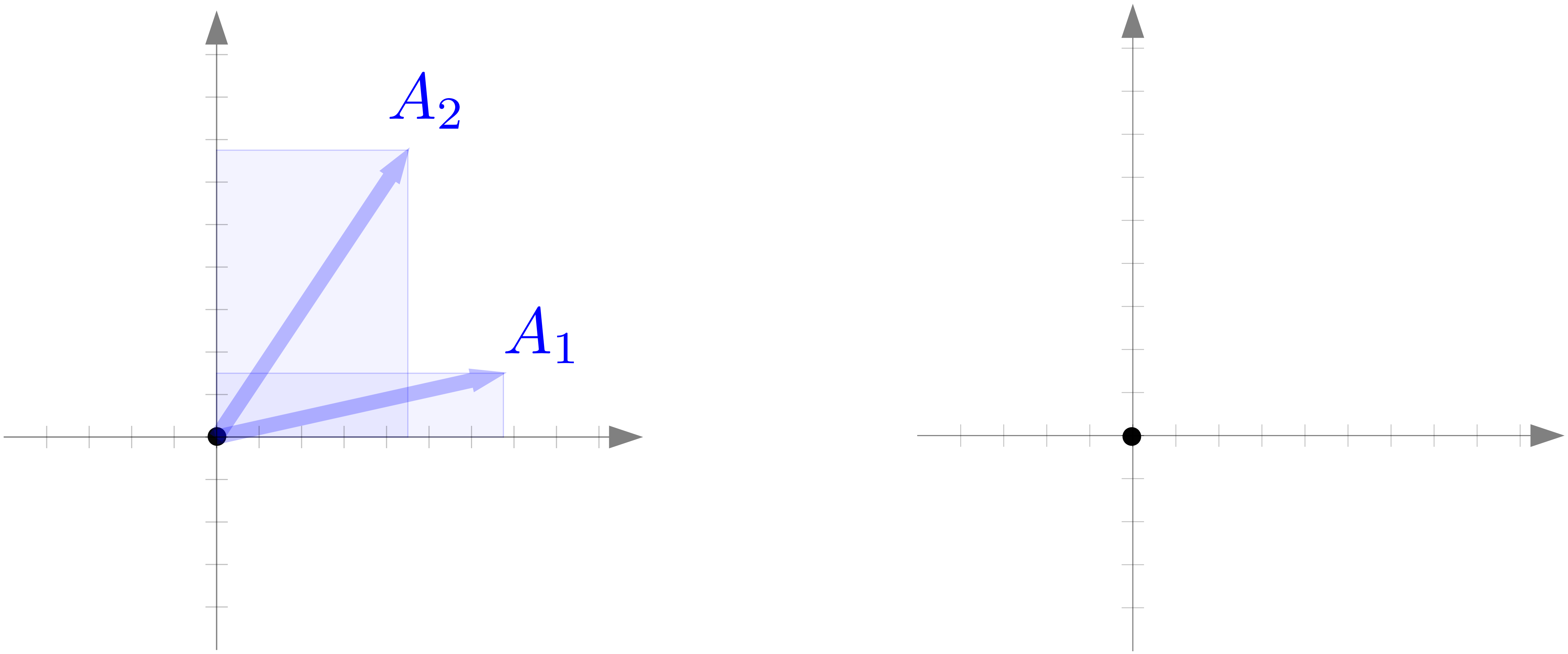
Column Geometry - Matrices

Linear Algebra

Summer 2023 - Dan Calderone

Square Matrices

Images of Sets



$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} | & | \\ A_1 & A_2 \\ | & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} | \\ A_1 \\ | \end{bmatrix} x_1 + \begin{bmatrix} | \\ A_2 \\ | \end{bmatrix} x_2$$

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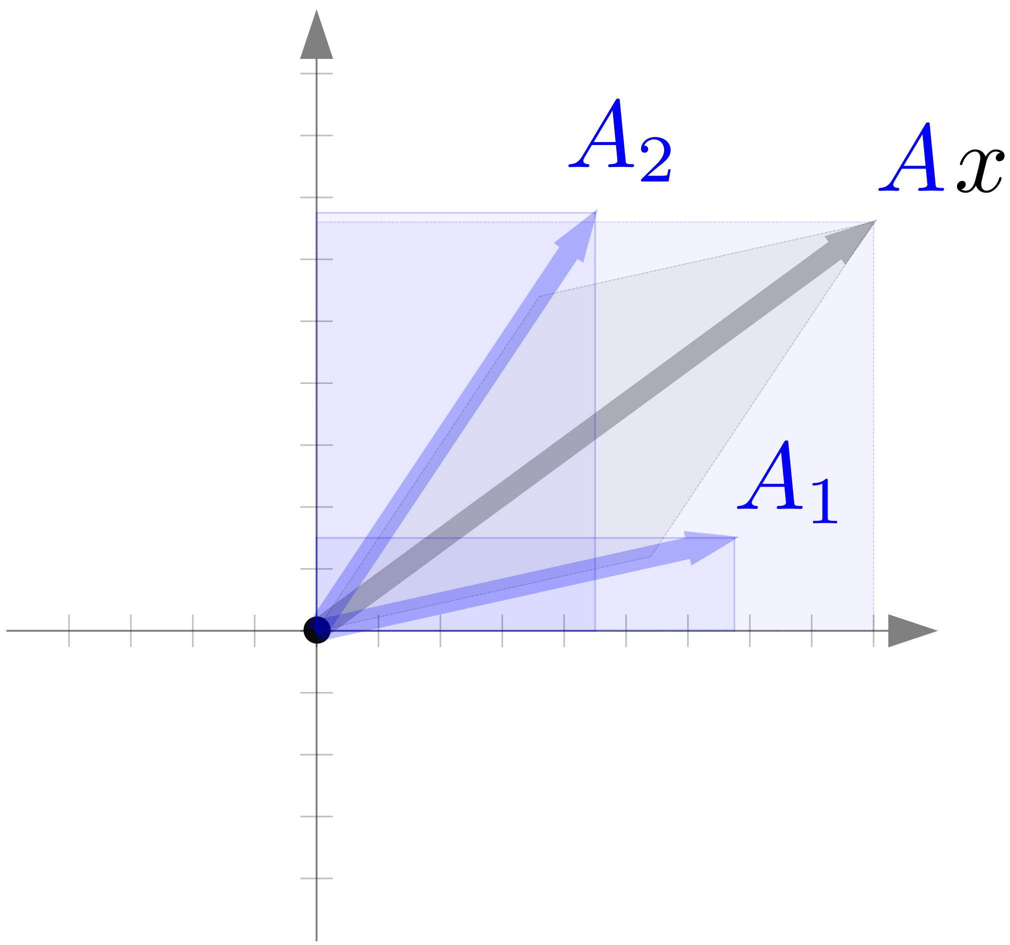

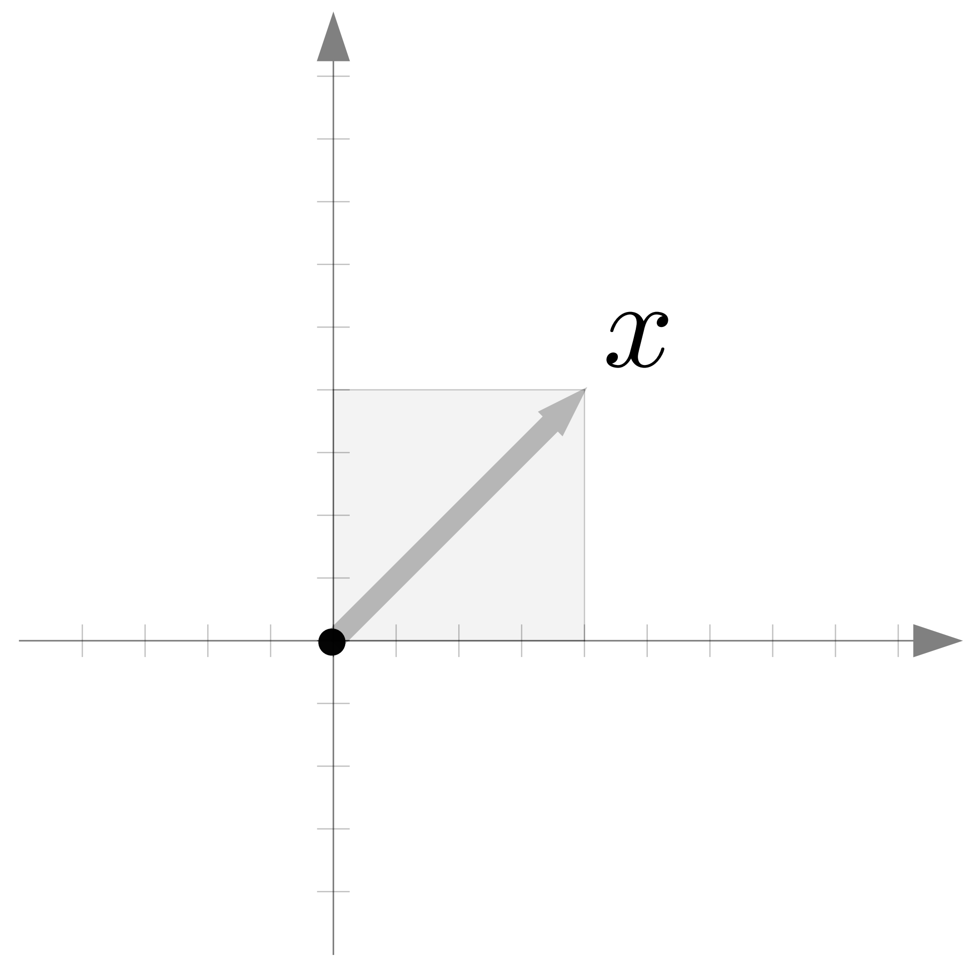

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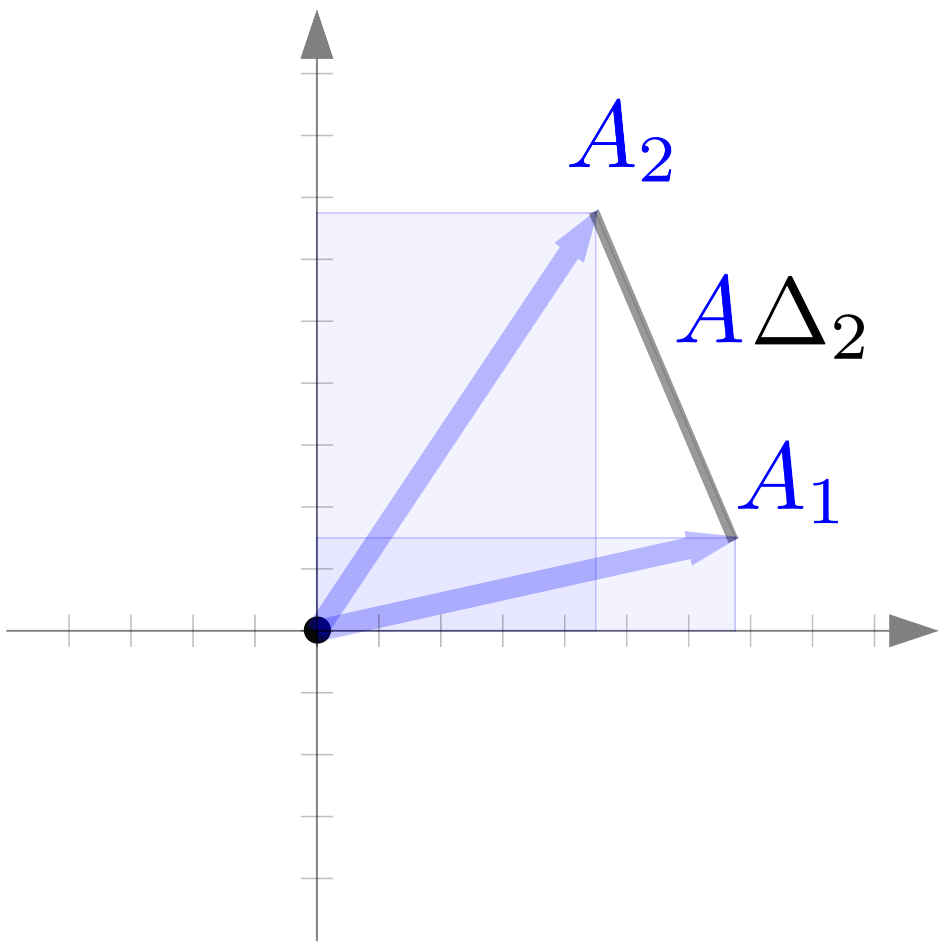

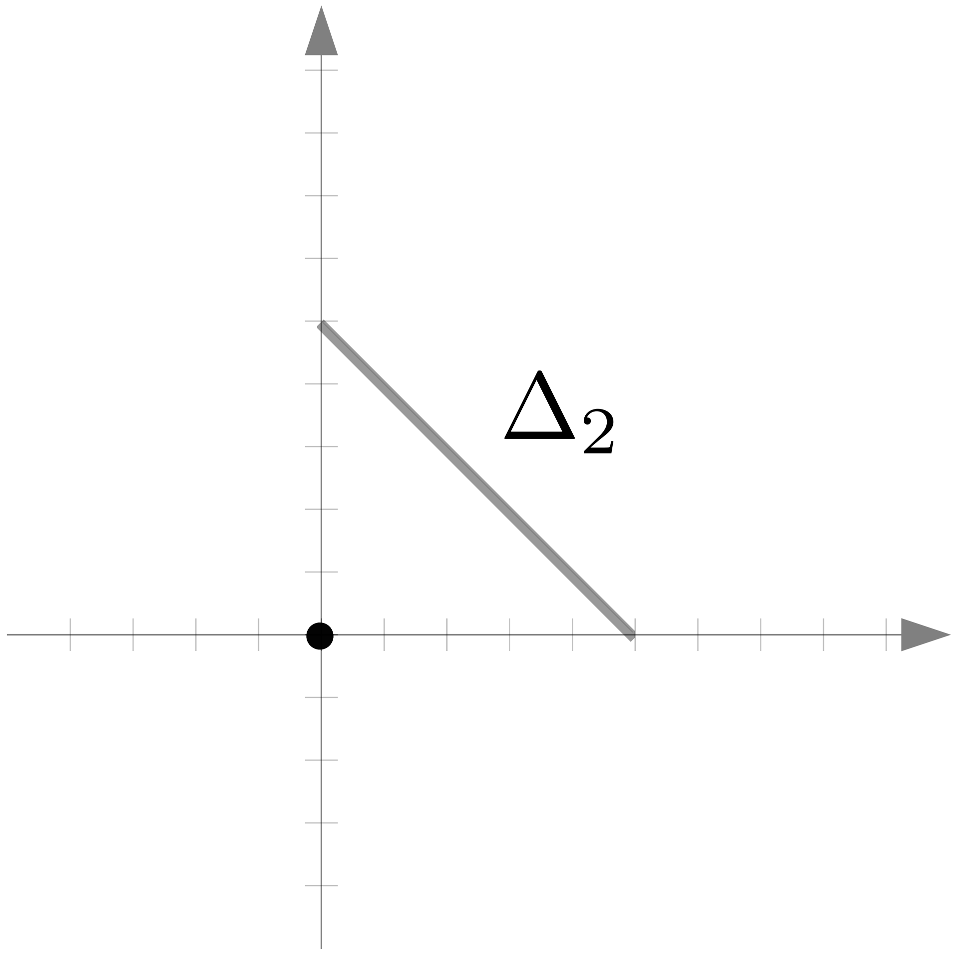

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$$\Delta_2 = \{x \in \mathbb{R}^2 \mid \mathbf{1}^\top x = 1, x \geq 0\}$$

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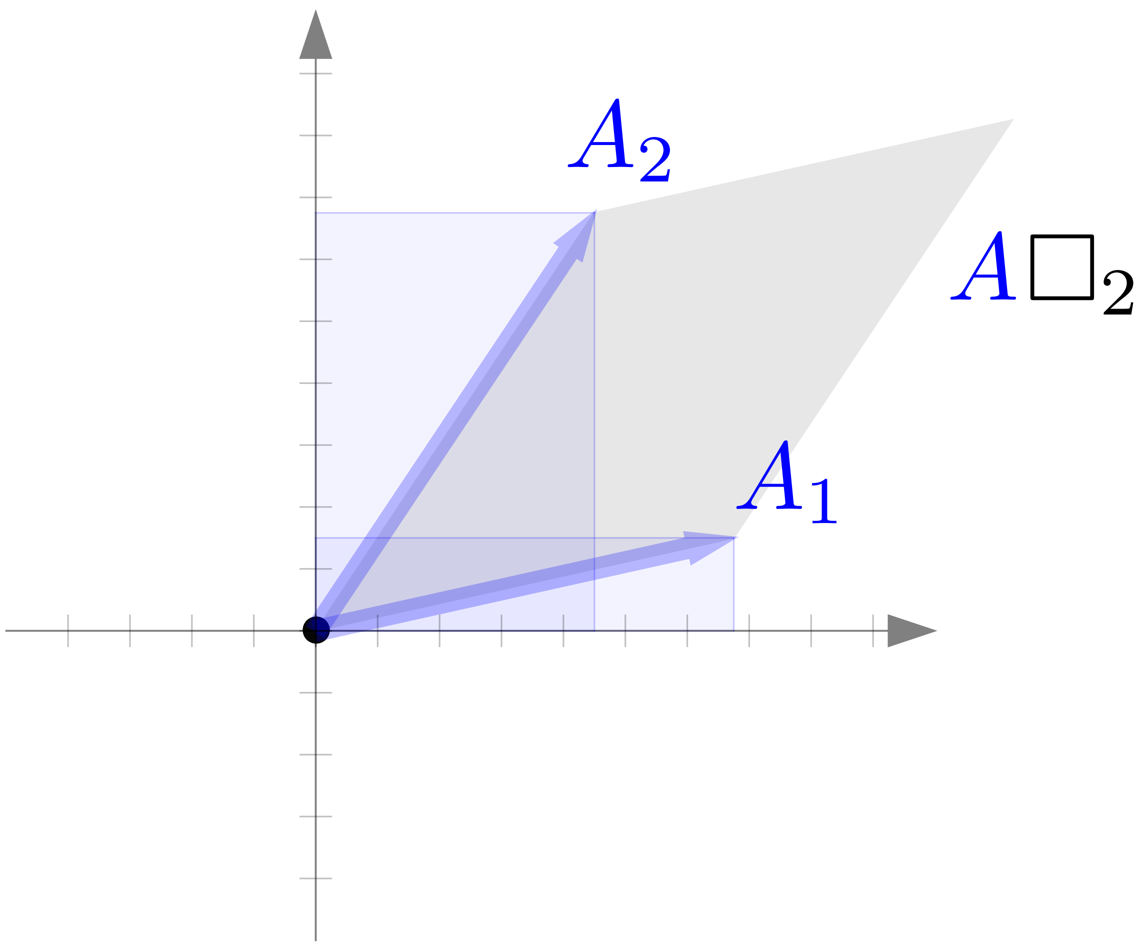

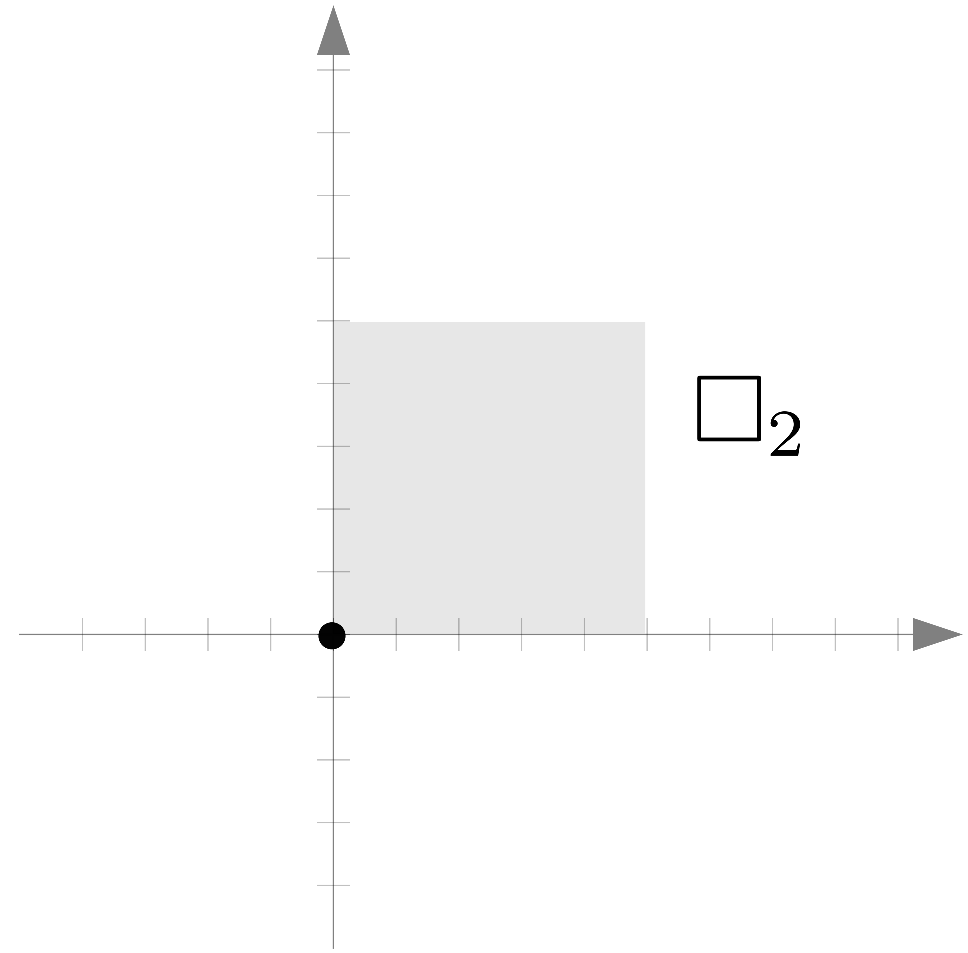

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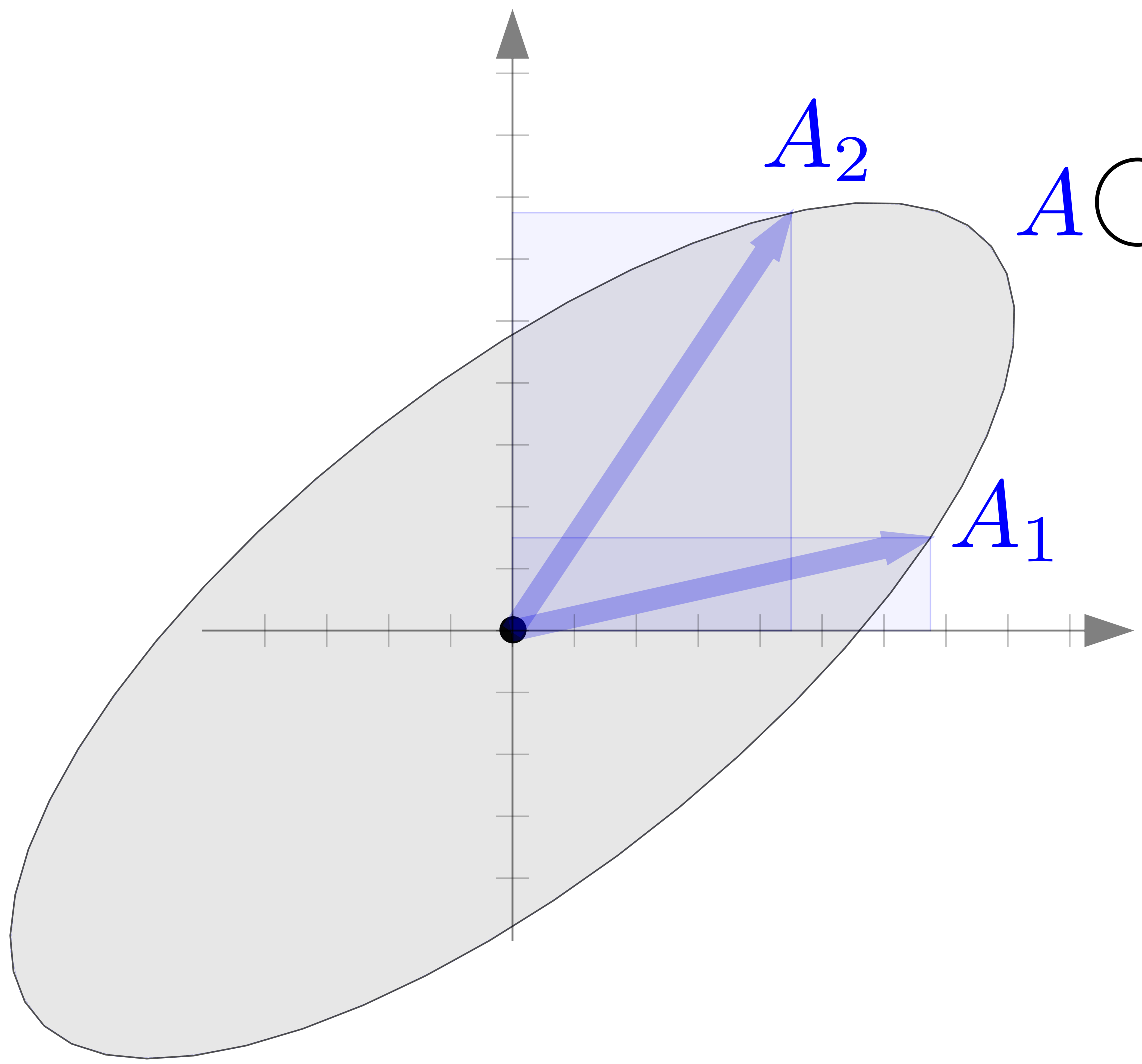

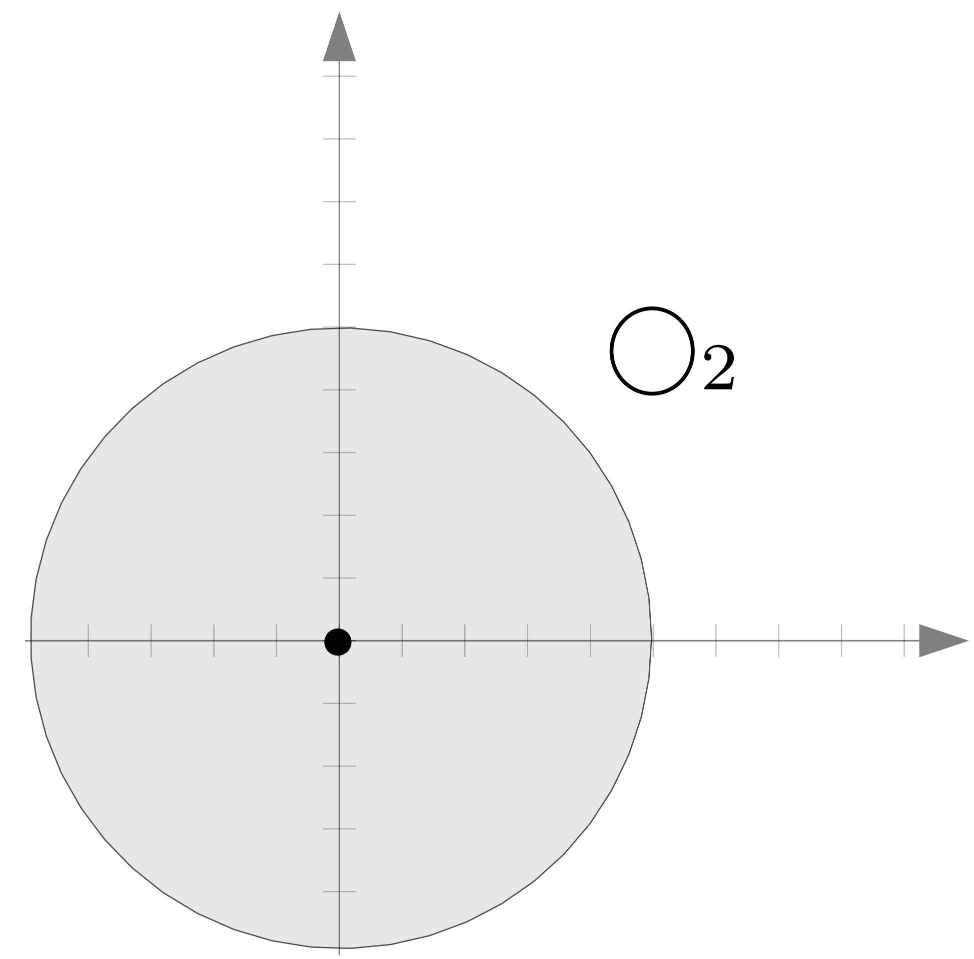

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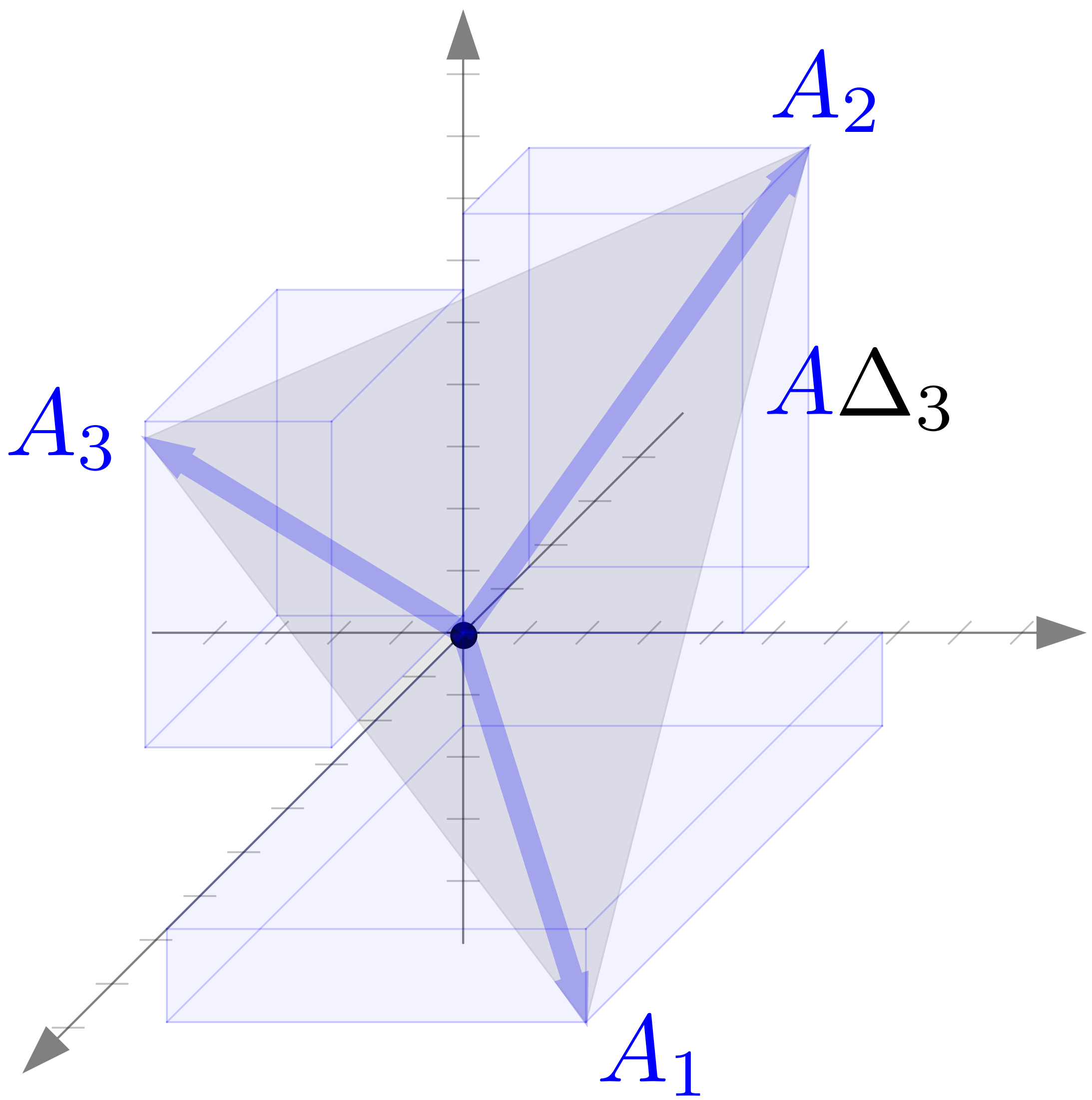

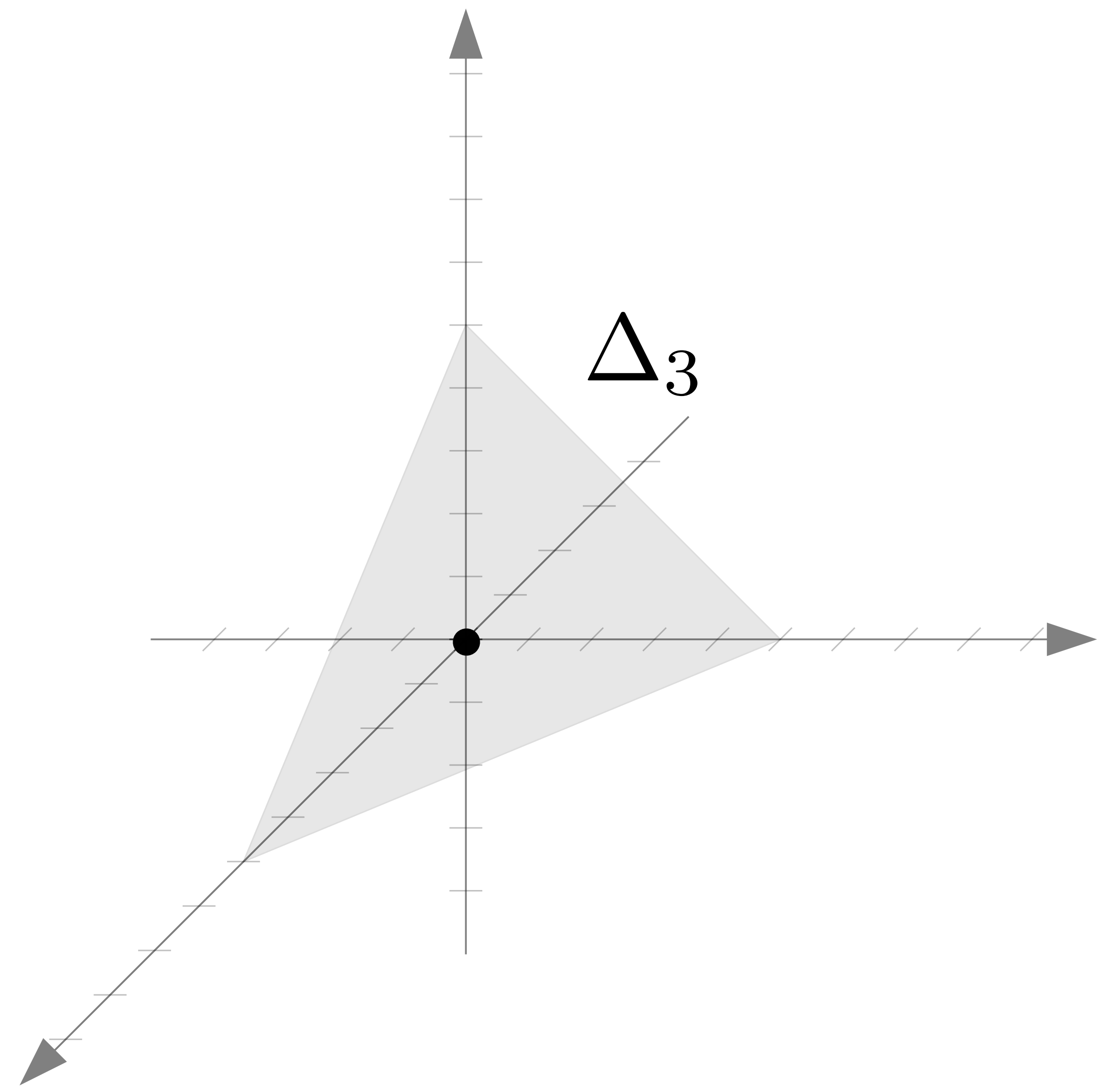

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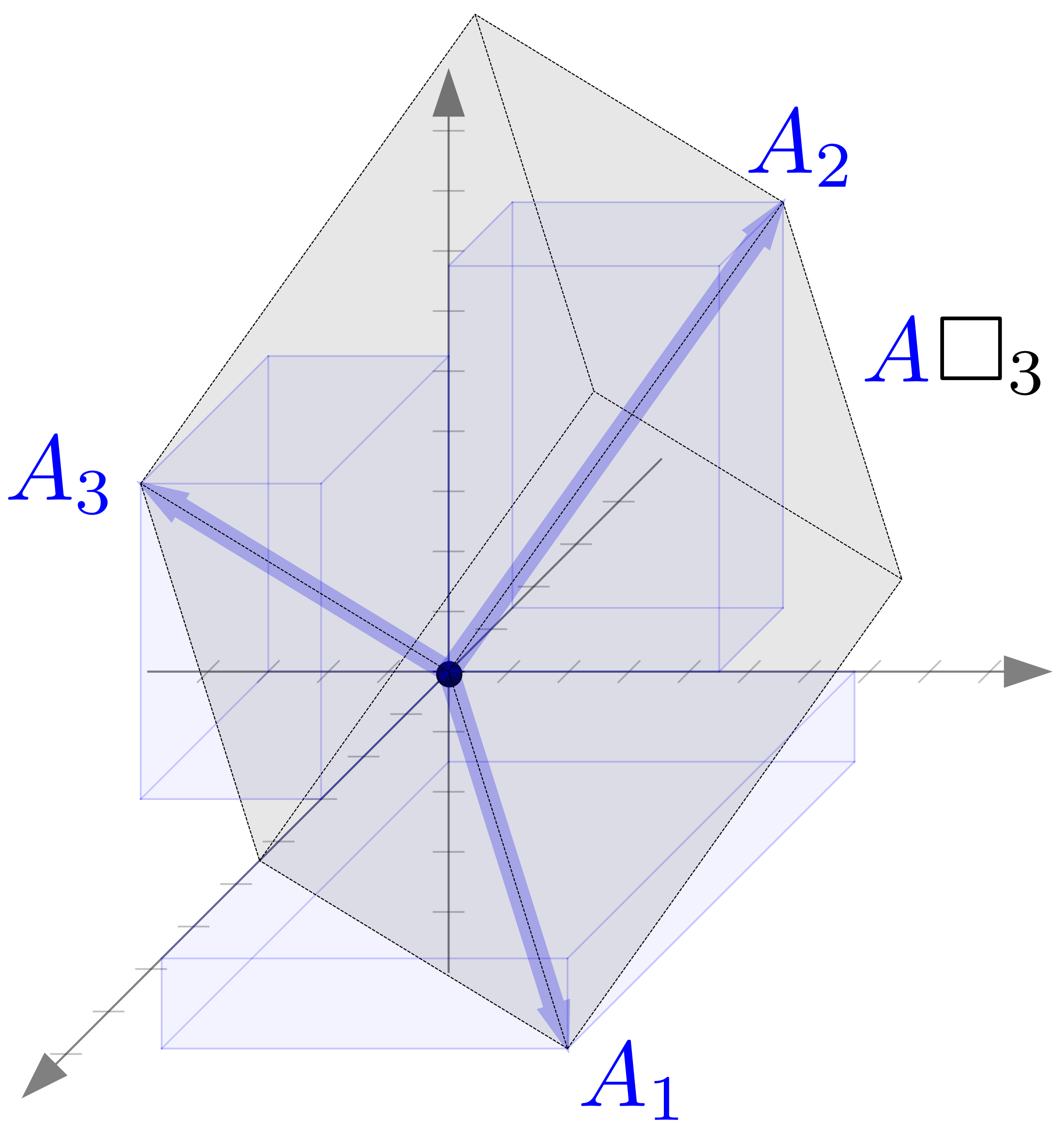

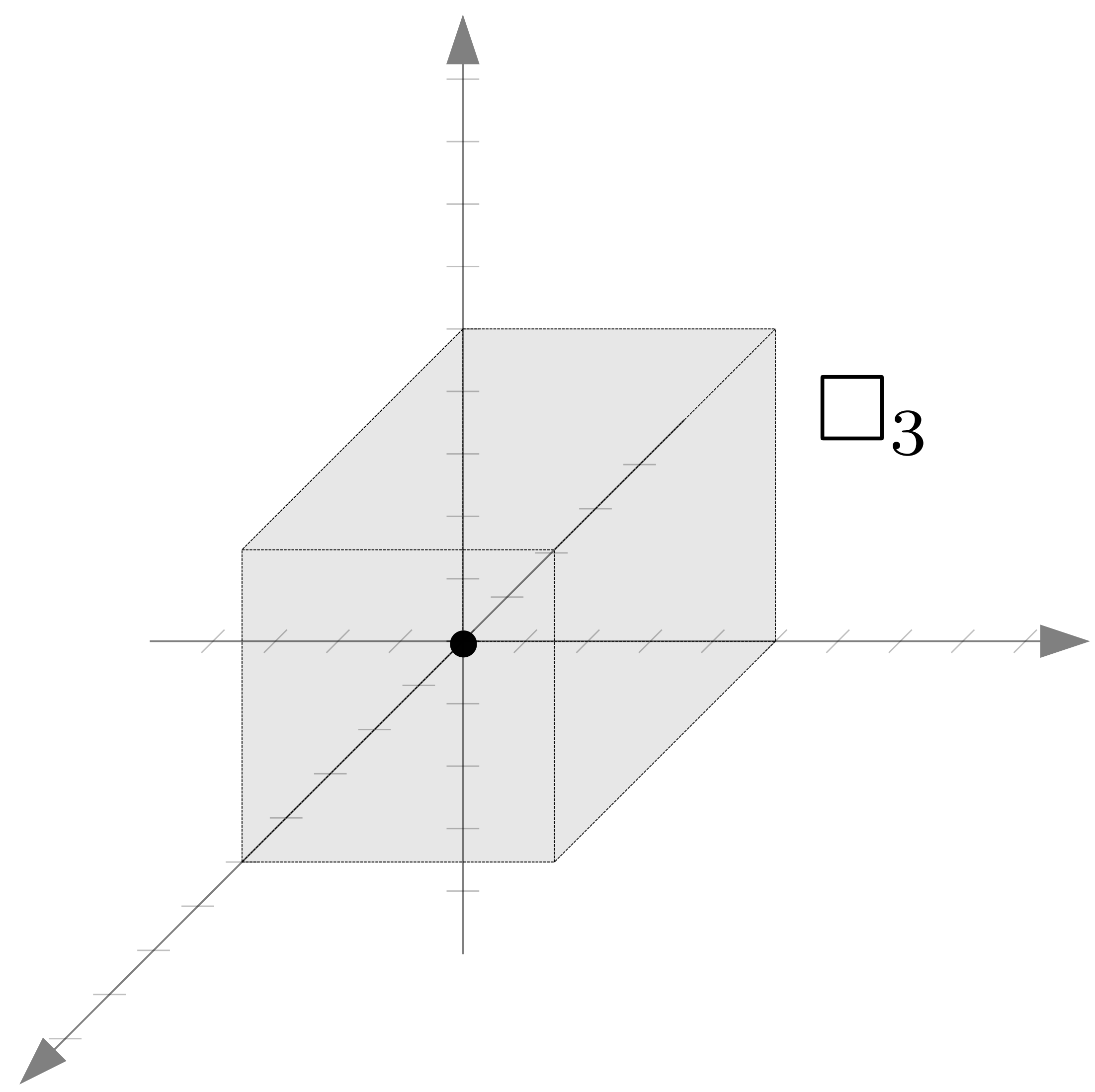

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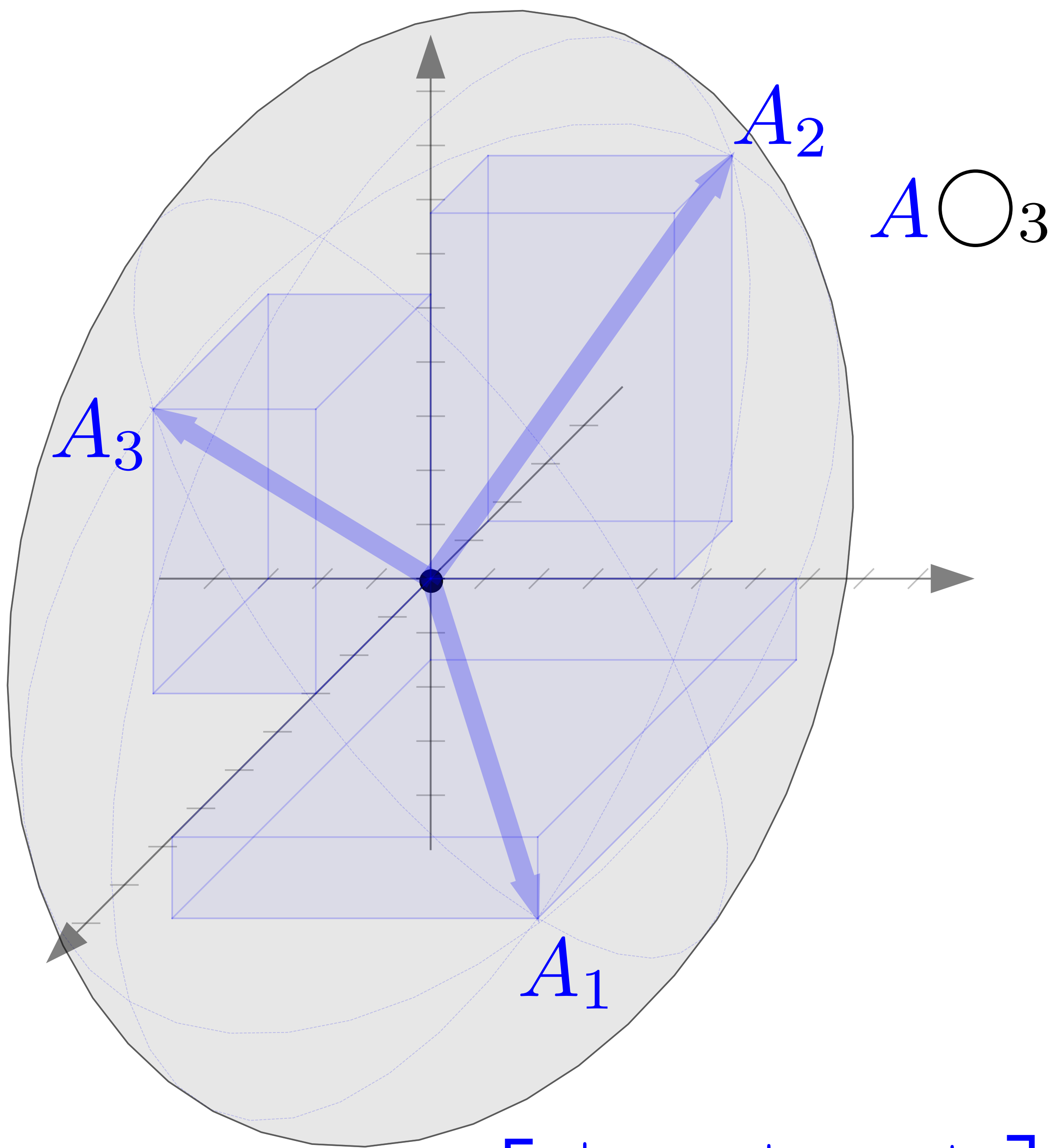

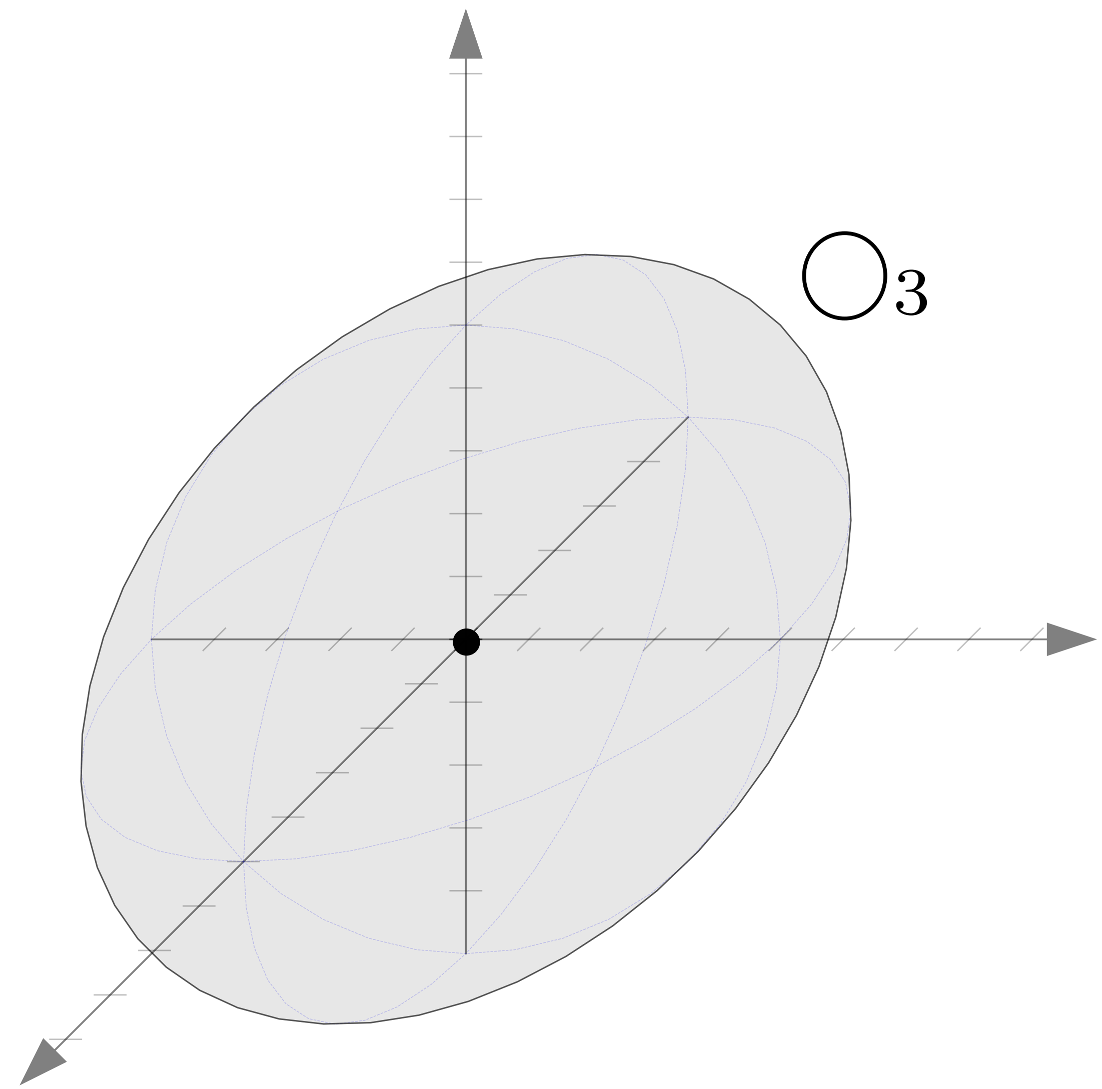

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Fat Matrices

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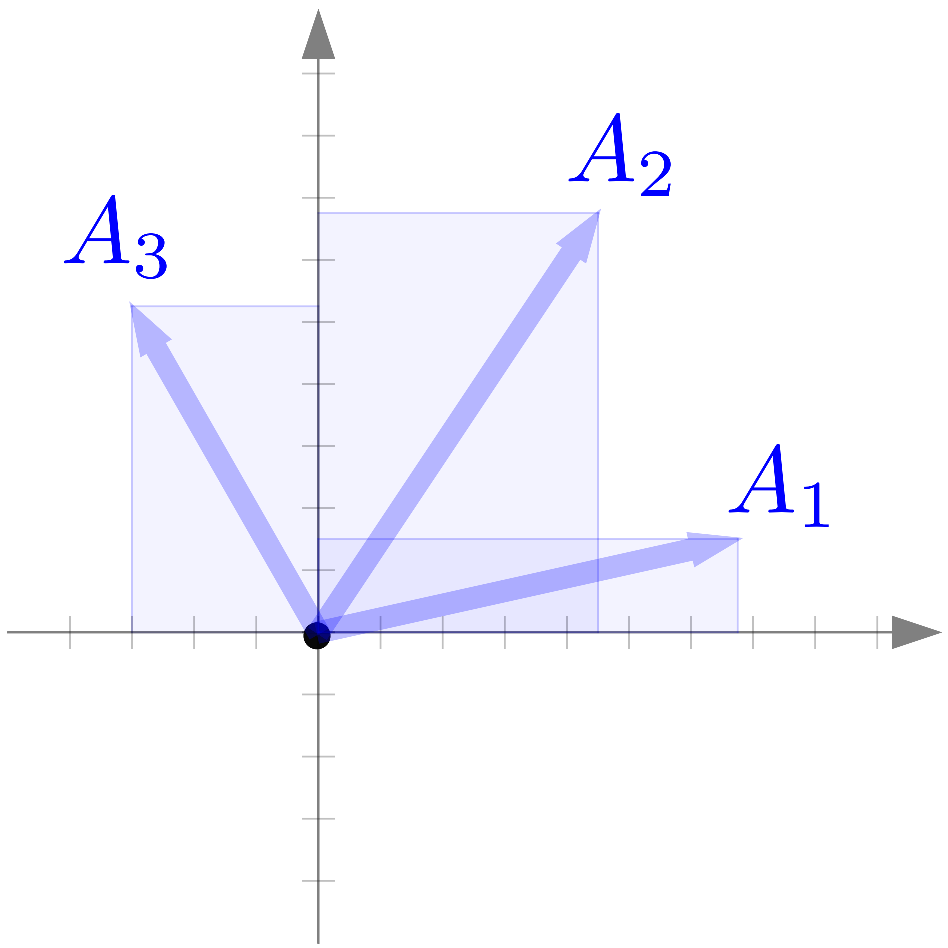

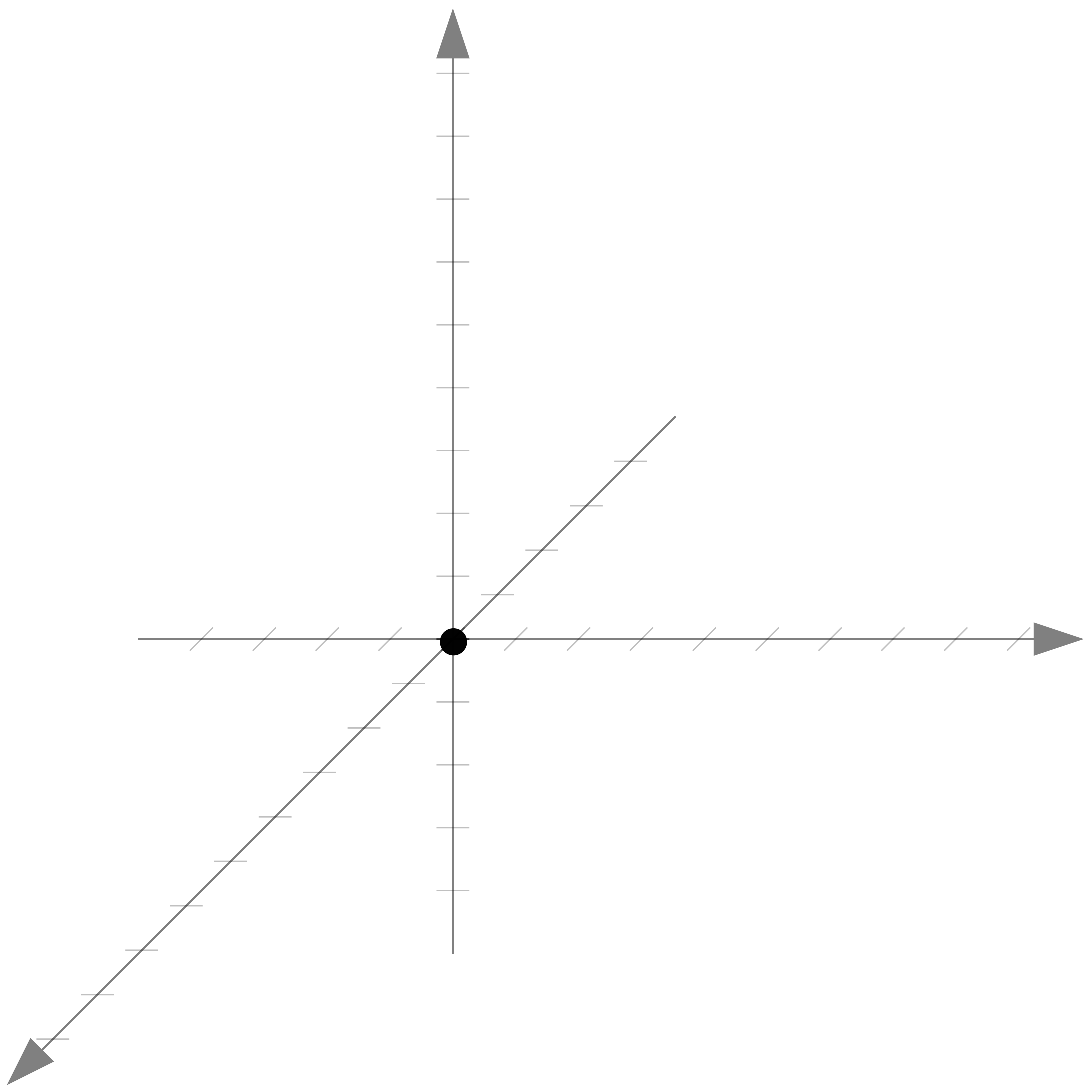

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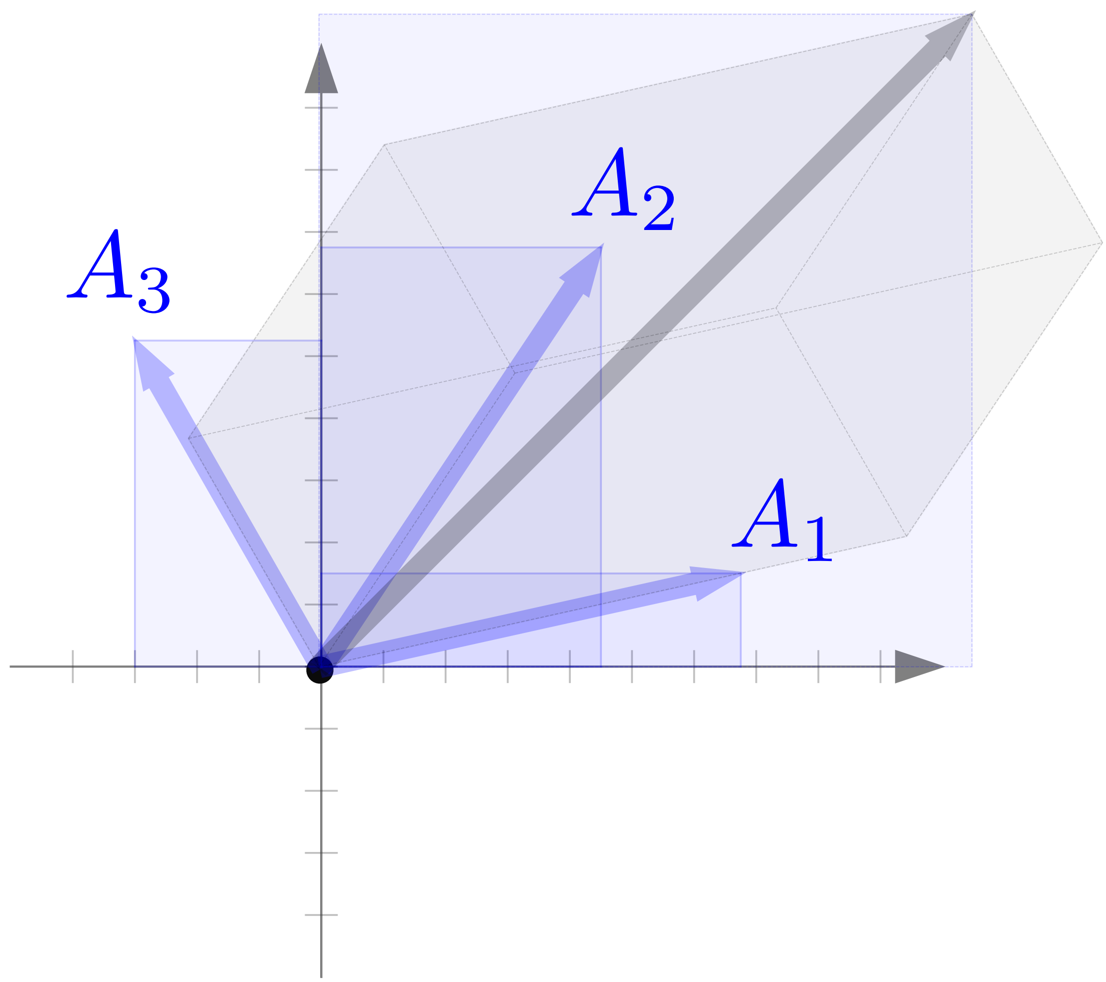

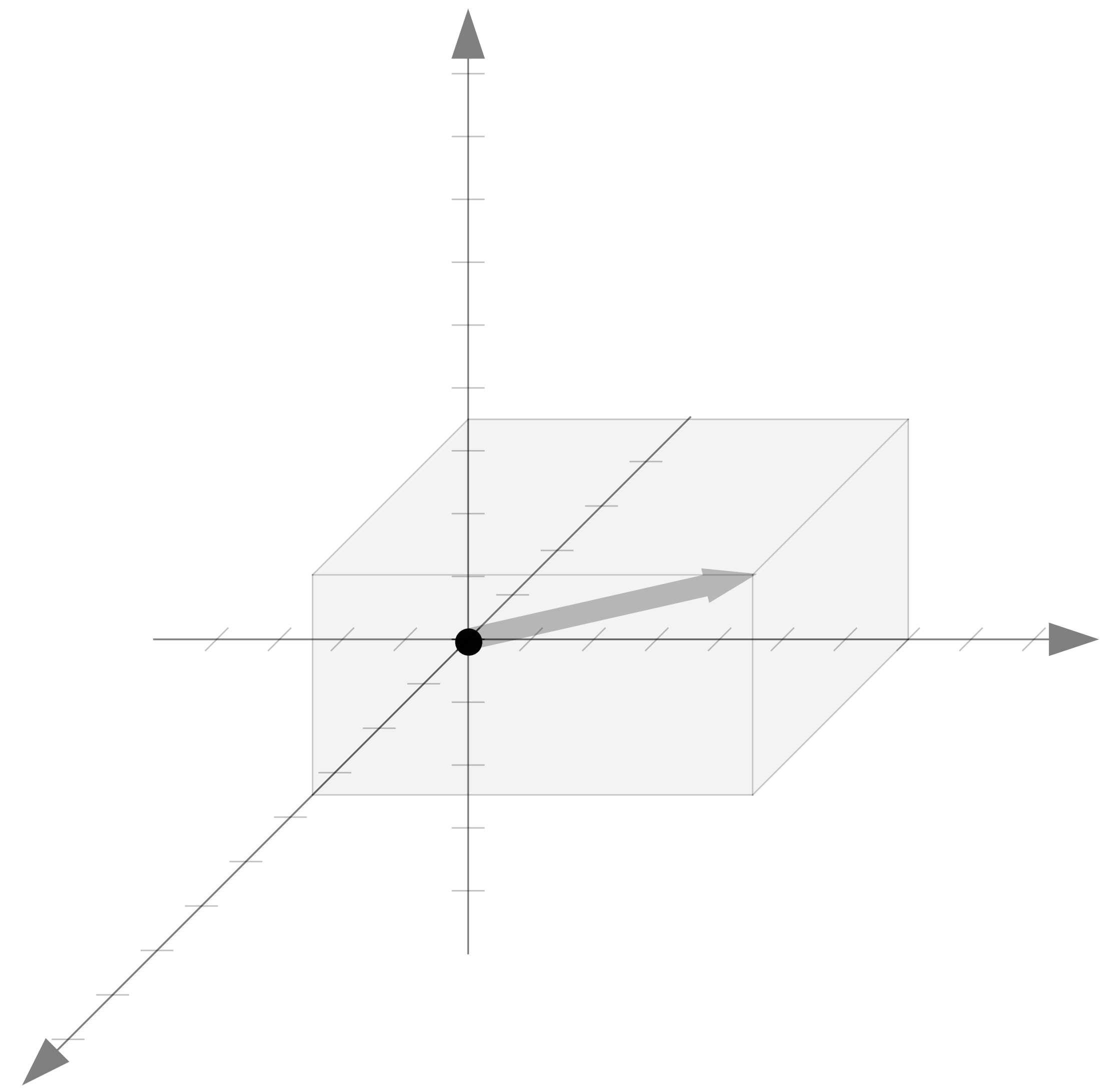

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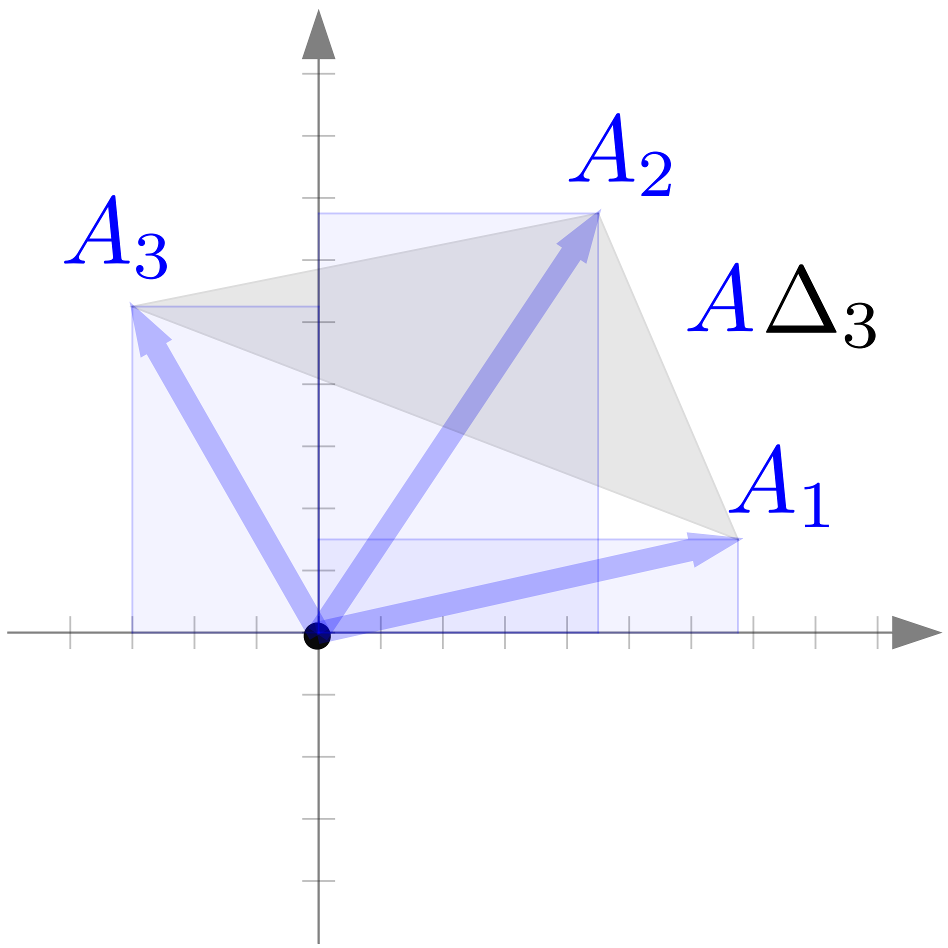

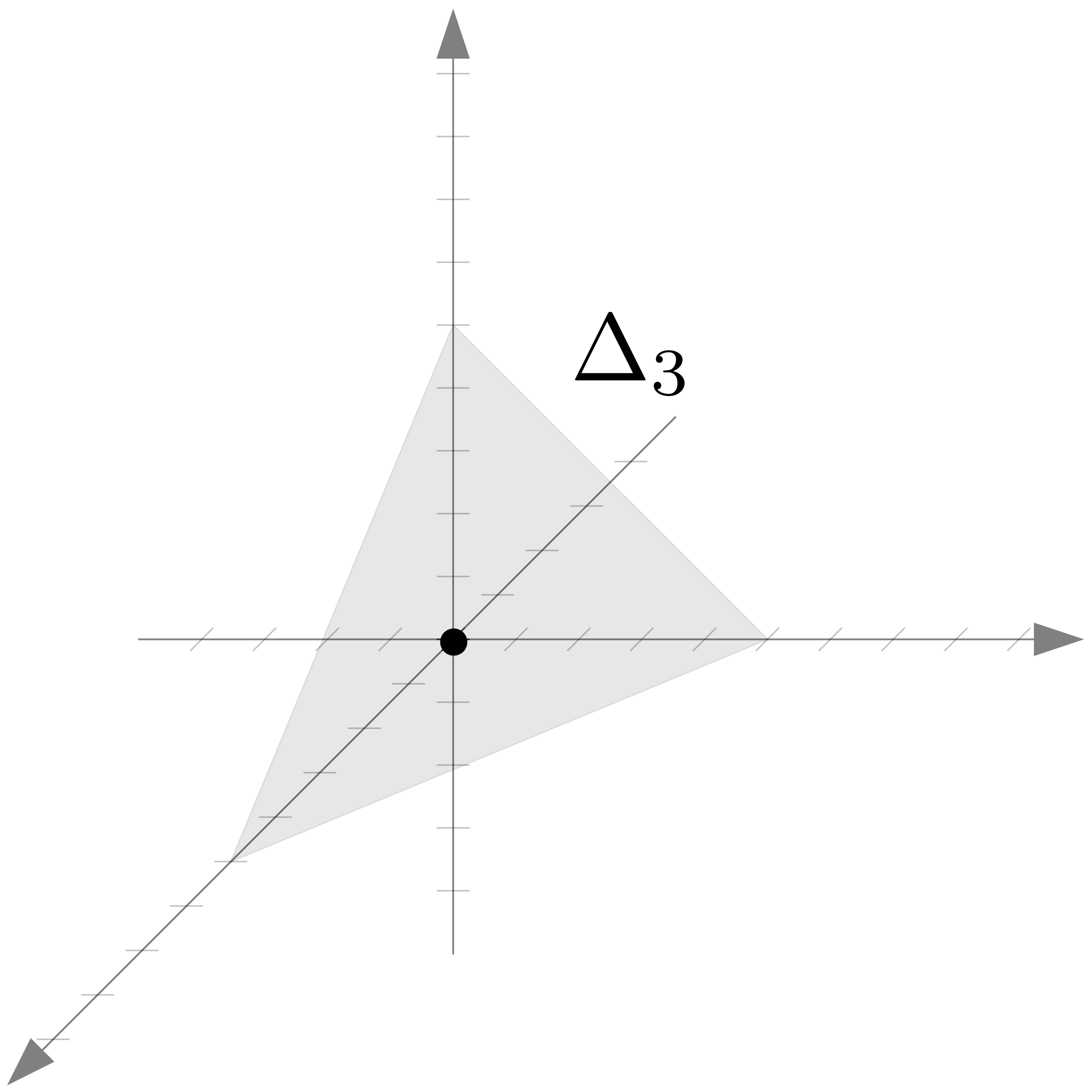

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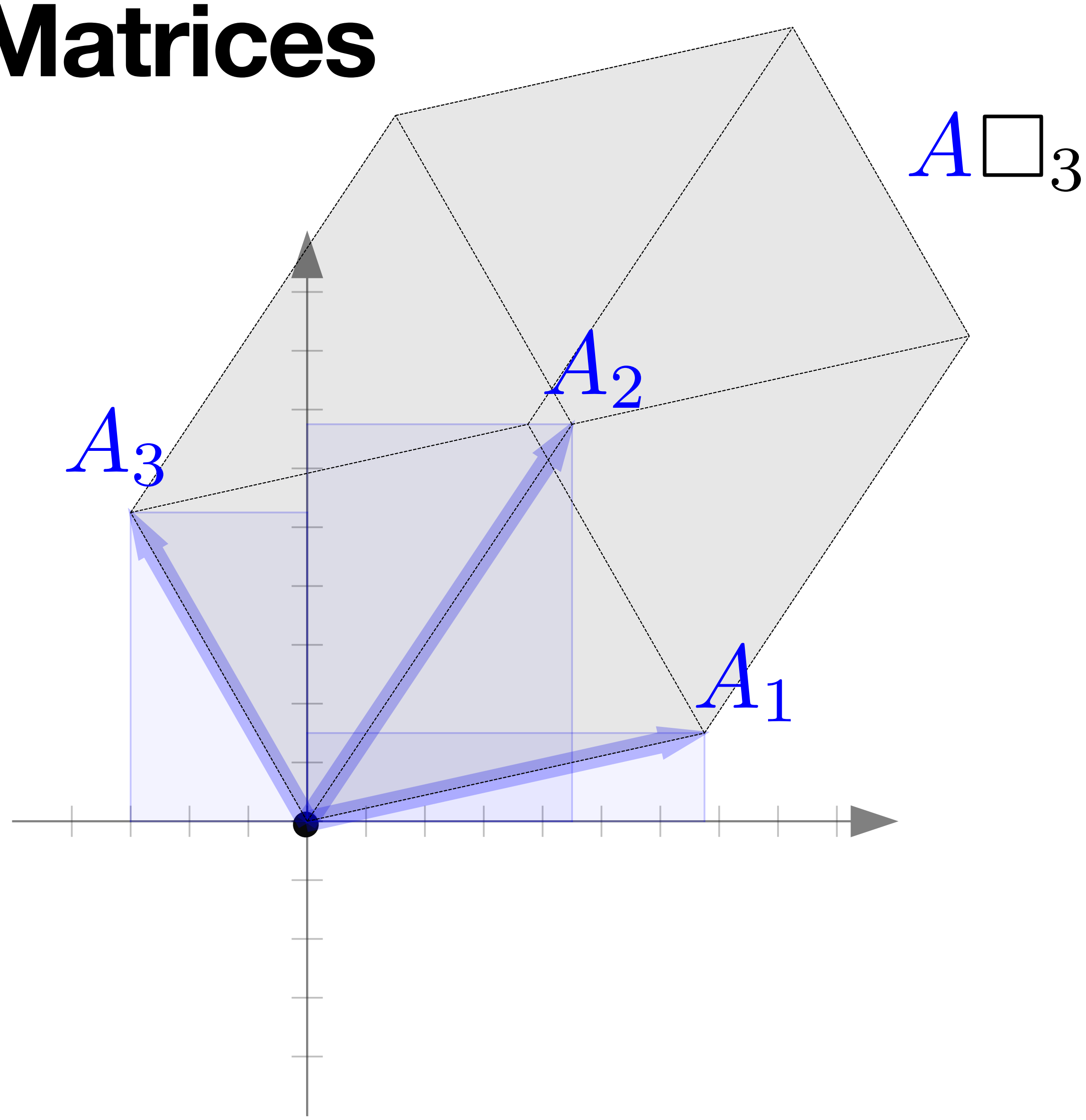

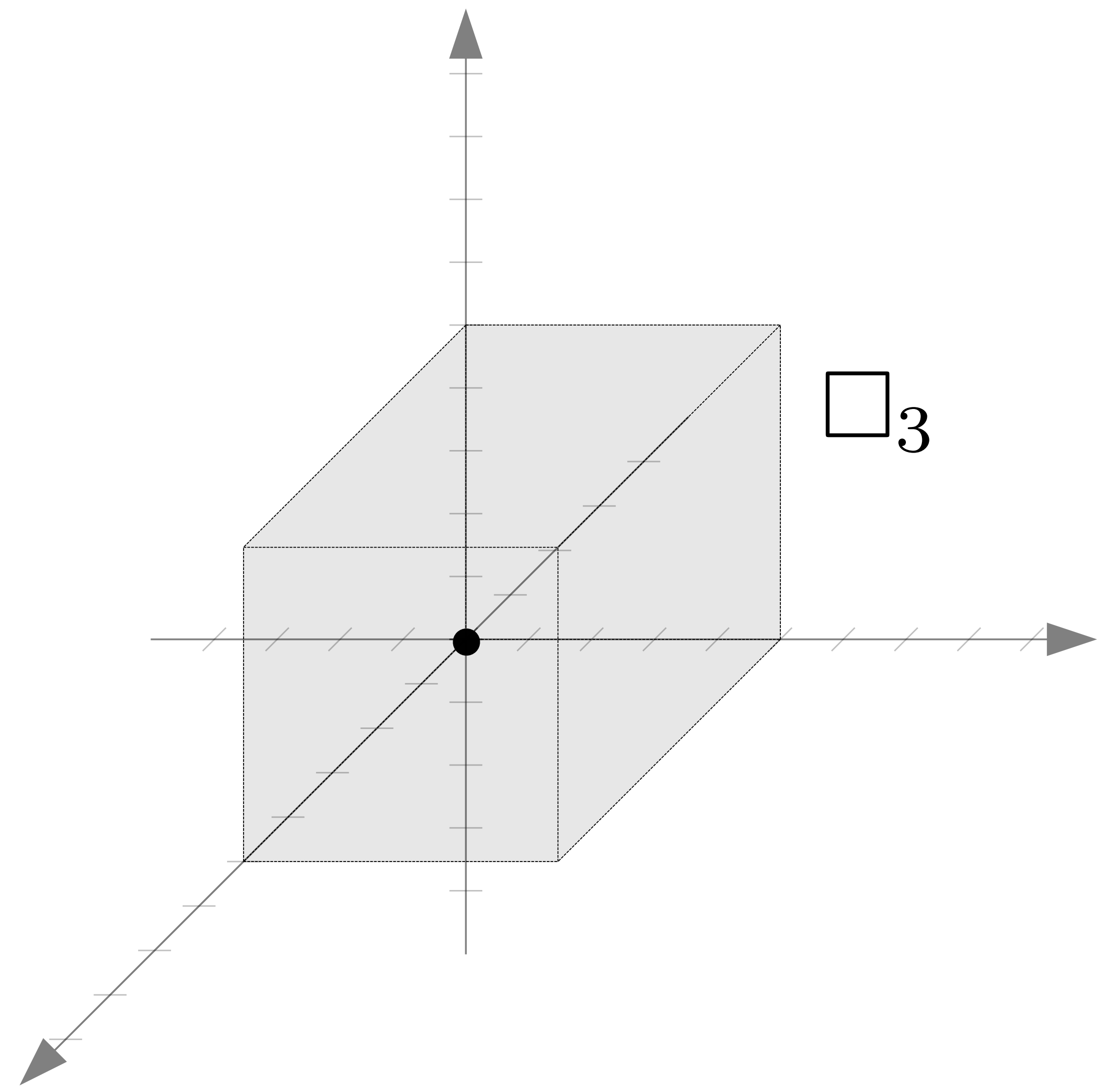

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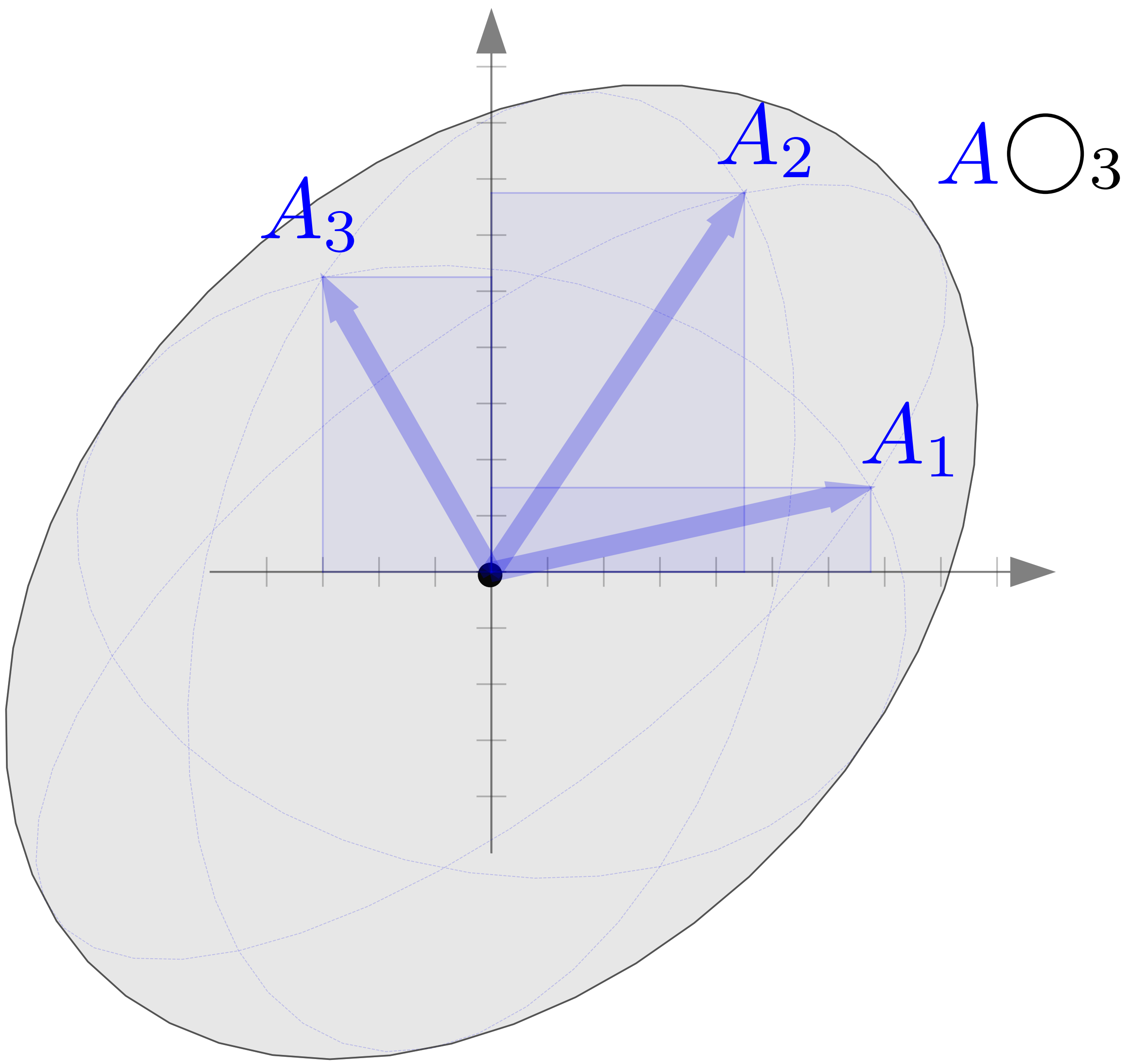

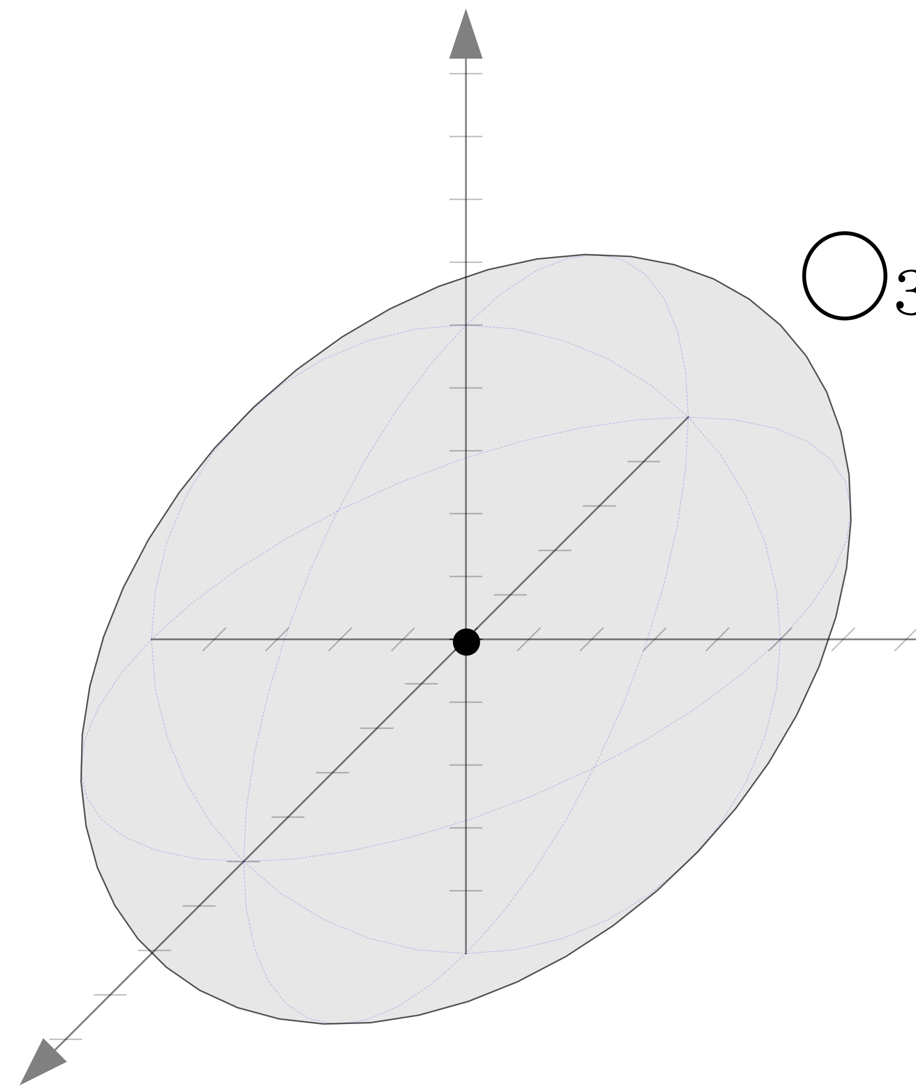

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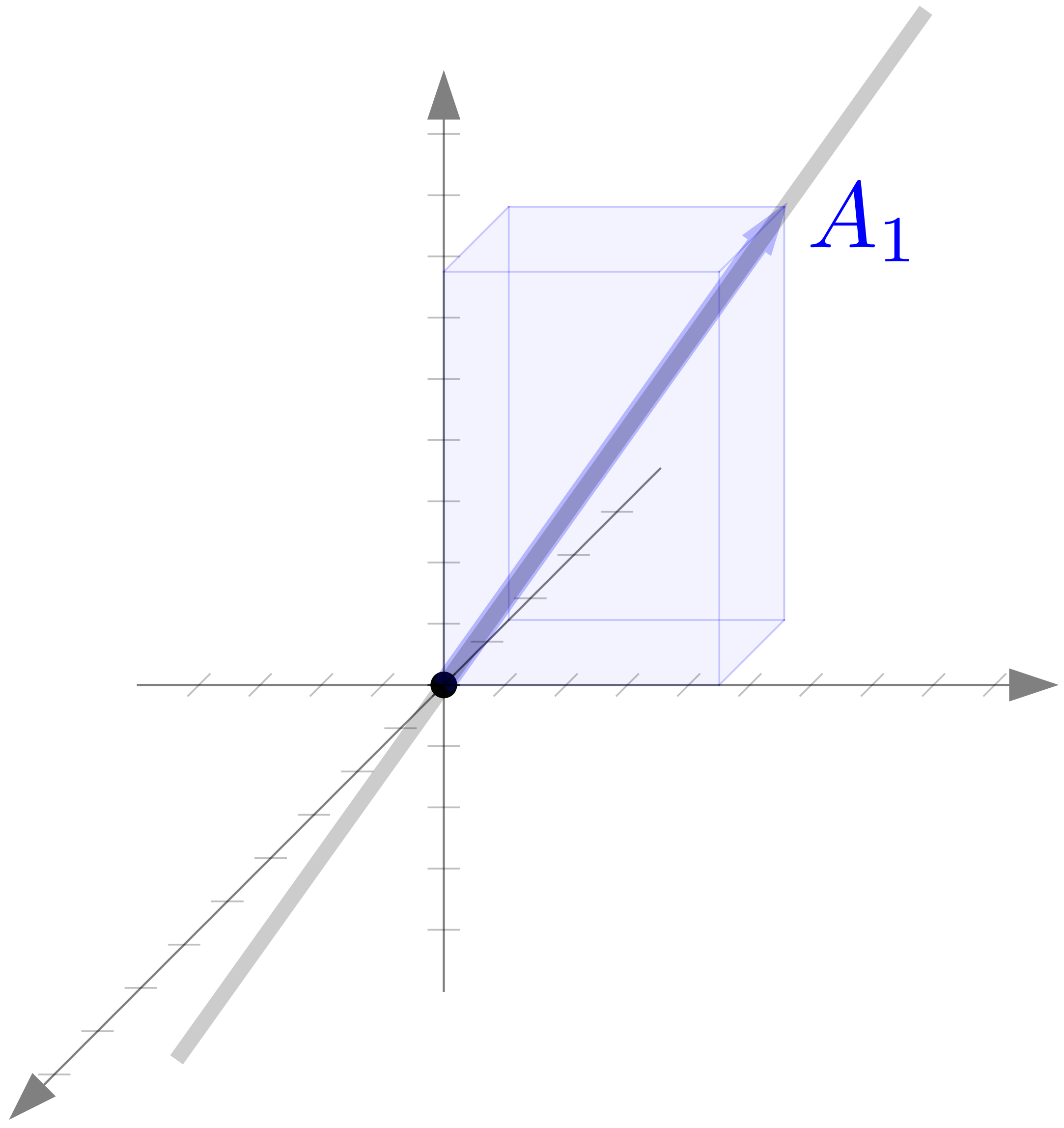

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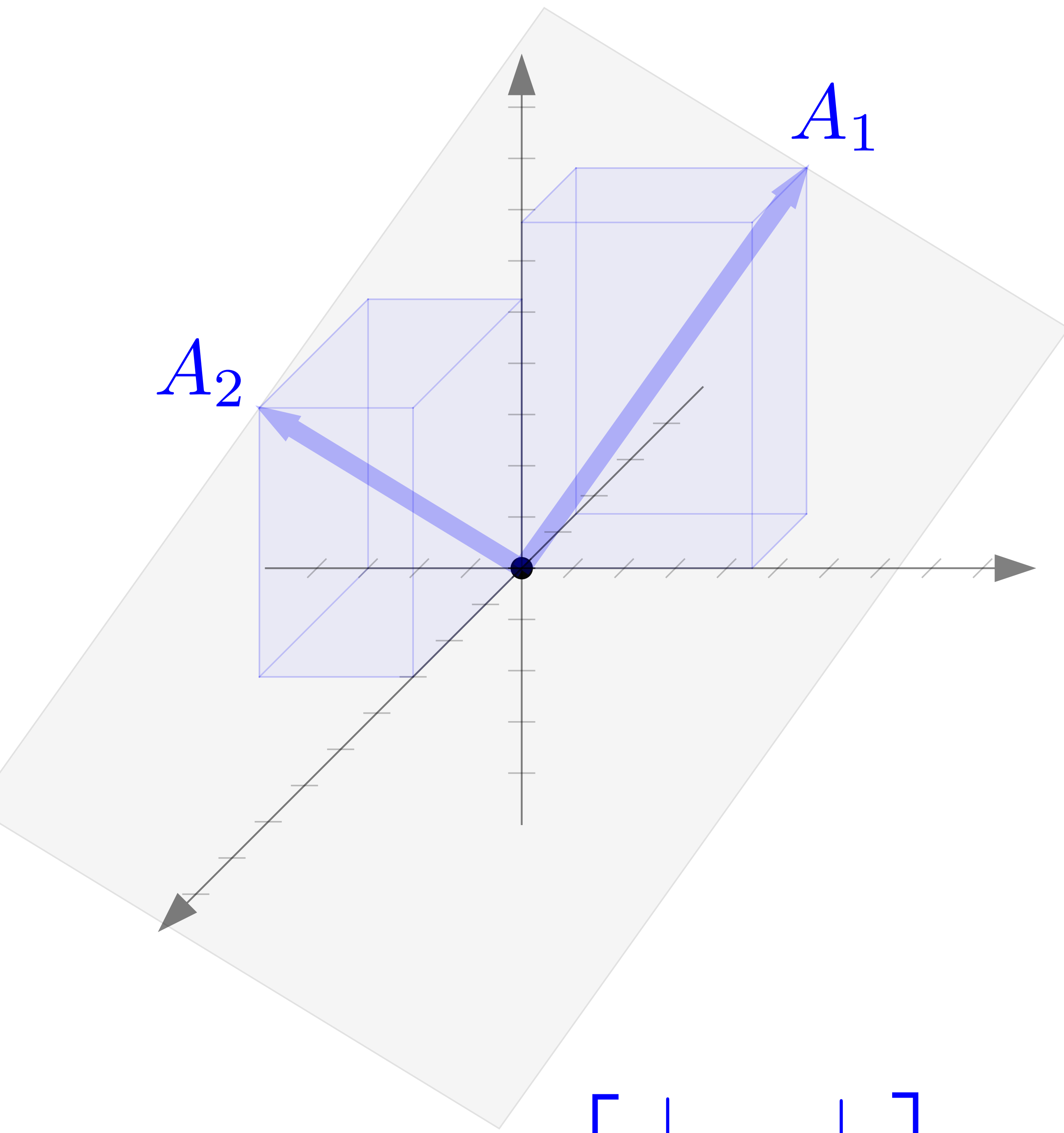

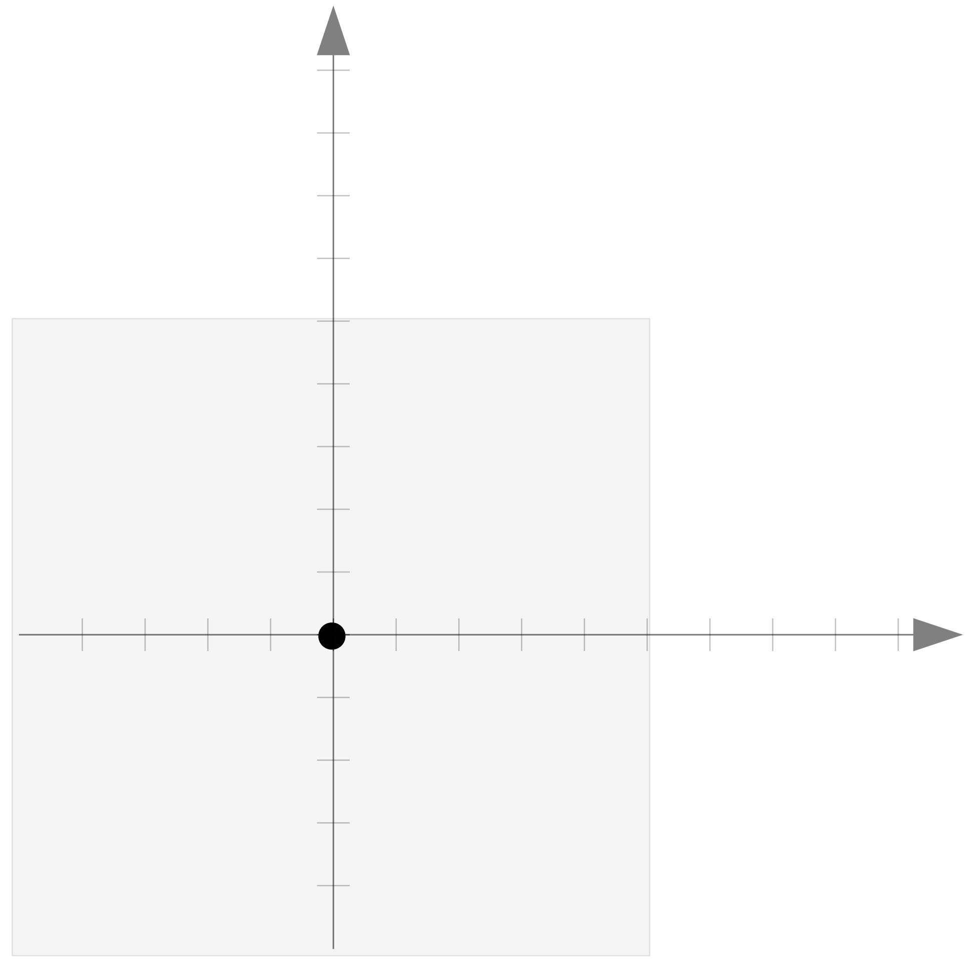

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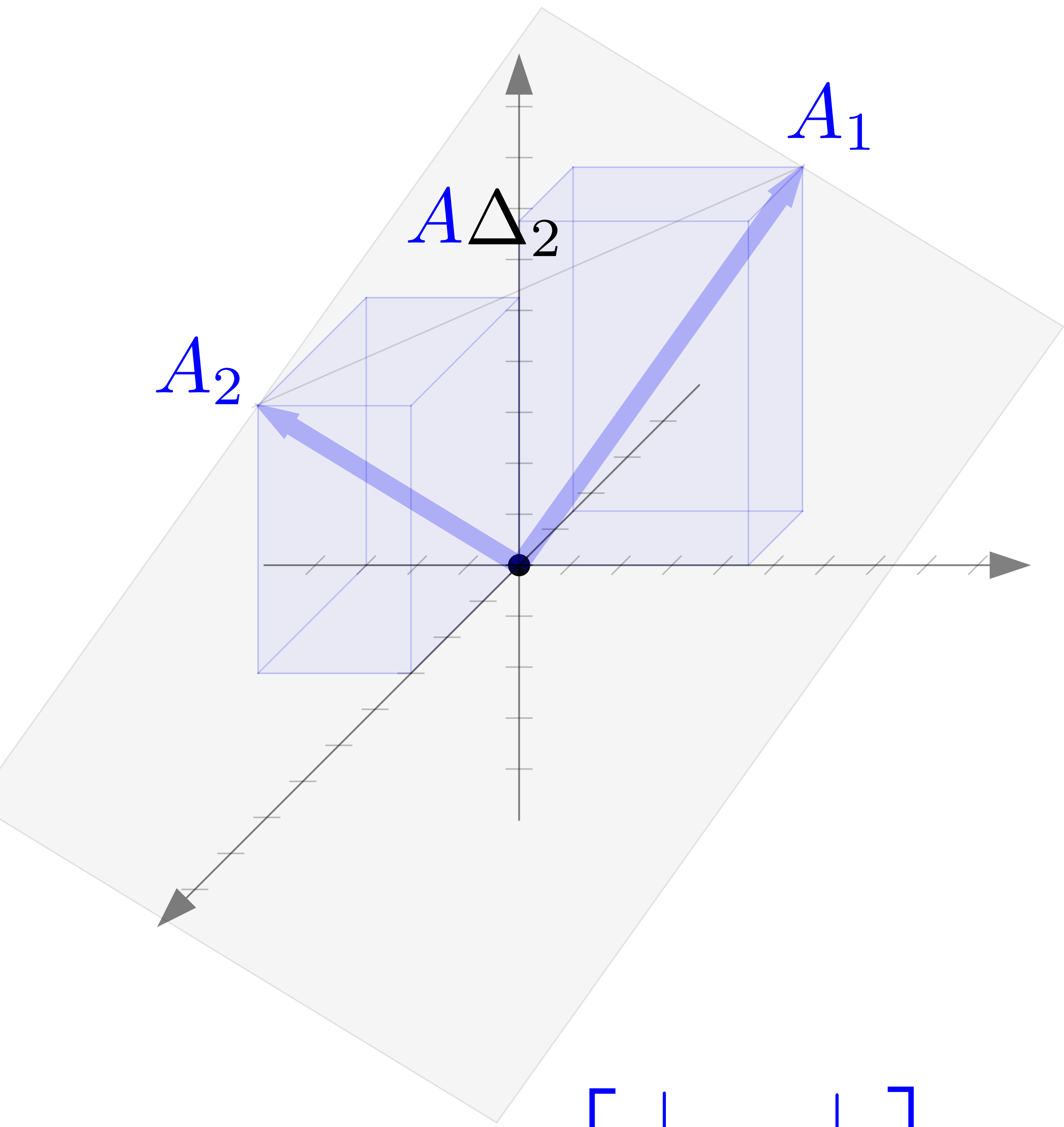

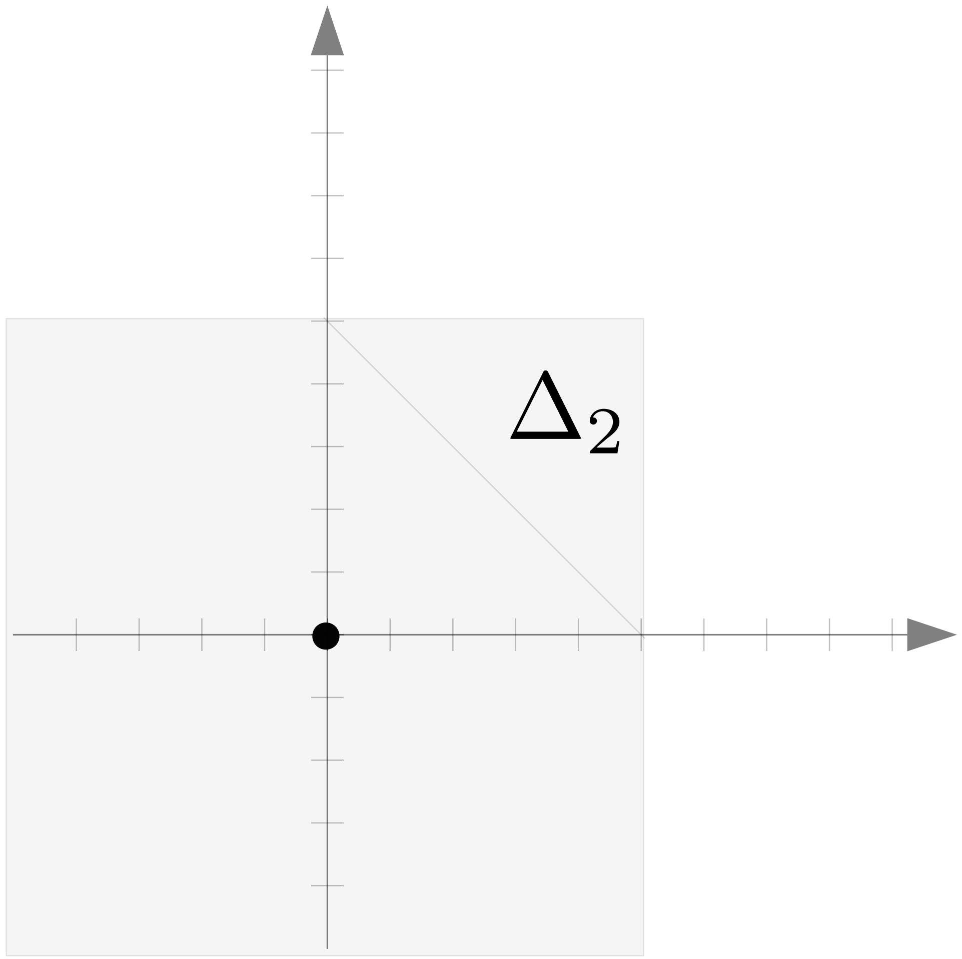

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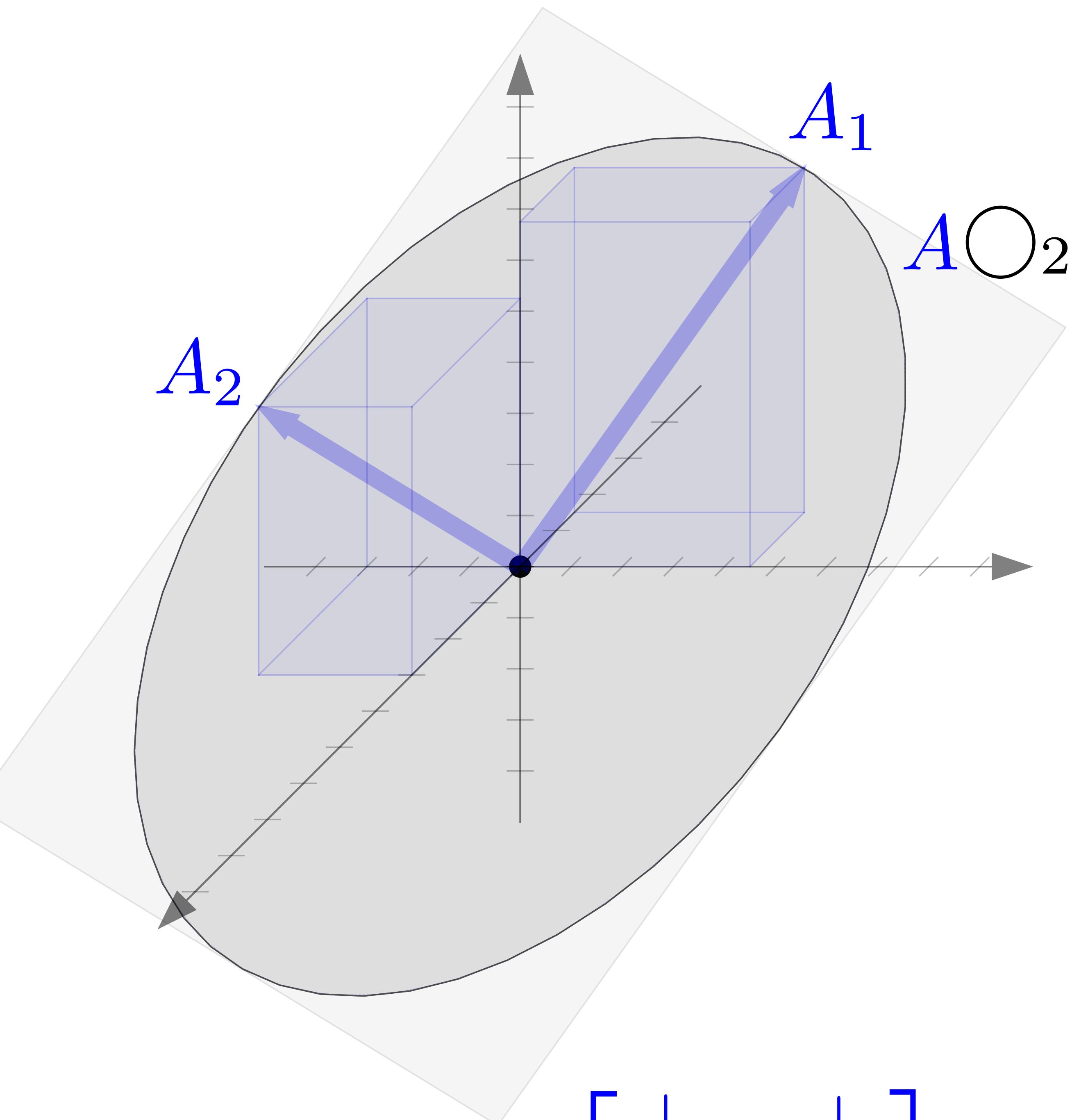

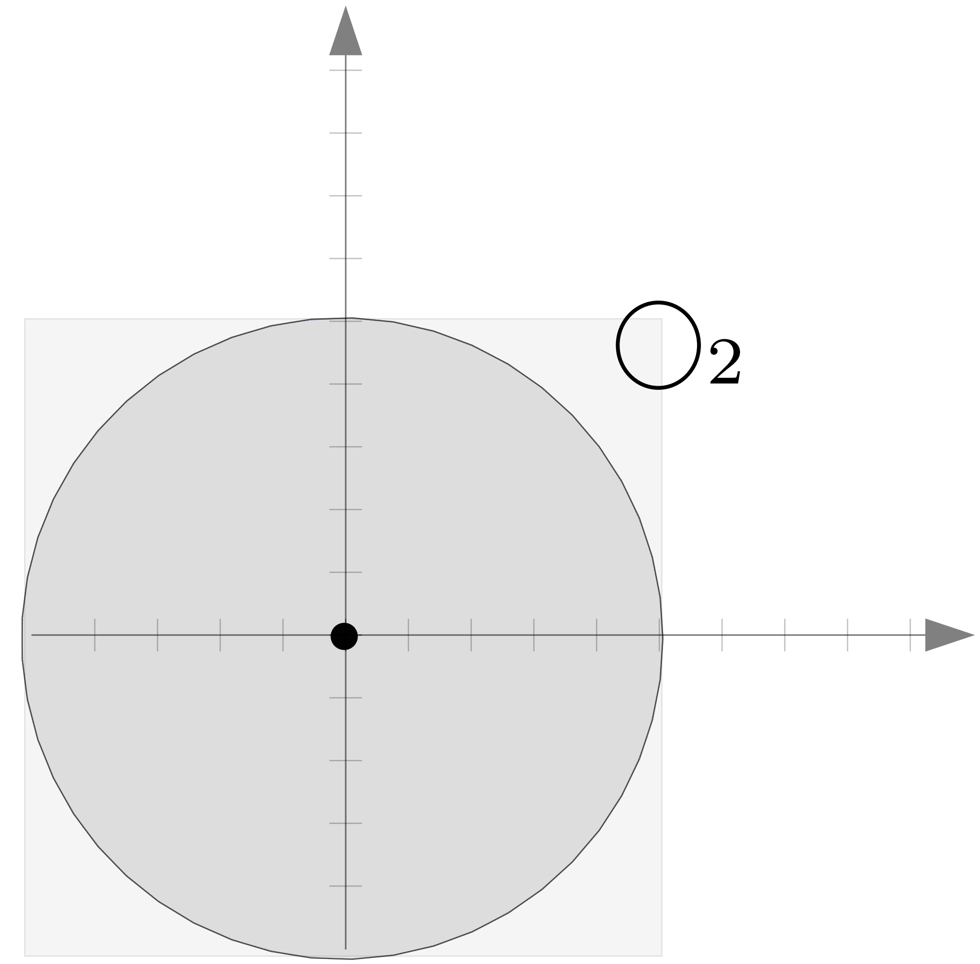

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Low-Rank Matrices

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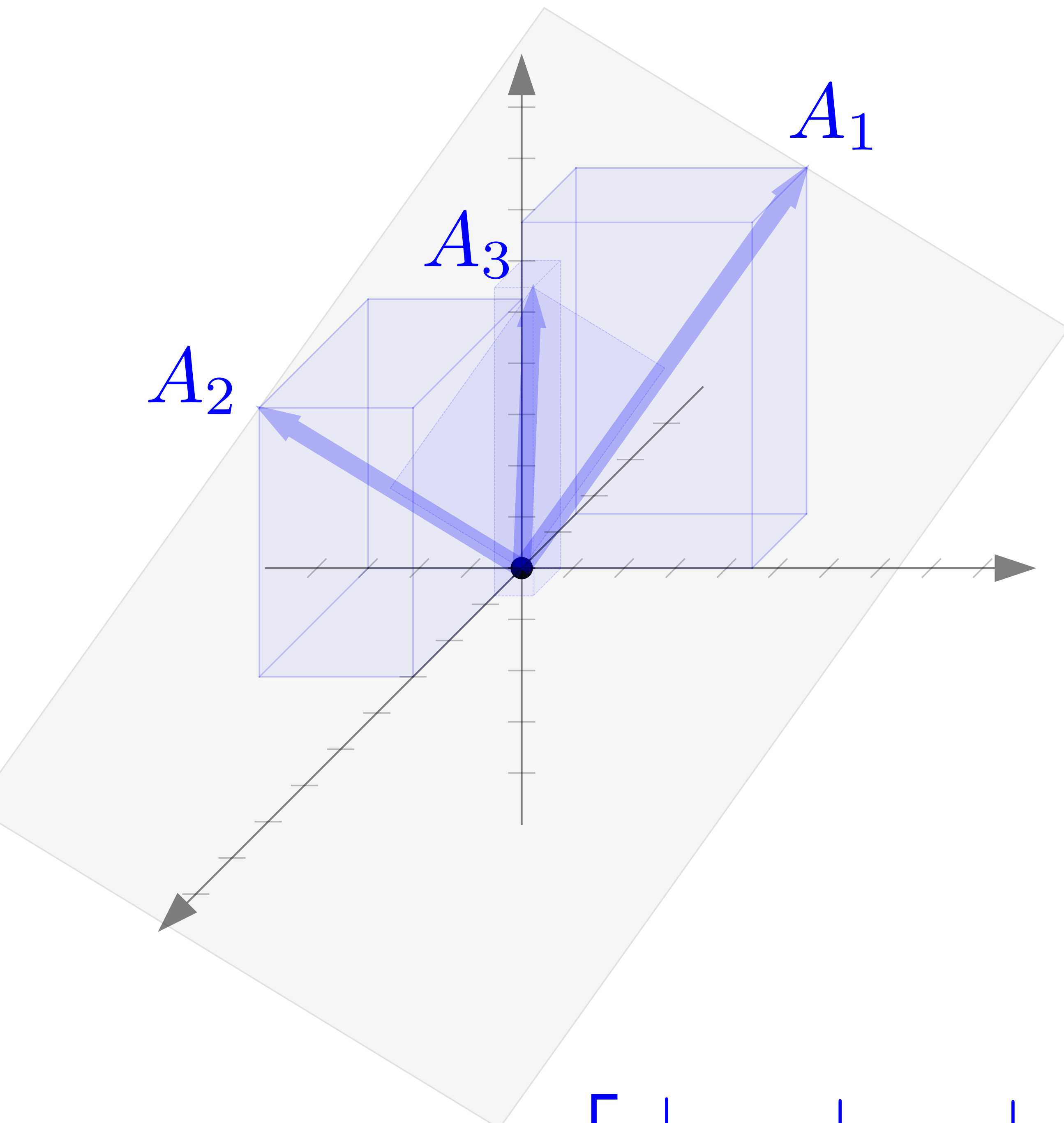

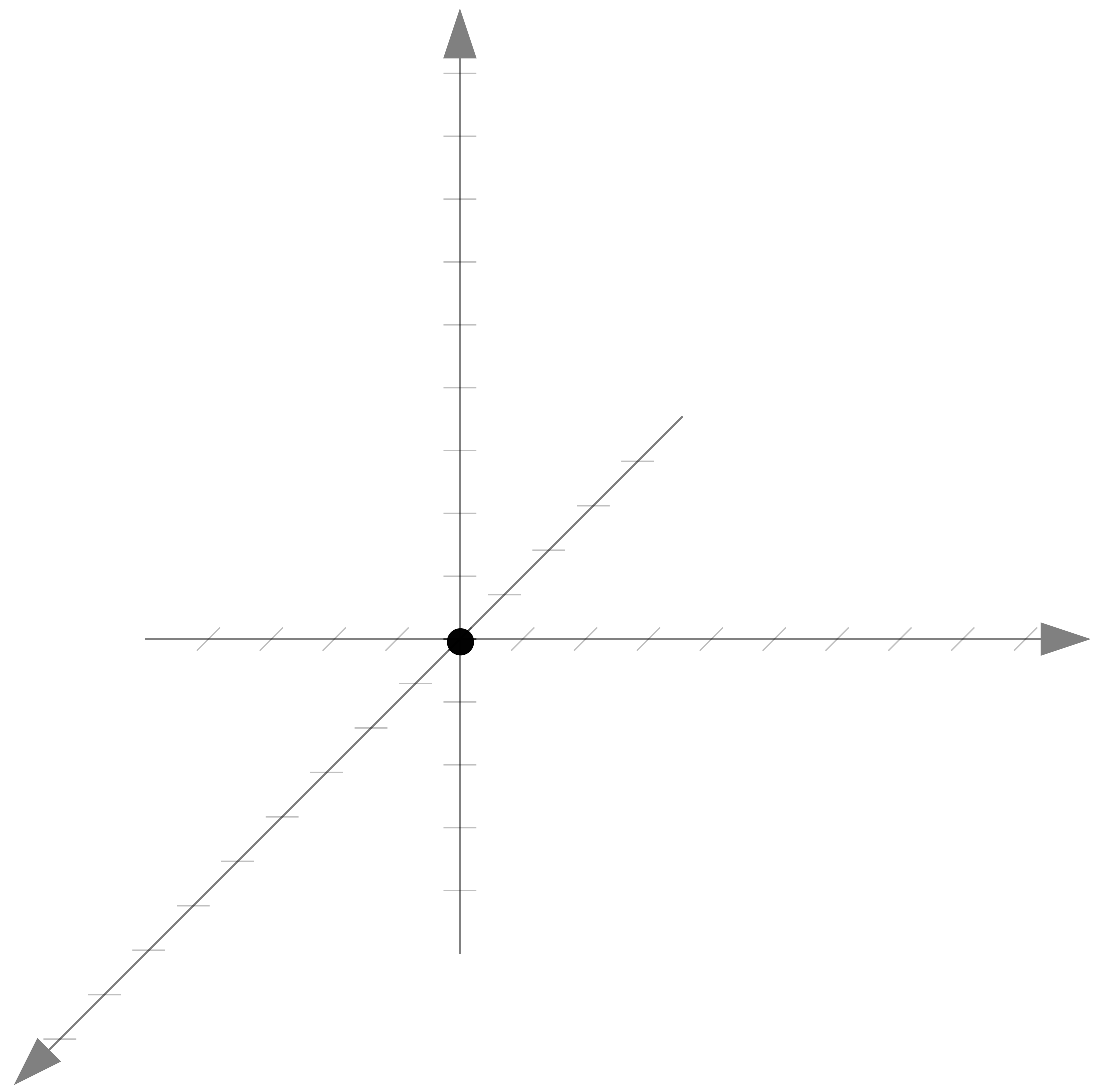

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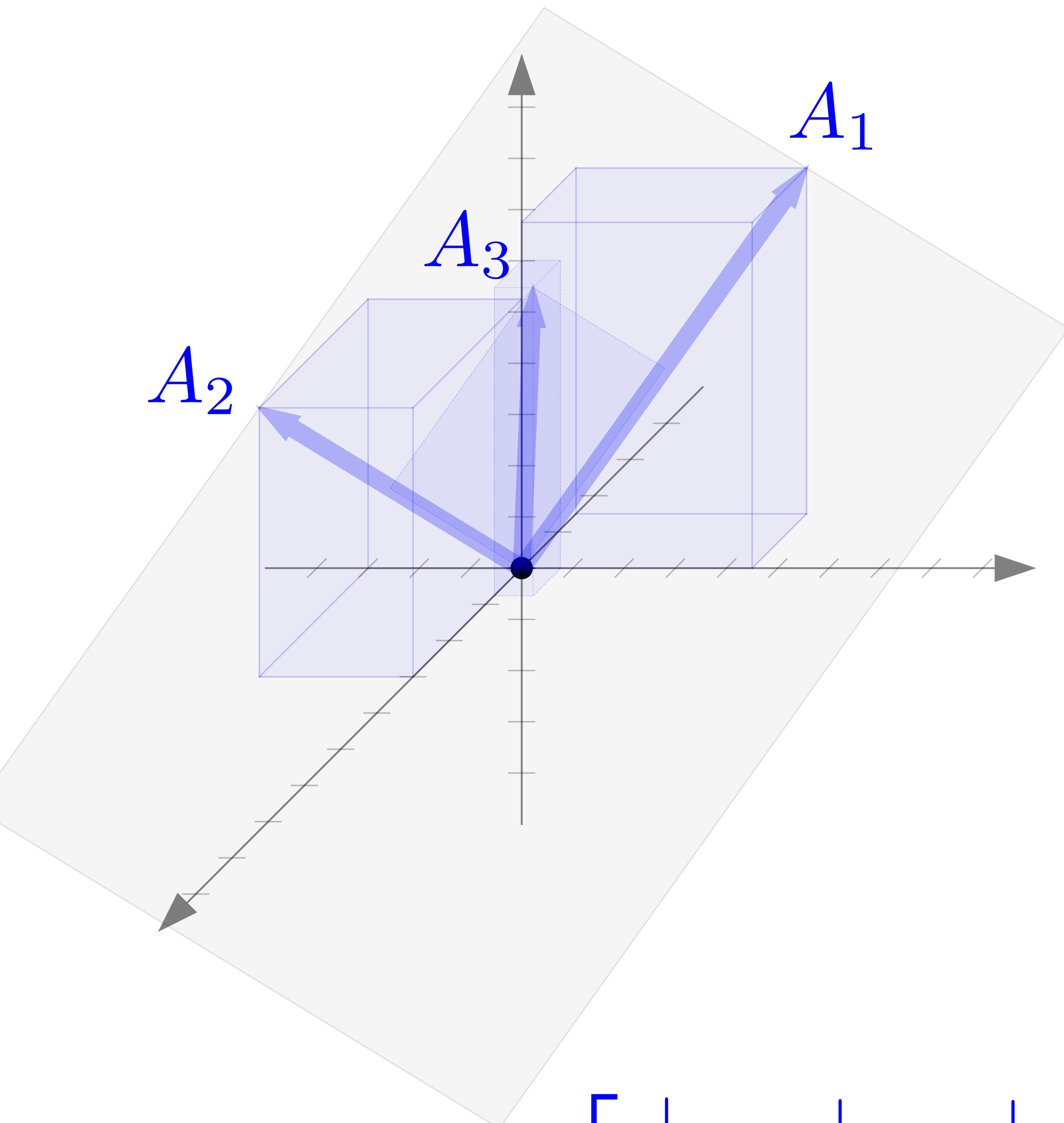

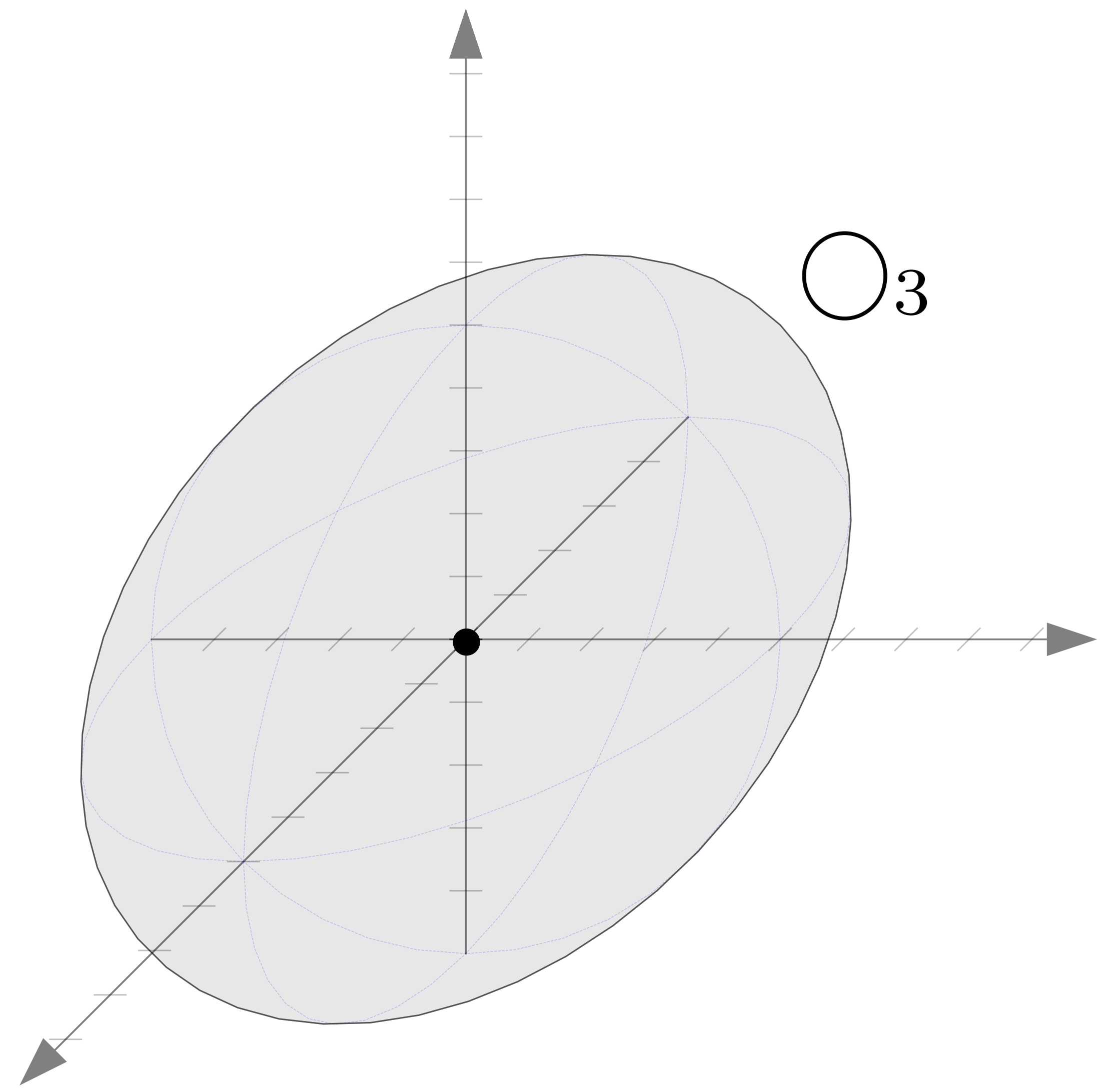

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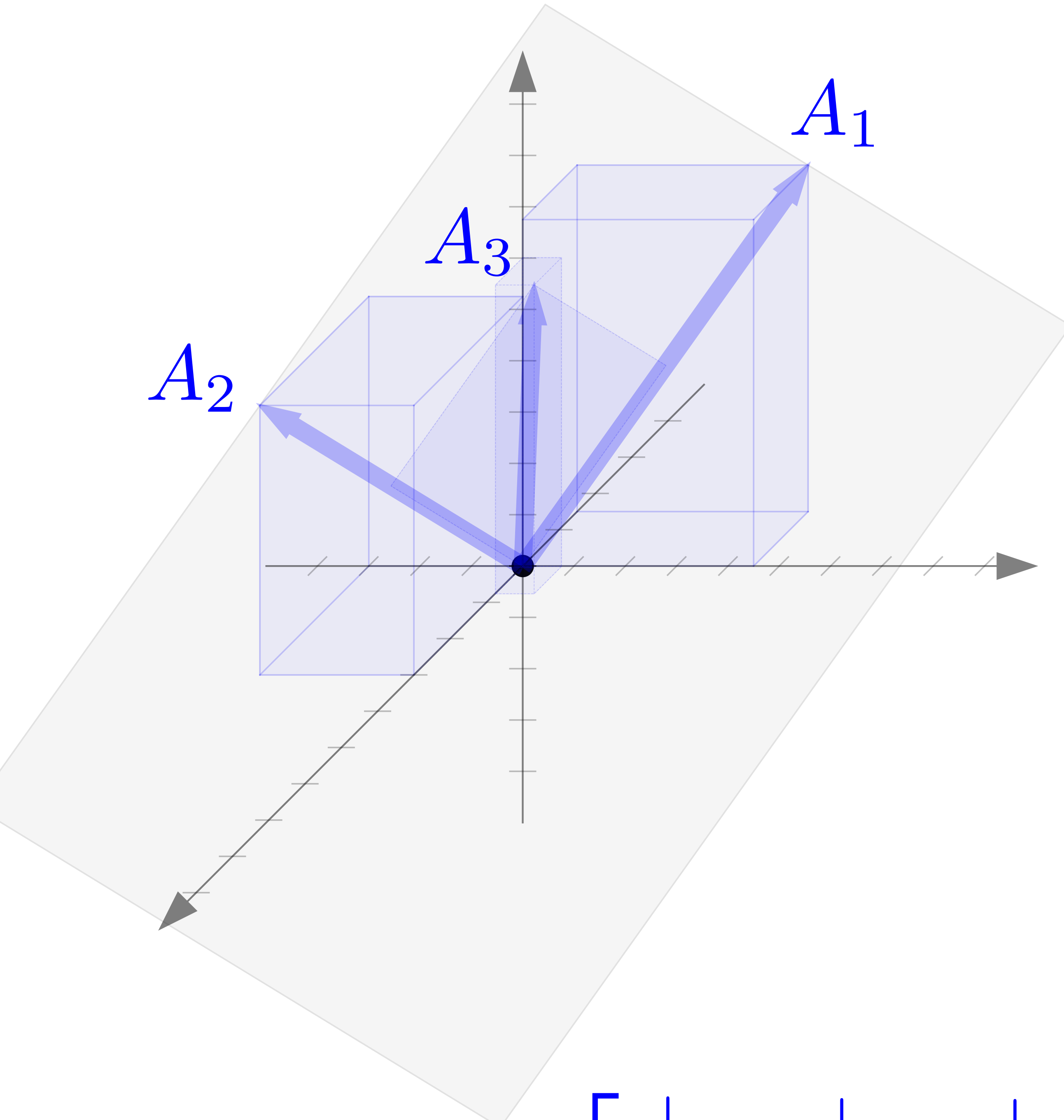

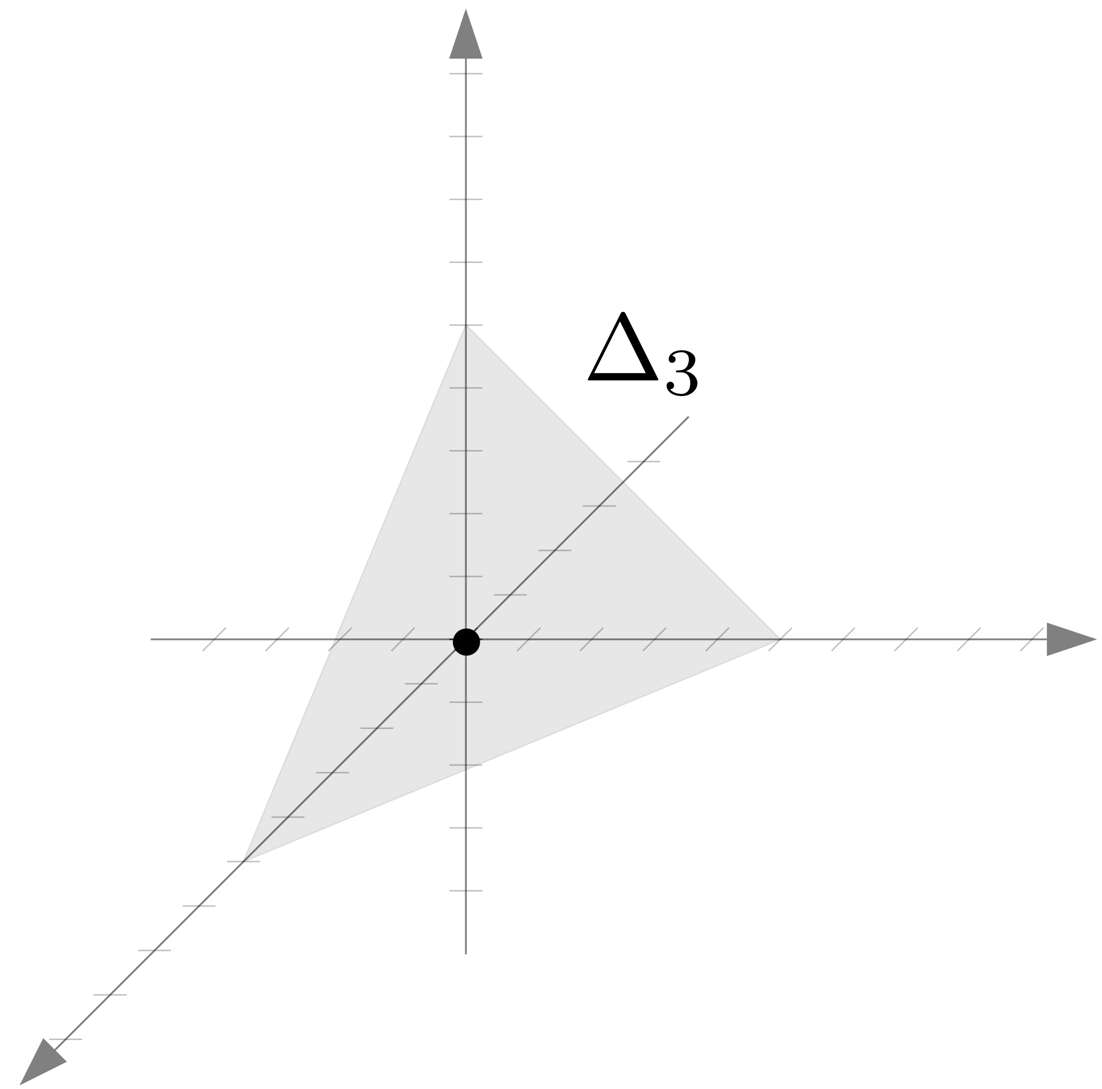

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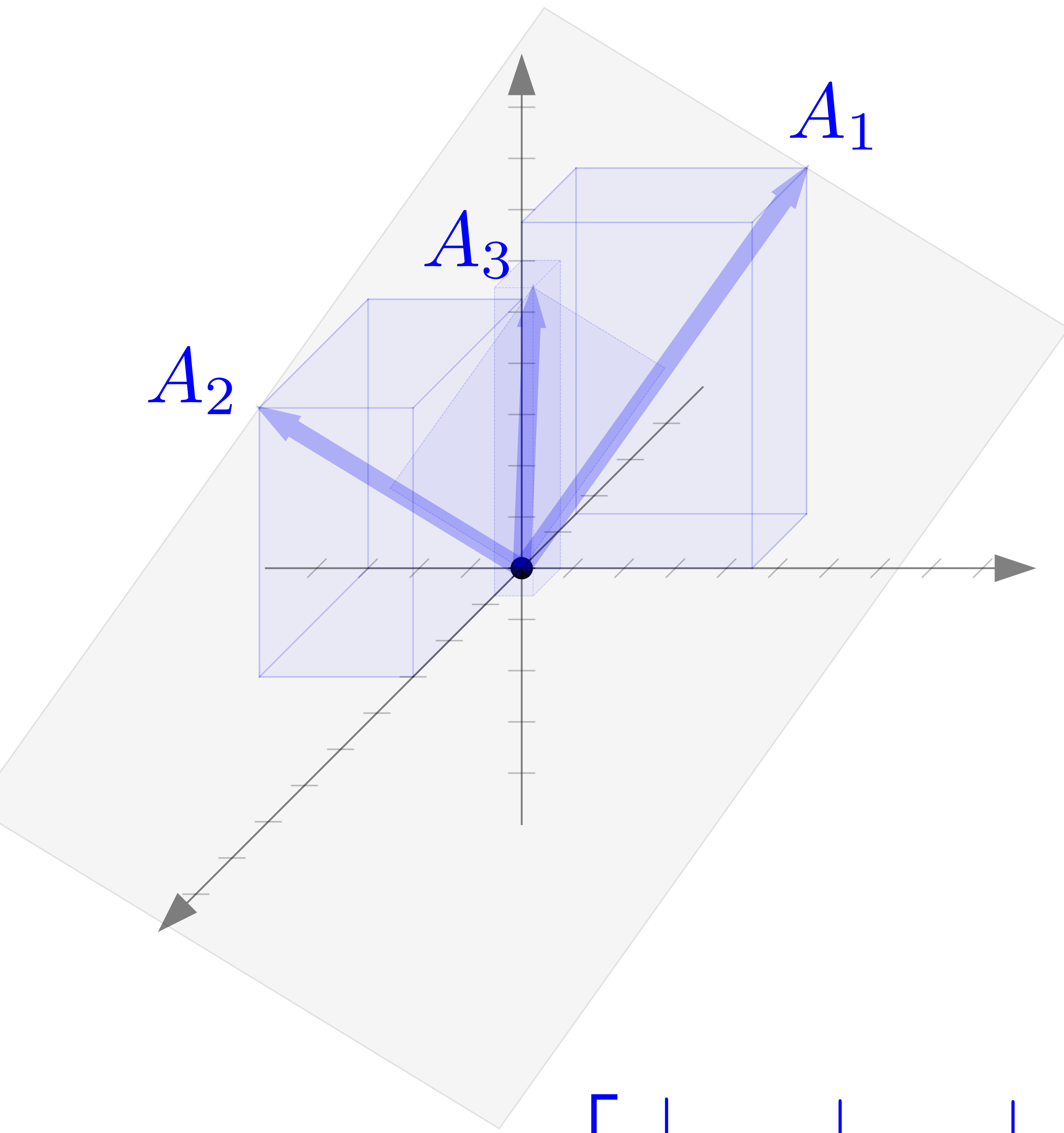

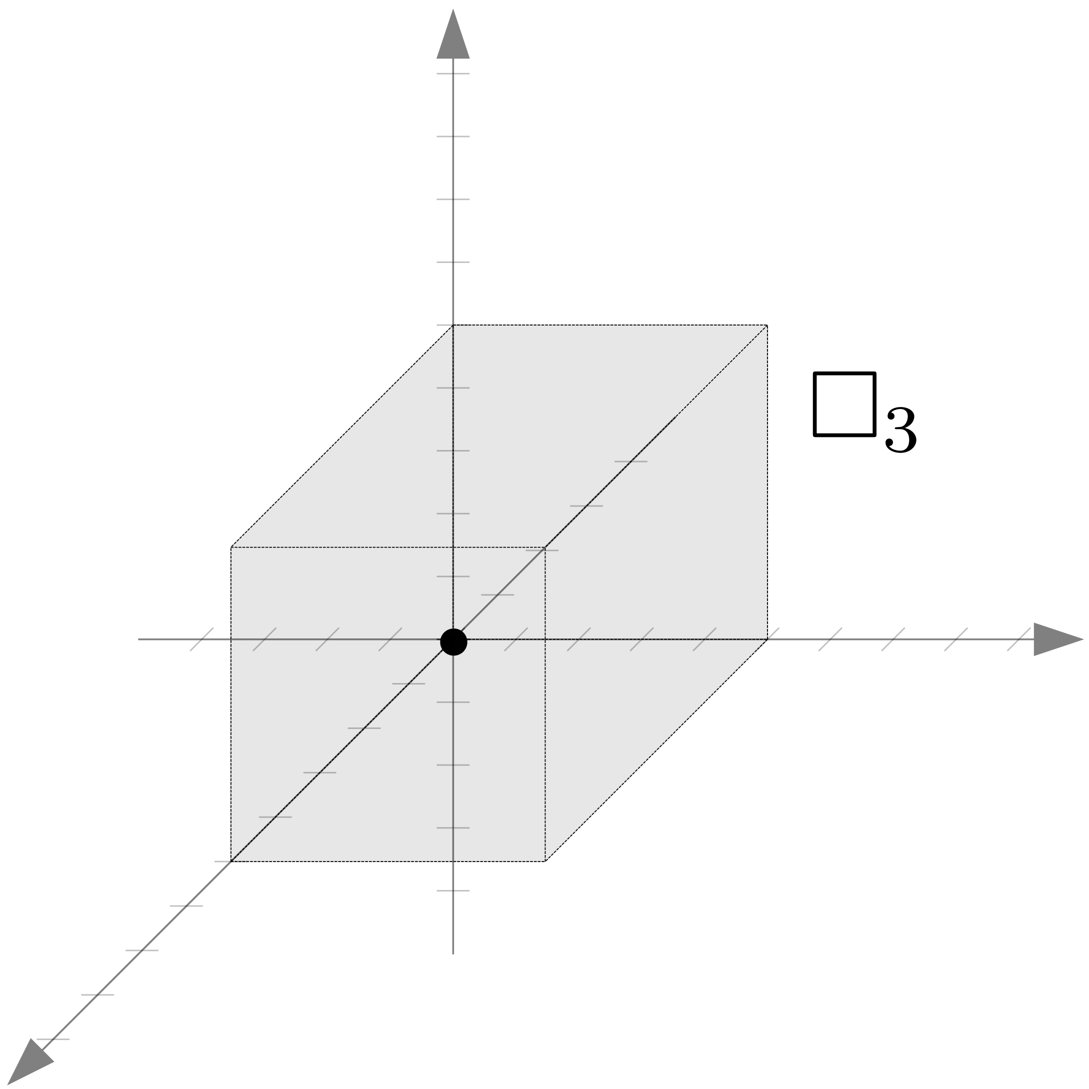

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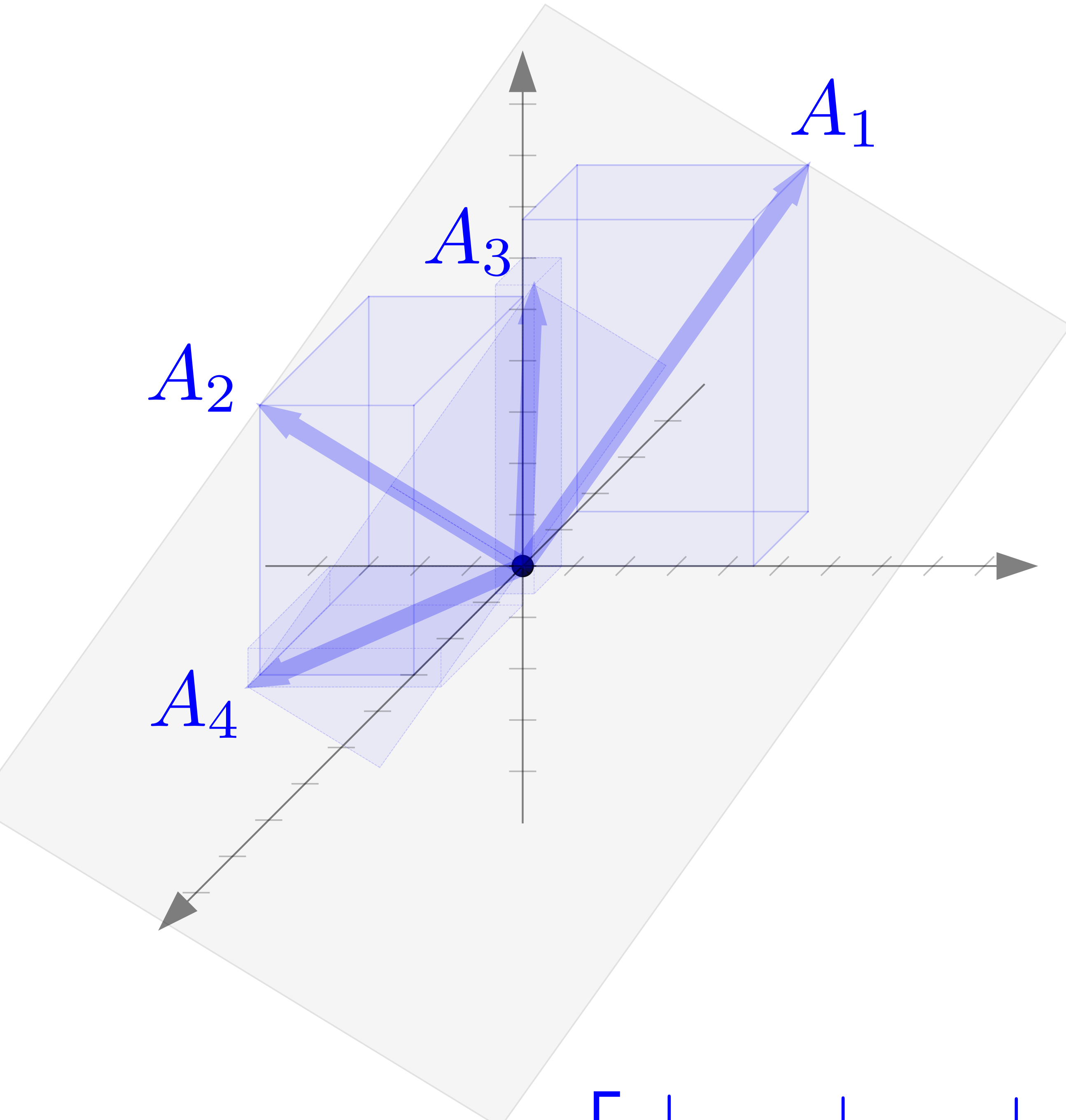

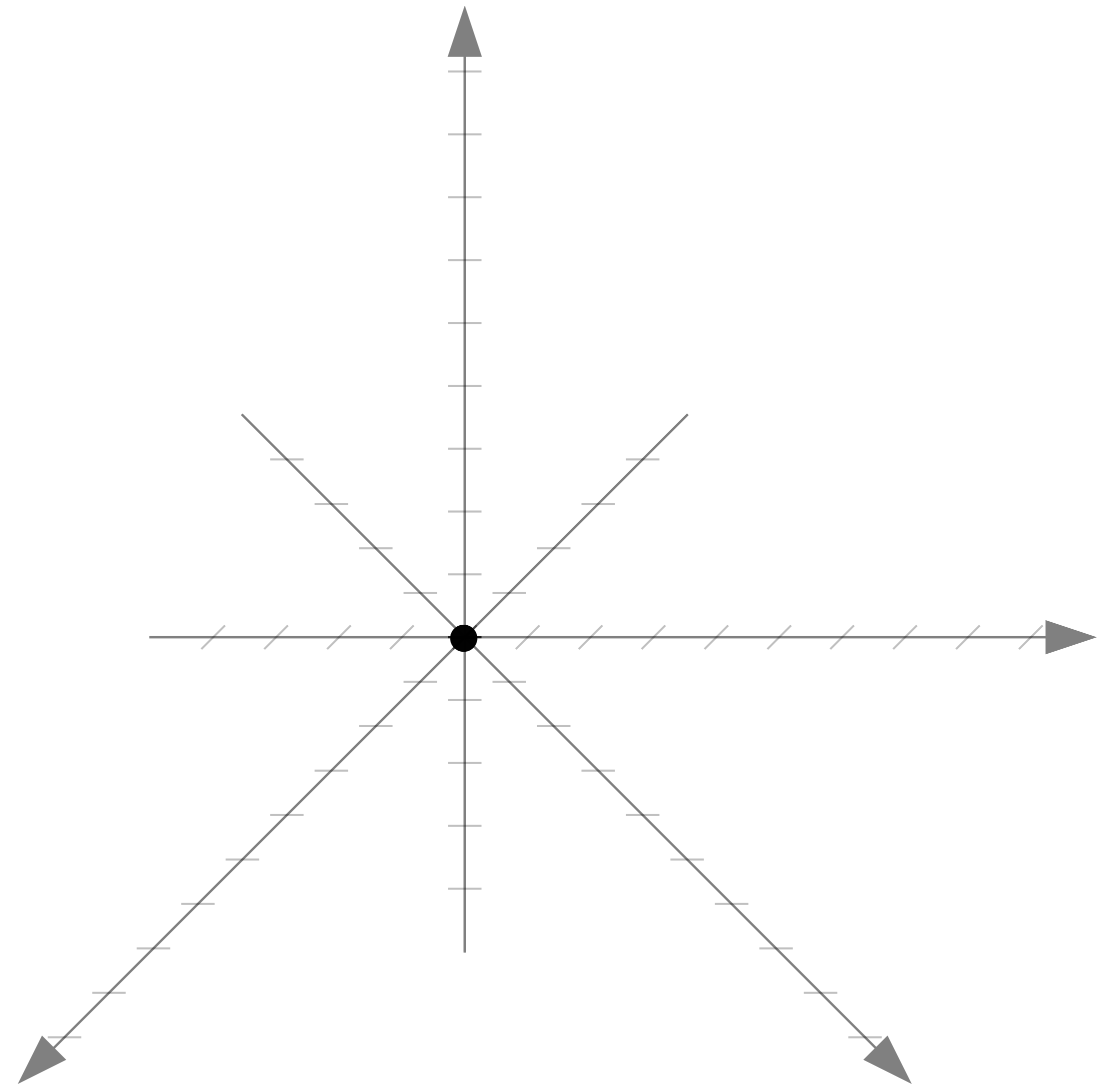

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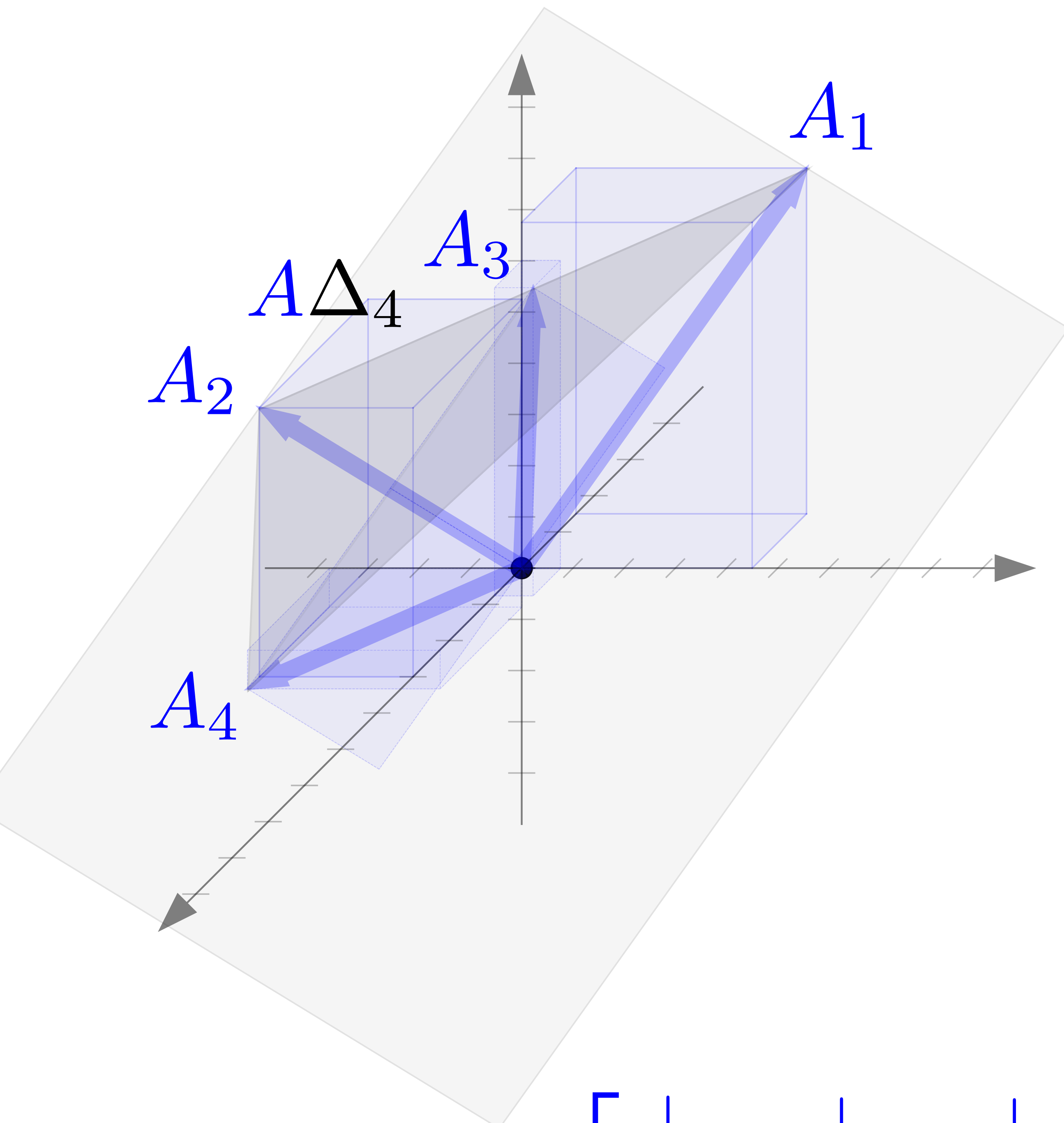

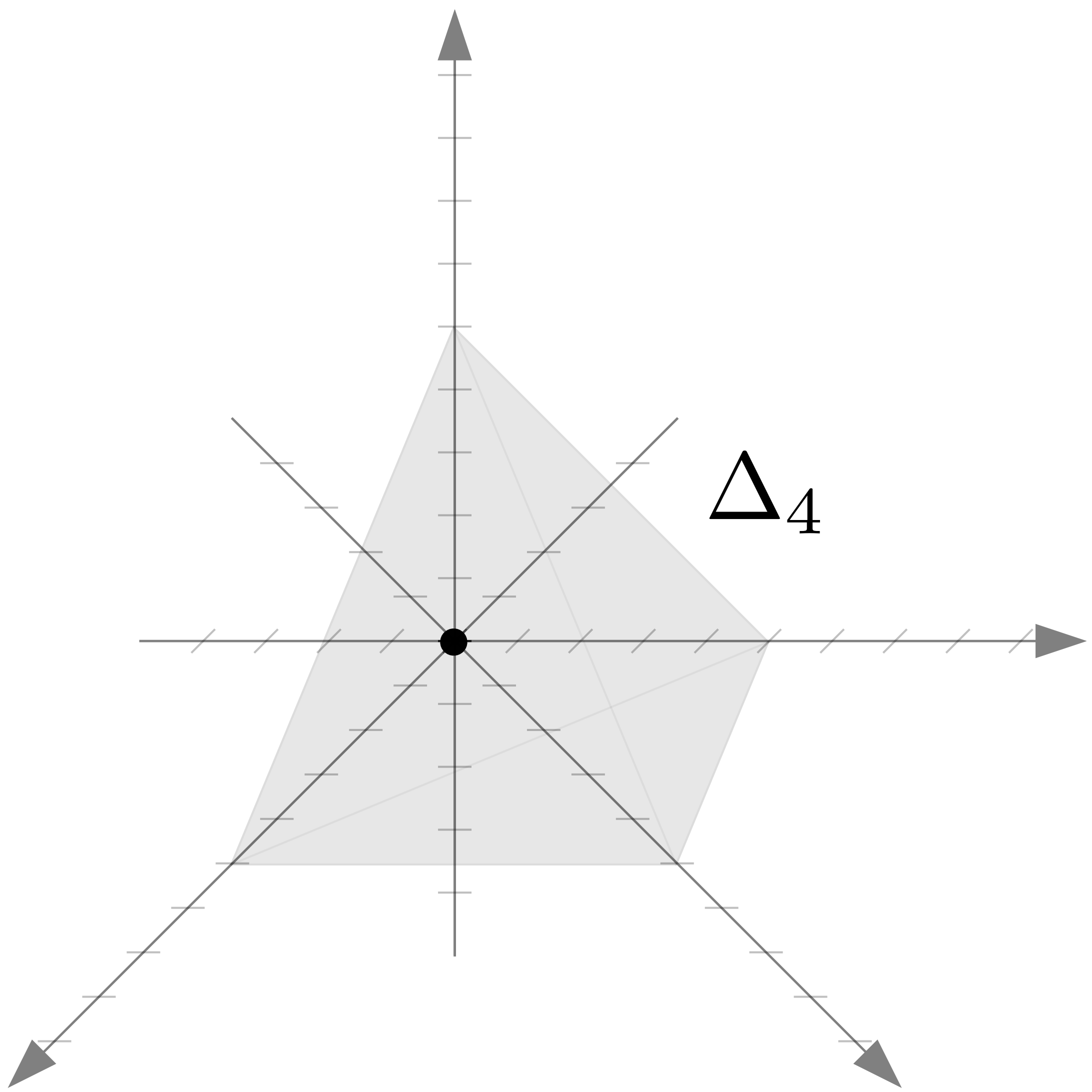

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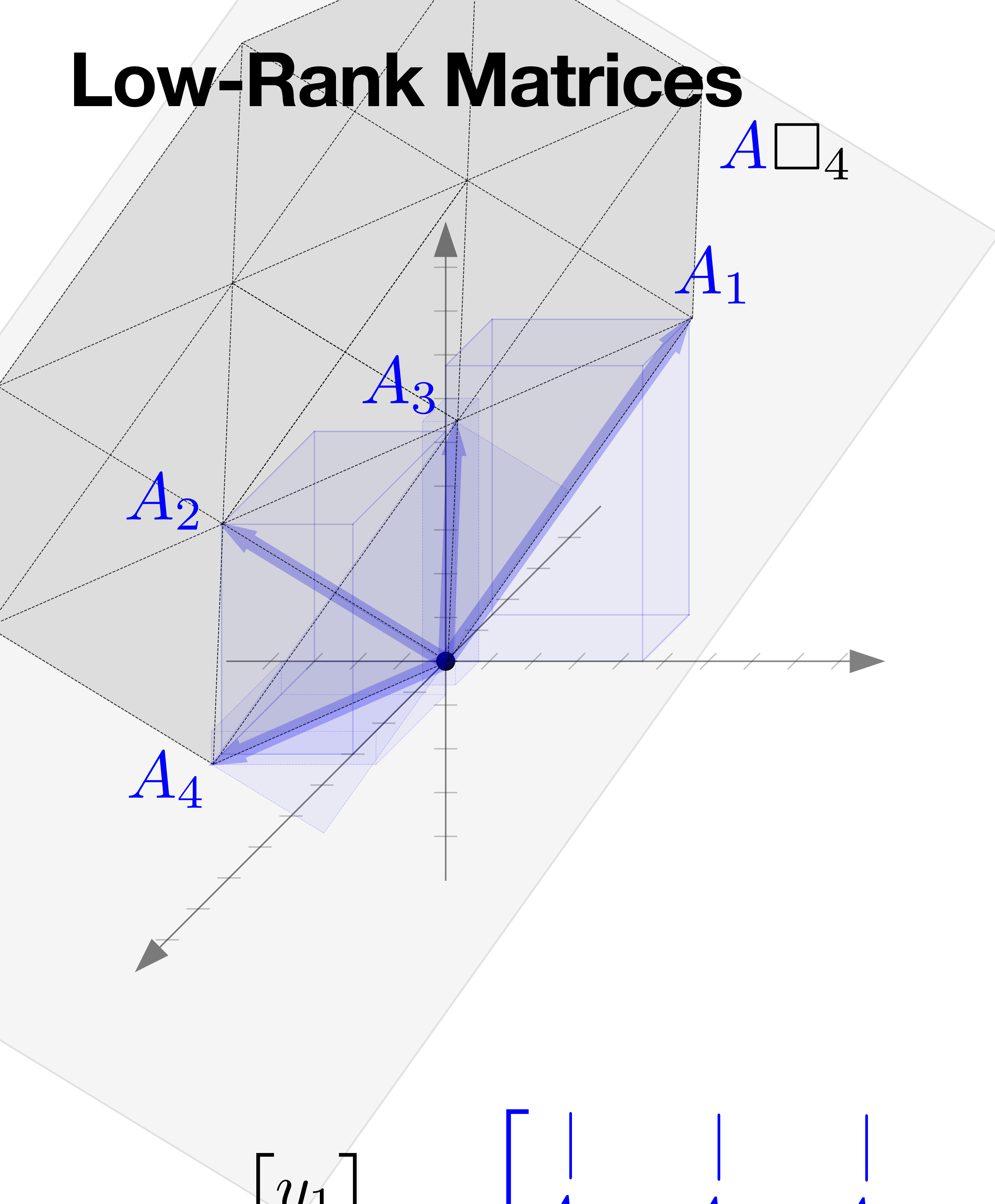

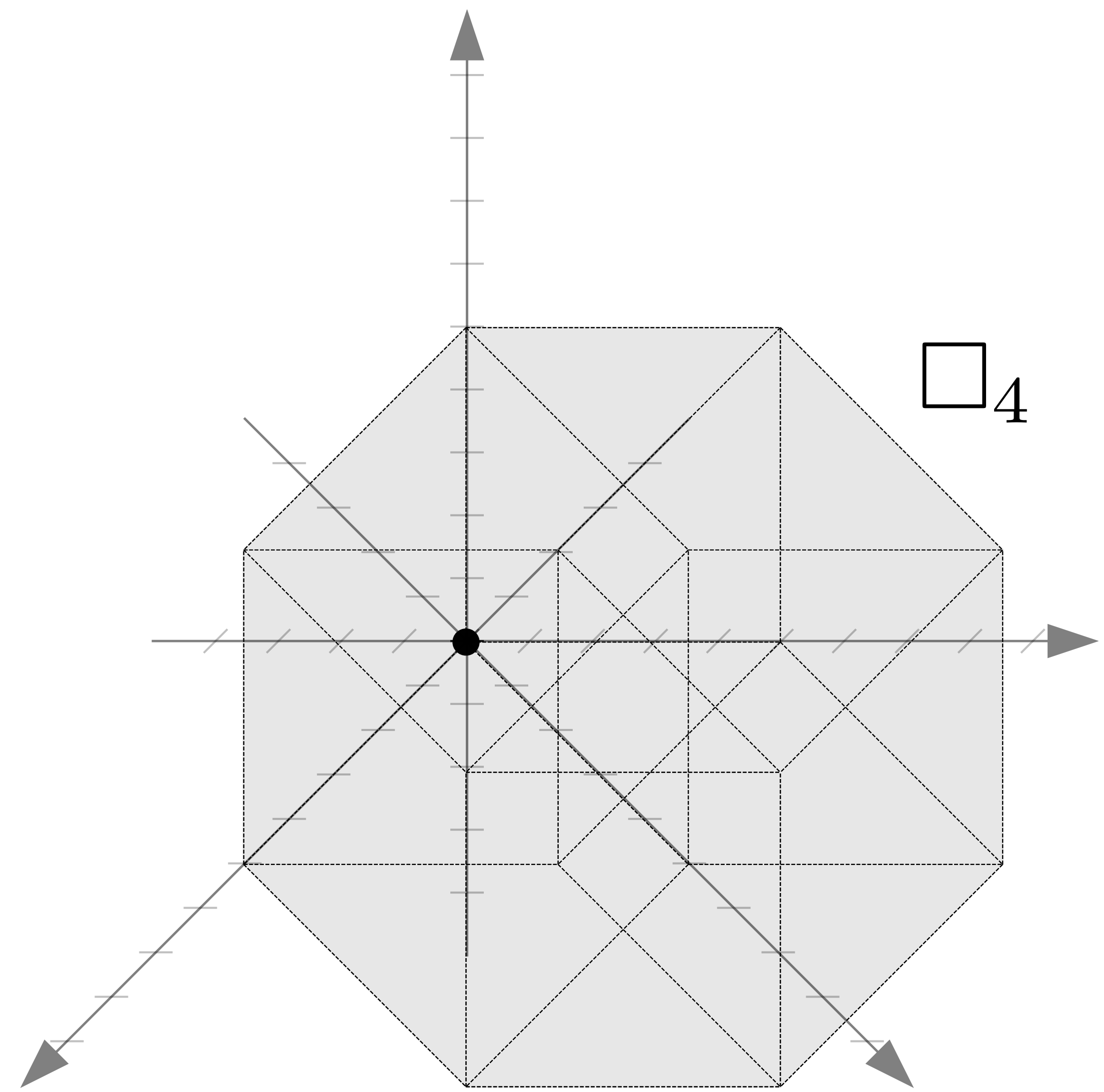

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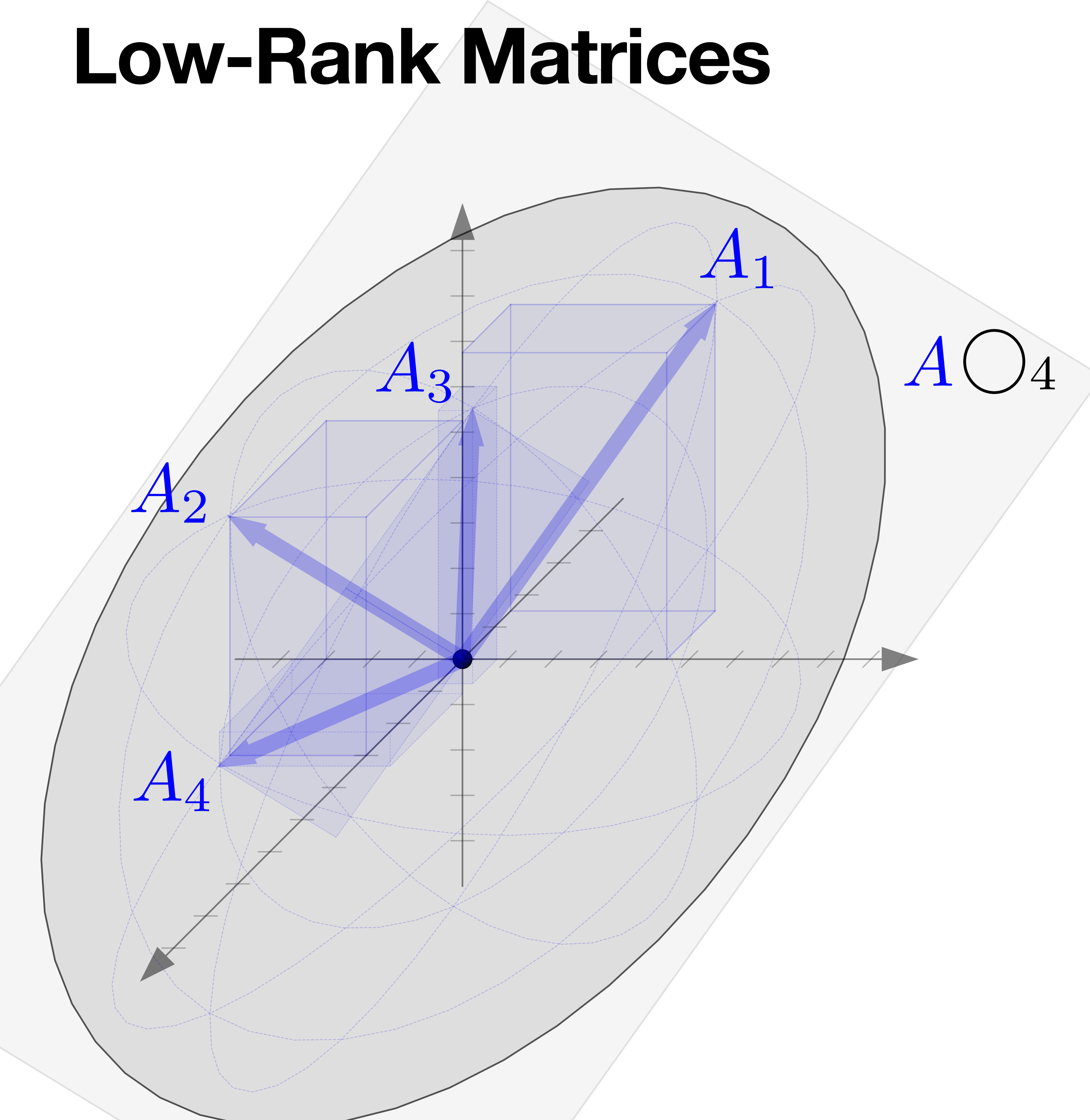

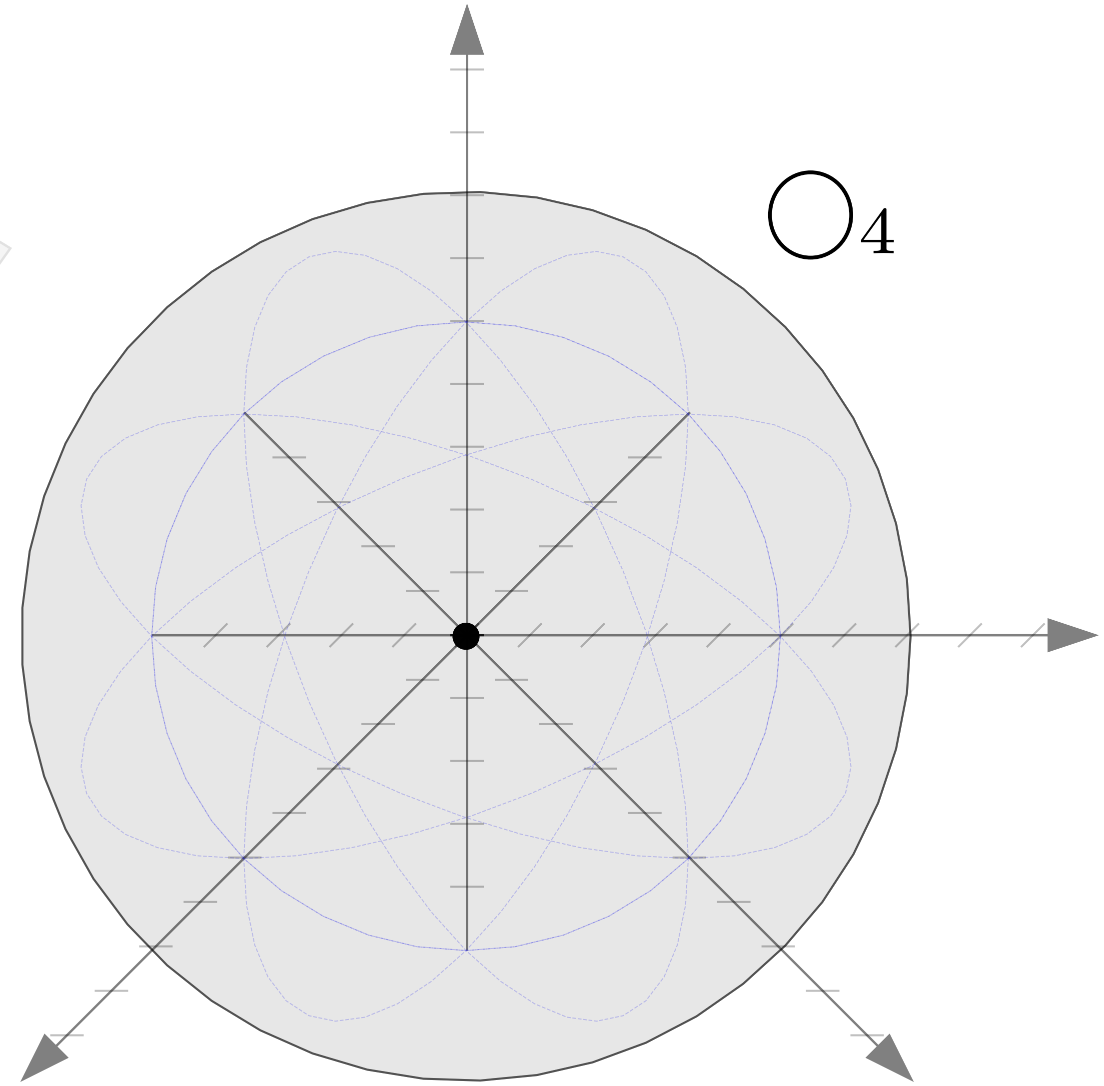

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