OBSERVABILITY

$$x^+ = Ax + B\hat{u} \rightarrow \text{real world}$$
$$y = Cx + D\hat{u} \rightarrow \text{measure}$$

$$x(k) = \left[A^k x(0) + \sum_{j=0}^{k-1} A^{k-1-j} B u(j)\right]$$

$$y(k) = Cx(k) = CA^k x(0) + \sum_{j=0}^{k-1} CA^{k-1-j} B u(j)$$

have an equation like this for $k = 0, \ldots, k$

usually C fat

$$\begin{aligned} y(0) &= I\, x(0) \\ y(1) &= CA\, x(0) + CBu(0) \\ y(2) &= CA^2 x(0) + \underline{CABu(0) + CBu(1)} \\ &\vdots \\ y(k) &= CA^k x(0) + \sum_{j=0}^{k-1} CA^{k-1-j} Bu(j) \end{aligned}$$

$y(0) = [1 \cdots 1]\, x(0)$

Y $\qquad\qquad$ Z

$$\xi + Y - Z = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^{n-1} \\ \hline CA^k \end{bmatrix} \boxed{x(0)}$$

M

$\longrightarrow$ least squares

$$x(0) = \left[(M^T M)^{-1} M^T (Y - Z)\right]$$

only invertible if M is full column rank

$k = \underline{n-1}$ and M has full col rank

then $\underline{x(0)} = \underline{M}^{-1}(Y - Z)$

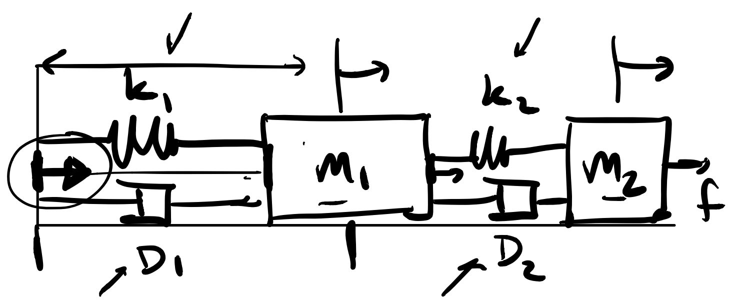
if $C \in \mathbb{R}^{1 \times n}$ $\quad$ if $C \in \mathbb{R}^{n \times n}$

$$\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} = n \text{ rows} \begin{bmatrix} \text{---} \\ \text{---} \\ \vdots \\ \text{---} \end{bmatrix}$$

if $C \in \mathbb{R}^{2 \times n}$

$$\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} = 2n \begin{bmatrix} \text{===} \\ \text{===} \\ \\ \text{===} \end{bmatrix} \begin{array}{l} \leftarrow C \\ \leftarrow CA \\ \\ \leftarrow CA^{n-1} \end{array}$$

Ex.



SENSORS
- accelerometer
-
- strain gauge
- LVD/RVD
- RPM counter

$\dot{x} = Ax + Bu$

$$x = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{array}{l} \leftarrow \\ \leftarrow \end{array}$$

$\underline{y} = Cx + \underline{Du} \longrightarrow y - \underline{Du} = Cx$

$\bar{y} = \underline{D}u$

$$\dot{x} = Ax + Bu \qquad \dot{x} = Ax + Bu$$
$$y = Cx + Du \qquad y = Cx$$

Controllability / OBSERVABILITY TESTS

CONTROLLABILITY

$\rightarrow \begin{bmatrix} A^{n-1}B & \cdots & AB & B \end{bmatrix}$

full row rank

OBSERVABILITY

$\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$

full column rank

suppose A diagonalizable

$A = PDP^{-1} \rightarrow$

P: right eigvectors

P^{-1}: row left eigenvectors

$$\begin{bmatrix} PD^{n-1}P^{-1}B & & PDP^{-1}B & B \end{bmatrix}$$

$$P\begin{bmatrix} D^{n-1}\underline{P^{-1}B} & \cdots & D\underline{P^{-1}B} & \underline{P^{-1}B} \end{bmatrix} = P\begin{bmatrix} \underbrace{\diagdown}_{D^{n-1}} B' & \cdots & \underbrace{\diagdown}_{D} B' & B' \end{bmatrix}$$

$$P^{-1}B = \begin{bmatrix} - q_1^T - \\ \vdots \\ - q_n^T - \end{bmatrix} B = \underbrace{\begin{bmatrix} * \\ * \\ 0 \\ * \end{bmatrix}}_{B'} \Rightarrow P\begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}$$

not full row rank

if $\boxed{q_j^T B = 0}$ for some j $\leftarrow$

q_i^T: left eigenvectors $\leftarrow$ input directions of A matrix another way to see... $\rightarrow q_j^T \underline{B} = 0$

$$q_j^T [A^{n-1}B \cdots AB \;\; B] = [\lambda_j^{n-1} \underbrace{q_j^T B}_{0} \cdots \lambda_j \underbrace{q_j^T B}_{0} \;\; \underbrace{q_j^T B}_{0}]$$

$$= 0$$

$$\dot{x} = Ax + Bu$$

coord transform : $x = Pz$ $\quad A = PDP^{-1}$

$$P\dot{z} = APz + Bu \Rightarrow \dot{z} = \underbrace{P^{-1}AP}_{D} z + \underbrace{P^{-1}B}_{B'} u$$

$$\begin{bmatrix} \dot{z}_1 \\ \vdots \\ \dot{z}_n \end{bmatrix} = \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix} \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix} + \underbrace{\begin{bmatrix} P^{-1}B \end{bmatrix}}_{B'} u$$

$$\dot{z}_i = \lambda_i z_i + \underbrace{B'_i}_{0} u \quad \text{if } B'_i = 0 \Rightarrow \boxed{\dot{z}_i = \lambda_i z_i}$$

if $\lambda_1 \neq \lambda_2 \neq \cdots \neq \lambda_n$ just check if $B'_i = 0$ for any i

if $\boxed{\lambda_1 = \lambda_2} \Rightarrow$ eigen subspace is 2D
not a unique basis for that subspace

$q_i^T B \neq 0$ for every q_j^T in that subspace

$\rightarrow A = \lambda I \longrightarrow$ all $\xi^T \lambda I = \lambda \xi^T$

$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad \begin{array}{l} \xi_1^T = [1 \; 0] \rightarrow \xi_1^T B = 1 \\ \xi_2^T = [0 \; 1] \quad\;\; \xi_2^T B = 1 \end{array}$

$\xi_1^T = [1 \; -1]$ $\quad \xi_1^T B = [1 \; -1]\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$

$\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$ $\quad A = PDP^{-1} \Rightarrow \begin{bmatrix} C \\ CPDP^{-1} \\ \vdots \\ CPD^{n-1}P^{-1} \end{bmatrix}$

right evecs

$CP = C\begin{bmatrix} p_1 \cdots p_n \end{bmatrix}$ $\quad = \begin{bmatrix} CP \\ CPD \\ \vdots \\ CPD^{n-1} \end{bmatrix} P^{-1}$

$\boxed{\text{if } CP_j = 0}$ for some j system $\longrightarrow$ not observable

$CP = [* \cdots * \; 0 \; * \cdots *]$

$\begin{bmatrix} CP \\ CPD \\ \vdots \\ CPD^{n-1} \end{bmatrix} P^{-1} = \begin{bmatrix} [\cdots 0 \cdots] \\ [\cdots 0 \cdots]D \\ \vdots \\ [\cdots 0 \cdots]D^{n-1} \end{bmatrix} P^{-1} \longrightarrow$ not full col rank

col of 0's

$\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} x(0) = \alpha \begin{bmatrix} CP_j \\ C\lambda_j P_j \\ \vdots \\ C\lambda_j^{n-1} P_j \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

$x(0) = \alpha P_j$

if $\lambda_1 = \lambda_2$ $\longrightarrow$ need to check all $\underline{p}$ in that 2D subspace

$A = \lambda I \Rightarrow$ $\quad P_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$C = \begin{bmatrix} 1 & -1 \end{bmatrix}$ $\quad CP = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \end{bmatrix}$

$P = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad C \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$

FEEDBACK CONTROLLER DESIGN:

$$\dot{x} = Ax + Bu$$

$u = \boxed{K} x \longrightarrow$ feed back controller

$u = Kx + \bar{u}$ feedback + reference controller

$$\dot{x} = Ax + B(Kx + \bar{u}) = (A + \underline{BK})x + B\bar{u}$$

feedback state matrix : $A + BK$

$B \in \mathbb{R}^{n \times m}$ $\quad u \in \mathbb{R}^m$ $\quad K \in \mathbb{R}^{m \times n}$ $\quad u = Kx$

PID Controllers in State Space:

state x_1: $$u = \underbrace{K_p}_{\text{proportional gain}} x_1 + \underbrace{K_D}_{\text{derivative gain}} \dot{x}_1 + \underbrace{K_I}_{\text{integral gain}} \int x_1$$

*

Proportional Integral Derivative controller : PID controller

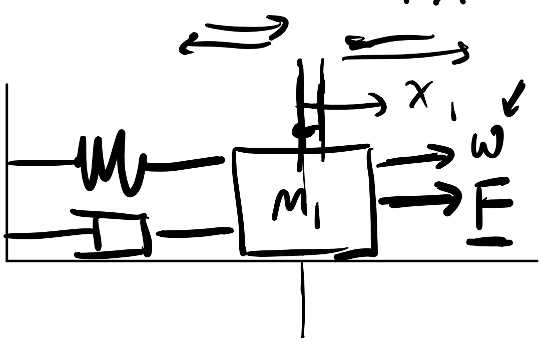
$$x = \begin{bmatrix} x_1 \\ \dot{x}_1 \end{bmatrix} \qquad \dot{x} = Ax + B_1 u + \boxed{B_2 \underline{w}}$$

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \end{bmatrix} = \underline{\begin{bmatrix} 0 & 1 \\ * & * \end{bmatrix}} \begin{bmatrix} x_1 \\ \dot{x}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ * \end{bmatrix} u$$

$$u = \underline{K} x = \begin{bmatrix} K_p & K_D \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \end{bmatrix} = K_p x_1 + K_D \dot{x}_1$$

$$x = \begin{bmatrix} \int x_1 \\ x_1 \\ \dot{x}_1 \end{bmatrix} \rightarrow \begin{bmatrix} x_1 \\ \dot{x}_1 \\ \ddot{x}_1 \end{bmatrix} = \underline{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & * & * \end{bmatrix}} \begin{bmatrix} \int x_1 \\ x_1 \\ \dot{x}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ * \end{bmatrix} u$$

$$u = \underbrace{\begin{bmatrix} K_I & K_p & K_D \end{bmatrix}}_{K} \begin{bmatrix} \int x_1 \\ x_1 \\ \dot{x}_1 \end{bmatrix} =$$



$u = K_p x_1$ pushing back against position

$+ K_D \dot{x}_1$ extra damping

$+ \underline{K_I} \int \underline{x_1}$ gets rid of small accumulated errors

Multivariable Feedback Control

Sigurd Skogestad
Ian Postlethwaite

Controllable Canonical Form

$\dot{x} = Ax + Bu \qquad A \in \mathbb{R}^{n\times n} \qquad B \in \mathbb{R}^{n\times 1}$

$$\begin{bmatrix} 0 & 1 & & 0 \\ & & \ddots & \\ 0 & 0 & & 1 \\ -\alpha_0 & -\alpha_1 & \cdots & -\alpha_{n-1} \end{bmatrix} \qquad \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$u = Kx = [k_0 \cdots k_{n-1}]\, x$

companion form

$\chi_A(s) = s^n + \alpha_{n-1} s^{n-1} + \cdots + \alpha_1 s + \alpha_0$

with feedback $\dot{x} = (A+BK)x$

$$A+BK = \begin{bmatrix} 0 & 1 & & 0 \\ & & \ddots & \\ 0 & 0 & & 1 \\ k_0-\alpha_0 & \cdots & & k_{n-1}-\alpha_{n-1} \end{bmatrix}$$

direct access to

$\chi_{A+BK}(s)$ thru K

if we want $A+BK$ to have evals

$\lambda_1 \cdots \lambda_n \rightarrow \chi_{A+BK}(s) = (s-\lambda_1)\cdots(s-\lambda_n)$

choose to be stable $\mathrm{Re}(\lambda_i) < 0$

$$\chi_{A+BK}(s) = (s-\lambda_1)\cdots(s-\lambda_n)$$
$$= s^n + \beta_{n-1}s^{n-1} + \cdots + \beta_1 s + \beta_0$$

if $\chi_A(s) = s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_1 s + \alpha_0$

if we set $K = [\beta_0 - \alpha_0 \quad \cdots \quad \beta_{n-1} - \alpha_{n-1}]$

pole placement method.

↳ picking the eigenvalues of $A+BK$

if $A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & & 1 \\ & & & \end{bmatrix} \quad \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$ $\quad x$ is $\quad x = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ \vdots \\ \frac{d^{n-1}x_1}{dt^{n-1}} \end{bmatrix}$

"chain of integrators
every state is the integral of previous state"

Now have a general $A \in \mathbb{R}^{n \times n}$ & $B \in \mathbb{R}^{n \times 1}$

Can we find a coordinate transformation not necessary
such that $x = Qz \rightarrow A' = Q^{-1}A \quad B' = Q^{-1}B$

$\dot{x} = Ax + Bu$

want A' & B' to be in controllable canonical form

$x = Qz$

$$\dot{z} = \underbrace{Q^{-1}AQ}_{A'}z + \underbrace{Q^{-1}B}_{B'}u$$

$\boxed{A'} = Q^{-1}AQ$ & $\boxed{A}$ have the same characteristic polynomial

$$\chi_A(s) = s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_1 s + \alpha_0$$

$$\boxed{A'} = \begin{bmatrix} 0 & I \\ -\alpha_0 & \cdots \; -\alpha_{n-1} \end{bmatrix} \qquad \boxed{B'} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

to find Q consider the controllability matrices..

$$\underbrace{\begin{bmatrix} A'^{n-1}B' & \cdots & A'B' & B' \end{bmatrix}}_{M'} = Q \underbrace{\begin{bmatrix} A^{n-1}B & \cdots & AB & B \end{bmatrix}}_{M}$$

if Q exists then $A = QA'Q^{-1}$ $\qquad [QA'^{n-1}Q^{-1}QB' \;\cdots\;]$

$$\boxed{Q = M'M^{-1}}$$

in order for Q to exist... M must be invertible

$\Updownarrow$

(A, B) is controllable

have A B.

compute $\boxed{Q = M'M^{-1}}$

then $A' = Q^{-1}AQ$ $\quad B' = Q^{-1}B$

controllable canonical form

Design controller in the z coordinates

$$\dot{z} = A'z + B'u$$

$$\begin{bmatrix} 0 & I \\ -\alpha_0 & \cdots & -\alpha_{n-1} \end{bmatrix} z + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

$u = K'z$

$A' + B'K'$

$$\begin{bmatrix} 0 & I \\ -\alpha_0 \cdots & -\alpha_{n-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix} K'$$

$$\begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ K'_1 & \cdots & K'_n \end{bmatrix}$$

want evals to be $\lambda_1 \ldots \lambda_n$

$$\chi_{A+BK}(s) = (s-\lambda_1)\cdots(s-\lambda_n)$$

$$= s^n + \beta_{n-1}s^{n-1} + \cdots + \beta_1 s + \beta_0$$

$$K' = [\alpha_0 - \beta_0 \quad \cdots \quad \alpha_{n-1} - \beta_{n-1}]$$

$$u = K'z \Rightarrow \dot{z} = [A' + B'K']z$$

$$= \begin{bmatrix} 0 & I \\ -\beta_0 & \cdots & -\beta_{n-1} \end{bmatrix} \leftarrow \chi_{A'+B'K'}(s) = \chi_{A+BK}(s)$$

$K'z = Kx = u$

$\Rightarrow K = K'Q^{-1}$

$$\dot{z} = A'z + B'K'z$$

plugging back in $z = Q^{-1}x$

$$Q^{-1}\dot{x} = A'Q^{-1}x + B'K'Q^{-1}x$$

$$\dot{x} = \underbrace{QA'Q^{-1}}_{A}x + \underbrace{QB'}_{B}K'Q^{-1}x$$

$$\boxed{K = K'Q^{-1}}$$

$$\dot{x} = Ax + B\underbrace{K'Q^{-1}}_{K}x$$

$u = Kx$

Result:

if $K = K' Q^{-1}$ where $Q = M' M^{-1}$

$\dot{x} = (A+BK)x$ has eigenvalues $\lambda_1 \cdots \lambda_n$

where $K' = [\alpha_0 - \beta_0 \; \cdots \; \alpha_{n-1} - \beta_{n-1}]$

where $\chi_A(s) = s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_1 s + \alpha_0$

desired $\rightarrow$ $\chi_{A+BK}(s) = s^n + \beta_{n-1}s^{n-1} + \cdots + \beta_1 s + \beta_0$

$= (s-\lambda_1) \cdots (s-\lambda_n)$

design choice

for stability

$Re(\lambda_i) < 0$

Pole placements

High Level Comments

Another method: Ackermann's Formula

if $A \in \mathbb{R}^{n \times n}$ $B \in \mathbb{R}^{n \times 1}$ (A, B) controllable

$K \Rightarrow$ unique

if $B \in \mathbb{R}^{n \times m}$ $K \Rightarrow$ not unique

$A + BK = W D W^{-1}$ $\leftarrow$ OLD SCHOOL

select eigenvalues

LQR controller: $\Leftarrow$ NEWER

AE513

Linear Quadratic Regulator

$$\min_{x(t)\, u(t)} \int_0^\infty x(t)^T Q\, x(t) + u(t)^T R\, u(t)\, dt$$

s.t. $\dot{x} = Ax + Bu$

$$[x_1\ x_2\ x_3]\underbrace{\begin{bmatrix} * & 0 & 0 \\ 0 & \boxed{*} & 0 \\ 0 & 0 & \boxed{*} \end{bmatrix}}_{Q}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\left.\begin{aligned} Q &= Q^T \succ 0 \quad 100 \\ R &= R^T \succ 0 \quad 100 \end{aligned}\right]$$

$$[u_1\ u_2]\underbrace{\begin{bmatrix} * & 0 \\ 0 & \boxed{*} \end{bmatrix}}_{R}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Solution: $u = Kx$

$$K = -R^{-1}BP$$

this K makes $A+BK$ stable and have "good" eigenvectors

$A+BK = WDW^{-1}$

where $P = P^T \succ 0$

and solves

$$A^TP + PA - PBR^{-1}B^TP + Q = 0$$

Algebraic Riccati eqn

Matlab:

pole placement: place(A, B, p) $\longrightarrow K$ $\quad B \in \mathbb{R}^{n \times m}$

$p = [\lambda_1 \cdots \lambda_n]$

LQR: lqr$(A, B, Q, R) \longrightarrow K$

Output Feed Back:

$u = K\boxed{x}$ → assumption : know x ← true state

more realistic know $y = Cx$

- estimate x : (use $y = Cx$) $\hat{x}$ ← our estimate of the true state
- apply $u = K\boxed{\hat{x}}$

$$\dot{x} = Ax + Bu$$ ← real world.

$$y = Cx \qquad C \in \mathbb{R}^{o \times n}$$ ← what we can measure

---

→ $\hat{x}$: estimate of x ← $\hat{y} = C\hat{x}$

→ $\dot{\hat{x}} = A\hat{x} + Bu + L(\hat{y} - y)$

estimator (computer)

design parameter

$L \in \mathbb{R}^{n \times o}$

$y = Cx$ } → actually is

$\hat{y} = C\hat{x}$

expect y to be from current estimate $\hat{x}$

Full controller

$$\left.\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + L(C\hat{x} - \boxed{y}) \\ u &= K\hat{x} \end{aligned}\right\} \to \text{controller}$$

how do x & $\hat{x}$ evolve together...

$u = K\hat{x}$

$$\begin{cases} \dot{x} = Ax + BK\hat{x} \\ \dot{\hat{x}} = A\hat{x} + BK\hat{x} + L(C\hat{x} - Cx) \end{cases}$$

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & BK \\ -LC & A+BK+LC \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

Coordinate transform

$e = \hat{x} - x \longrightarrow \begin{bmatrix} x \\ e \end{bmatrix} = \begin{bmatrix} I & 0 \\ -I & I \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$

$$\begin{bmatrix} x \\ \hat{x} \end{bmatrix} = \begin{bmatrix} I & 0 \\ I & I \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}$$

$\hat{x} = x + e$

$$\begin{cases} \dot{x} = Ax + Bu \\ \dot{e} = \dot{\hat{x}} - \dot{x} = A(\underbrace{\hat{x} - x}_{e}) + L(C\hat{x} - Cx) \\ \quad = (A + LC)e \end{cases}$$

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A+LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u$$

apply $u = K\hat{x} = K(x+e)$

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A+LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \begin{bmatrix} K & K \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} \boxed{A+BK} & BK \\ 0 & \boxed{A+LC} \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}$$

error dynamics independent of x

if we design L correctly $\Rightarrow$ $e \to 0$, $\hat{x} \to x$

$\to$ $(A+LC)$ stable

if we design K correctly $\Rightarrow$ first $\hat{x} \to x$, then $x \to 0$

$\to$ $(A+BK)$ stable

designing $A+BK$ to have specific eigenvalues

now designing $A+LC$ to " " "

$\to$ $A^T + C^T L^T$ " " " "

pole placement techniques work for designing L also

$\underline{L}^T = \text{place}(A^T, C^T, \underline{\quad\quad})$

↳ evals

optimal way to design L ...

related to LQR

↳ Kalman Filtering