AA/EE/ME 510 - Linear Systems

Linear systems theory is the cornerstone of control theory and a prerequisite for more advanced courses in control, robotics, and optimization. Prerequisites for such a course include undergraduate-level differential equations and linear algebra.

Contact Info

Instructor: Dan Calderone
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Lecture

Date & Time: TBA
Location: TBA
Zoom link: TBA

Materials

Notes/Slides

Collected materials can be accessed here.
Teaching website: https://danjcalderone.github.io/teaching.html

Textbooks

Linear System Theory, Chen (main textbook) - free through the library
Linear Systems Theory, Hespanha (good linear system reference)
Feedback Systems, Murray & Astrom, (good frequency domain reference)
Linear Algebra Done Right, Axler
Linear Systems Theory, Callier & Desoer (classic reference, thorough/dense/hard to read)
The Matrix Cookbook (matrix formula reference)

Video Resources

*some of these videos are the best references on the subject
3blue1brown: https://www.3blue1brown.com/
Brian Douglas: https://www.youtube.com/channel/UCq0imsn84ShAe9PBOFnoIrg
Steve Brunton: https://www.youtube.com/channel/UCm5mt-A4w61lknZ9lCsZtBw

Graphing Tools

- https://www.geogebra.org/graphing

Course Schedule

- **Week 1:** Vectors, Bases, Coordinates Matrix multiplication, inner products, projections, norms, vector derivatives, column geometry, row geometry, subspaces, transformation of sets, span bases intuition, coordinate transformations, rotation matrices,

- **Week 2:** Matrix Rank and Fundamental Theorem of Linear Algebra linear dependence, span, row and column rank, row and column geometry, range nullspace, nullspace computation, fundamental theorem of linear algebra,
- **Week 3:** Systems of Equations, Least Squares, Minimum Norm Gaussian elimination, elementary matrices, systems of equations, least squares, minimum norm solutions
- **Week 4:** Eigenvalues & Eigenvectors Similarity transformations, traces and determinants, eigenvalues, left and right eigenvectors, diagonalization, polynomial functions of matrices, spectral mapping theorem, complex eigenvalues and eigenvectors, Cayley-Hamilton theorem, matrix exponential.
- **Week 5:** Special Matrices Jordan Form Nilpotent matrices and Jordan canonical form, special matrix structures: symmetric and skew-symmetric, quadratic forms, positive definiteness, congruent transformations, shape matrices $A^T A$ and AA^T, polar decomposition, singular value decomposition (SVD)
- **Week 6:** Dynamical Systems Modeling dynamics, Newton's laws (mechanics), Kirchhoff's Laws (circuits), 2nd order systems, vector fields, autonomous vs. control systems, linear vs. nonlinear dynamics, linearization, LTI LTV systems, state-space models
- **Week 7:** LTI Systems Laplace transforms, frequency domain, transfer functions, state-space to frequency domain computation, solutions to linear ODEs, state transition matrix, matrix exponential, DT linear dynamics, CT to DT conversion, solutions to ODEs with control inputs
- **Week 8:** Stability Review (state-space models and Lecture 7), qualitative behavior of first- and second-order systems, BIBO/asymptotic/exponential stability, eigenvalue tests for stability in CT and DT
- **Week 9:** Controllability/Observability Controllability (reachability)/observability, controllability matrix, PBH test, Gramians, controllability/observability metrics, duality, controllable/observable canonical form
- **Week 10:** Feedback control/Observer design/Preview topics Controllable/observable/Kalman decompositions, stabilizability/detectability, transfer function to state-space (minimal realizations), feedback control, pole placement, observer design, separation principle, Introduction to Lyapunov theory, LQR, Kalman filters
- **Extra Topics:** Fourier transforms, causality in frequency domain, poles and zeros, effects of RHP and time delays, Bode plots/transfer function visualization, Nyquist plot, controllability and observability in frequency domain, state-space to transfer, Z-transforms, roots of unity, shift-matrices, circulant matrices, diagonalization via DFT diagonalization by the DFT,

Grades

- **Homework:** 80%
- **Midterm (optional):** - replaces 2 lowest homework scores (see below)
- **Final:** 20%

Homeworks

General Policy

Homeworks will be assigned each week after class and due 10 days later. Self-grades/error explanations for the previous week's homework will be due at the same time. Your writeup must be submitted as a single self-contained `.pdf` document, including commented source code for any computational tools used to complete the assignment. Every student can turn in one homework late for full credit with no questions asked. Other extensions can be given for exceptional circumstances but must be requested well in advance (teacher discretion) of the deadline. Homeworks will be worth 80% of your grade. Solutions will be provided after each assignment is due.

Self-assessment/Rubric

Each homeworks will divided into sub-problems worth 2 points each. To provide training and feedback that helps cultivate self-reflection, you will grade your homework assignments and receive oversight and feedback from the instructional staff regarding the accuracy of your self-assessment. Self-assessment will be due with the next assignment. We will provide detailed solutions on the day of the homework submission deadline, and you will have until the following due date to grade each of your homework sub-problems on a 0,1,2 point scale:

- **0 points:** - no effort / not attempted
- **1 points:** - attempted, but incomplete or incorrect solution
- **2 points:** - complete and correct solution

For any sub-problem that earns a 1, you have the opportunity to explain the error in your solution and how to correct it; if this explanation is correct, you earn the full 2 points on the problem. You do not give yourself the 2 points if you got an incorrect problem and corrected it. The grader will assign you the extra point after going through your homework. To specify grades and provide explanations of any errors, use the Comment feature in Canvas's Assignment page for the homework.

Notes and caveats intended to ensure the integrity of this process:

- if you did not attempt the problem initially, you will receive 0 points;
- if you do not grade a problem, you will receive 0 points;
- if you grade incorrectly (i.e. initial solution is incomplete or incorrect and your explanation is incomplete or incorrect), you will receive 1 or 0 at the discretion of the instructional staff (this ensures you cannot simply assign all ”2”s, nor can you receive full credit for incomplete or incorrect explanations).

Frequently Asked Questions (FAQ)

Q: Should I download my.pdf, add comments (e.g. via Adobe Acrobat), and re-upload the .pdf?
A: That's fine, but it's probably easier to use the in-browser ”CrocoDoc” viewing and annotation pane provided in Canvas.

Q: Does my .pdf need to contain all of my homework writeup materials, or can some be in an .m file?
A: Your writeup must be submitted as a single self-contained .pdf document, including commented source-code for any computational tools used to complete the assignment.

Submission guidelines

Please make homeworks neat and organized. Points may be deducted if not. You are welcome (and we encourage you) to typeset your homework assignments rather than write them by hand. We will provide both .pdf and .m files for homework assignments and solutions. If you write your solutions by hand, you must create a legible scan; if you have any doubts about the fidelity of your scans, send a sample to the TA / Prof in advance of the homework deadline.

Exams

Midterm: There will be an optional midterm. It will cover up through homework 5. You will have multiple days to work on it. There will be no resubmission. Your midterm score can replace your two lowest homework scores (assuming that is an improvement). If you do worse on the midterm than on your homeworks, you will not be penalized, ie. your midterm score will be dropped.

Final: There will be a mandatory take-home final exam worth 20% of your grade. The exam will be open-book/open notes/open internet. You will not be allowed to collaborate with anyone on the exams. Exams will be graded by the professors (rather than being self-graded).

1 Coding Resources

Coding can be done in Matlab or Python.

- Control Tutorials for MATLAB and Simulink: http://ctms.engin.umich.edu/CTMS/index.php?aux=Home
- Python: https://colab.research.google.com/