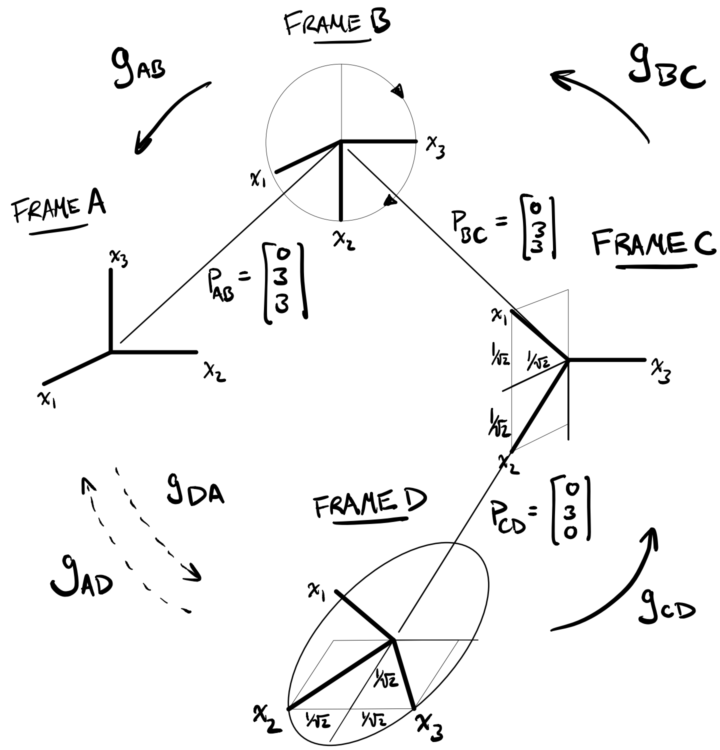
AA/EE/ME 510 - Linear Systems Theory - Autumn 2020

Homework 8

Due Date: Sunday, Dec 13^{th}, 2020 at 11:59pm

1. **Homogeneous transformations**



- **(PTS: 0-2)** From the diagram, write down the homogeneous transformations g_{AB}, g_{BC}, g_{CD}.
- **(PTS: 0-2)** Use these three homogeneous transformations to write g_{DA}. Then use g_{DA} to compute g_{AD}.

2. **Forward Kinematics**

   Use the product of exponentials to compute the forward kinematics for each manipulator.

   (a) **SCARA manipulator**

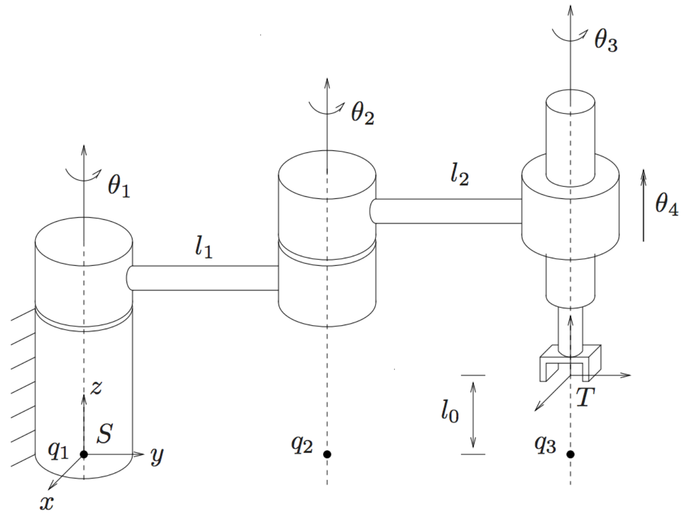


Figure 3.3: SCARA manipulator in its reference configuration.

- **(PTS: 0-2)** Write the homogeneous transformation for each joint for $l_0 = 1$, $l_1 = 1$, and $l_2 = 1$.
- **(PTS: 0-2)** Compute g_{ST} using the product of exponentials formula.

(b) **Elbow Manipulator**

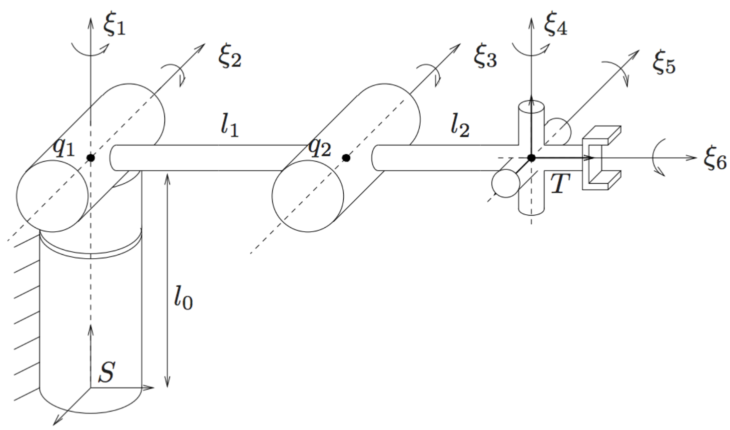


Figure 3.4: Elbow manipulator.

- **(PTS: 0-2)** Write the homogeneous transformation for each joint for $l_0 = 1$, $l_1 = 1$, and $l_2 = 1$.
- **(PTS: 0-2)** Compute g_{ST} using the product of exponentials formula

3. **Numerical Linear Algebra**

- QR**-Decomposition**

$$A = \begin{bmatrix} | & | & | & | \\ A_1 & A_2 & A_3 & A_4 \\ | & | & | & | \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$$

  - **(PTS: 0-2)** Use the Gram-Schmidt process on the columns of A to compute a QR-factorization of A
  - **(PTS: 0-2)** Use Householder reflections to compute a QR-factorization of A

- LU**- Decomposition**

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

(PTS: 0-2) Use elementary matrices to compute an LU-decomposition of A.

4. **Circulant Matrices**

Consider the *shift* matrix

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

and the *circulant matrix* (for the vector $\begin{bmatrix} c_0 & c_1 & c_2 & c_3 & c_4 & c_4 c_5 \end{bmatrix}$).

$$C = \begin{bmatrix} c_0 & c_5 & c_4 & c_3 & c_2 & c_1 \\ c_1 & c_0 & c_5 & c_4 & c_3 & c_2 \\ c_2 & c_1 & c_0 & c_5 & c_4 & c_3 \\ c_3 & c_2 & c_1 & c_0 & c_5 & c_4 \\ c_4 & c_3 & c_2 & c_1 & c_0 & c_5 \\ c_5 & c_4 & c_3 & c_2 & c_1 & c_0 \end{bmatrix}$$

(a) **(PTS: 0-2)** Check that the columns of V are right eigenvectors of P. **(You can just check**

3 of them.)

$$V = \begin{bmatrix} | & \cdots & | \\ V_1 & \cdots & V_6 \\ | & \cdots & | \end{bmatrix} = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & e^{\frac{i2\pi(1\times1)}{6}} & e^{\frac{i2\pi(1\times2)}{6}} & e^{\frac{i2\pi(1\times3)}{6}} & e^{\frac{i2\pi(1\times4)}{6}} & e^{\frac{i2\pi(1\times5)}{6}} \\ 1 & e^{\frac{i2\pi(2\times1)}{6}} & e^{\frac{i2\pi(2\times2)}{6}} & e^{\frac{i2\pi(2\times3)}{6}} & e^{\frac{i2\pi(2\times4)}{6}} & e^{\frac{i2\pi(2\times5)}{6}} \\ 1 & e^{\frac{i2\pi(3\times1)}{6}} & e^{\frac{i2\pi(3\times2)}{6}} & e^{\frac{i2\pi(3\times3)}{6}} & e^{\frac{i2\pi(3\times4)}{6}} & e^{\frac{i2\pi(3\times5)}{6}} \\ 1 & e^{\frac{i2\pi(4\times1)}{6}} & e^{\frac{i2\pi(4\times2)}{6}} & e^{\frac{i2\pi(4\times3)}{6}} & e^{\frac{i2\pi(4\times4)}{6}} & e^{\frac{i2\pi(4\times5)}{6}} \\ 1 & e^{\frac{i2\pi(5\times1)}{6}} & e^{\frac{i2\pi(5\times2)}{6}} & e^{\frac{i2\pi(5\times3)}{6}} & e^{\frac{i2\pi(5\times4)}{6}} & e^{\frac{i2\pi(5\times5)}{6}} \end{bmatrix}$$

(b) **(PTS: 0-2)** What are the eigenvalues associated with each eigenvector? Which eigenvectors are conjugate pairs of each other?

(c) **(PTS: 0-2)** Show that the circulant matrix C can be written as

$$C = c_0 I + c_1 P + c_2 P^2 + c_3 P^3 + c_4 P^4 + c_5 P^5$$

(d) **(PTS: 0-2)** Show that the columns of V are orthogonal to each other, ie. $V_i^* V_j = 0$ for $i \neq j$ where V_i and V_j are columns of V. What does this say about V^{-1}?

(e) **(PTS: 0-2)** Use the spectral mapping theorem to compute the eigenvalues of C.

(f) **(PTS: 0-2)** Write out a diagonalization of C.