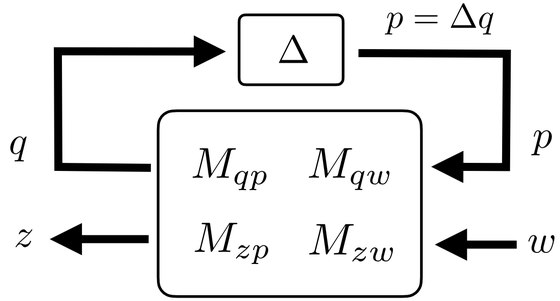
Examples: LFR Uncertainty

$$\begin{bmatrix} q \\ z \end{bmatrix} = \begin{bmatrix} M_{qp} & M_{qw} \\ M_{zp} & M_{zw} \end{bmatrix} \begin{bmatrix} p \\ w \end{bmatrix} \qquad p = \Delta q$$



General uncertainty…

pull signal out from wherever…

add uncertainty… $\Delta(s)$ stable, causal

put it back in wherever…

$$q = M_{qp}p + M_{qw}w$$
$$q = M_{qp}\Delta q + M_{qw}w$$
$$q = (I - M_{qp}\Delta)^{-1}M_{qw}w$$

$$z = \left(M_{zp}\Delta(I - M_{qp}\Delta)^{-1}M_{qw} + M_{zw}\right)w$$

Analyzing structure

1. Restructuring to see form better…

if Δ invertible…

if $M_{zp} = I$

$$z = \left(I - M_{qp}\Delta\right)^{-1}\left(\Delta M_{qw} + (I - M_{qp}\Delta)M_{zw}\right)w$$

if $M_{qw} = I$

$$z = \underbrace{\left(M_{zp}\Delta + M_{zw}(I - M_{qp}\Delta)\right)}_{\text{linear in } \Delta}\underbrace{\left(I - M_{qp}\Delta\right)^{-1}}_{\text{Inverse of linear in } \Delta}w$$

2. If all elements are scalars…

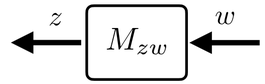
$$z = \frac{(M_{zp}M_{qw} - M_{qp}M_{zw})\Delta + M_{zw}}{-M_{qp}\Delta + I}w$$

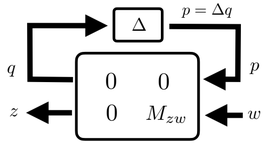
Specific cases:

Nominal case

$$z = \left(\cancel{M_{zp}}^{0}\Delta(I - \cancel{M_{qp}}^{0}\Delta)^{-1}\cancel{M_{qw}}^{0} + M_{zw}\right)w$$

$$z = M_{zw}w$$

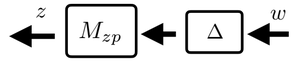


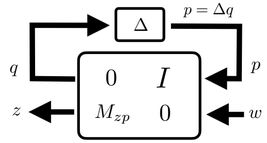


Multiplicative Uncertainties

$$z = \left(M_{zp}\Delta(I - \cancel{M_{qp}}^{0}\Delta)^{-1}\cancel{M_{qw}}^{I} + \cancel{M_{zw}}^{0}\right)w$$

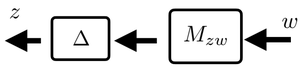
$$z = M_{zp}\Delta w$$

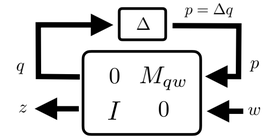




$$z = \left(\cancel{M_{zp}}^{I}\Delta(I - \cancel{M_{qp}}^{0}\Delta)^{-1}M_{qw} + \cancel{M_{zw}}^{0}\right)w$$

$$z = \Delta M_{qw}w$$

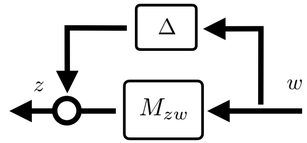


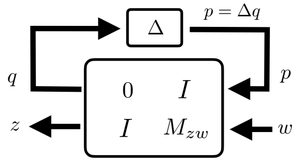


Nominal + multiplicative uncertainty

$$z = \left(\cancel{M_{zp}}^{I}\Delta(I - \cancel{M_{qp}}^{0}\Delta)^{-1}\cancel{M_{qw}}^{I} + M_{zw}\right)w$$

$$z = \left(M_{zw} + \Delta\right)w$$





Feedback uncertainty

$$z = \left(\cancel{M_{zp}}^{I}\Delta(I - \cancel{M_{qp}}^{M_{zw}}\Delta)^{-1}\cancel{M_{qw}}^{M_{zw}} + \cancel{M_{zw}}^{0}\right)w$$

$$z = (I - M_{zw}\Delta)^{-1}M_{zw}w$$