Mechanical Systems & RLC Circuits

Dynamical Systems

Major sources:

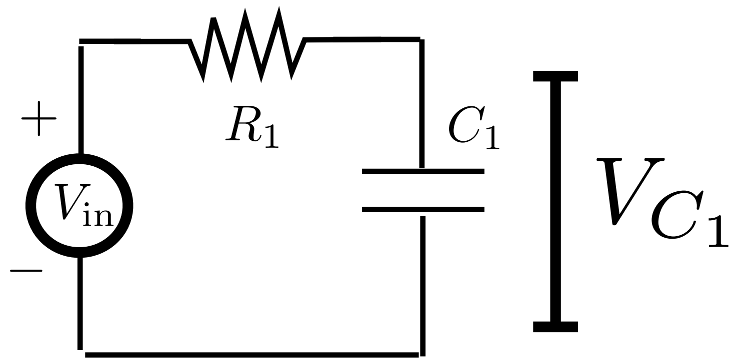
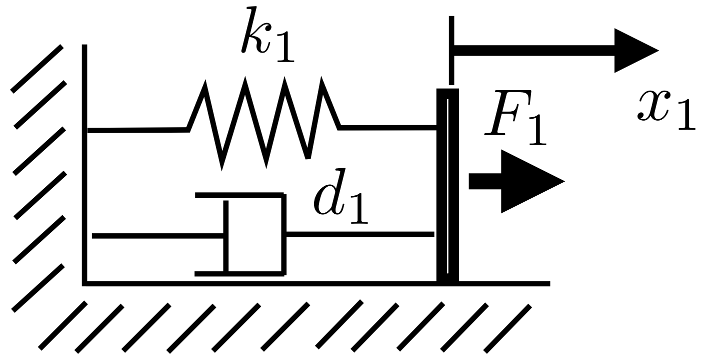
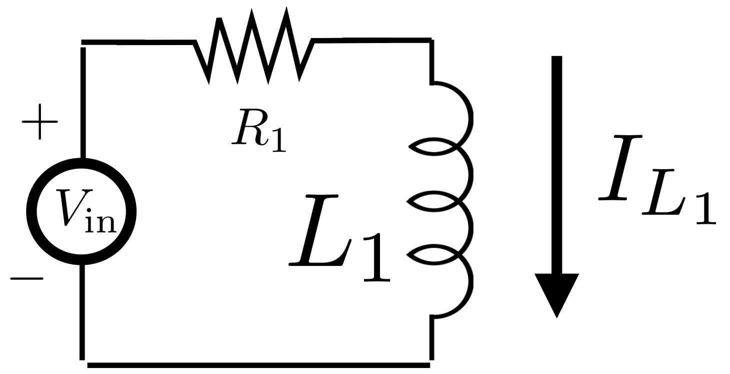
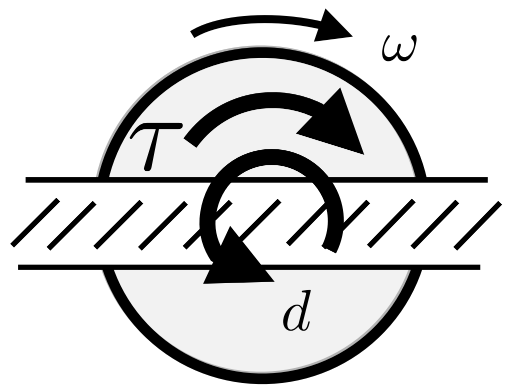
Major references:

Winter 2022 - Dan Calderone

RLC Circuits

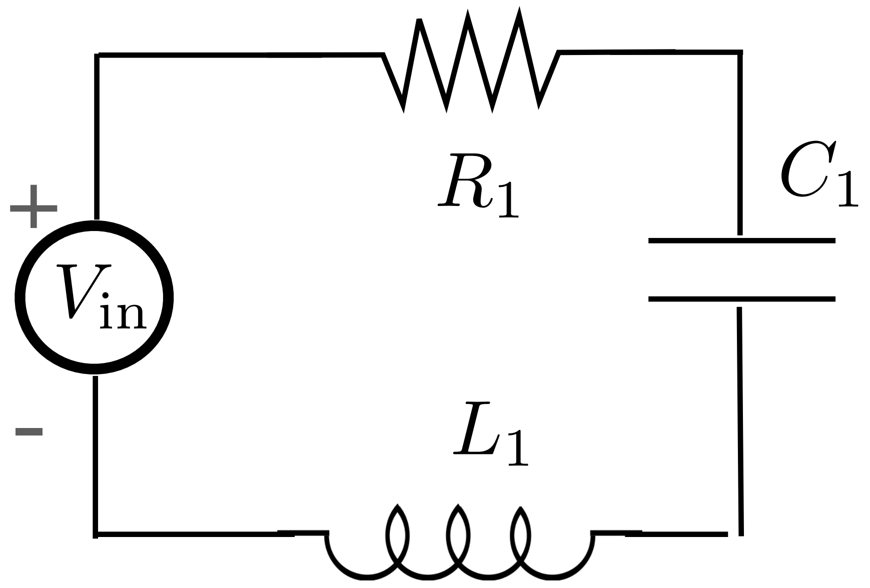
FIRST ORDER SYSTEMS

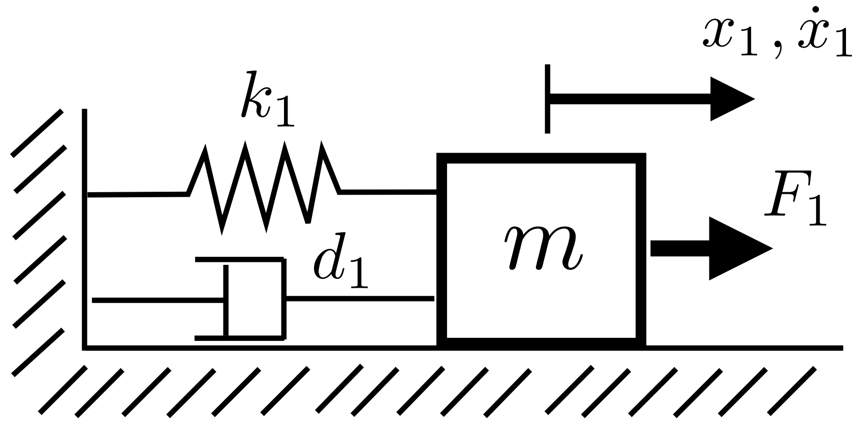
(1st order derivatives)

		Diagrams	Dynamics	Mechanical Analogs	
RC Circuits	Capacitor state: charge or voltage $V_C = CQ_C$ **(proportional)**		$\dot{Q} = -\frac{C}{R}Q + \frac{1}{R}V_{\text{in}}$		Compression spring (no mass, stores energy)
RL Circuits	Inductor state: current I_L		$\dot{I} = -\frac{R}{L}I + \frac{1}{L}V_{\text{in}}$		Flywheel with angular momentum (no position, only angular velocity)

SECOND ORDER SYSTEMS

(2nd order derivatives)





RLC Circuits

… one state per object with dynamics

States:

Capacitor state: charge or voltage **(proportional)** …analogous to position

Inductor state: current …analogous to velocity

Dynamics:	**Graph Relations:**		**Elements:**
$\dot{Q}_C = I_C$	**KVL:**	**sum of voltage around a loop = 0**	$V_C = CQ_C$
			$V_R = RI_R$
$L\dot{I}_L = V_L$	**KCL**	**sum of currents into a node = 0**	$V_L = L\dot{I}_L$

RLC Circuits

FIRST ORDER SYSTEMS

(1st order derivatives)

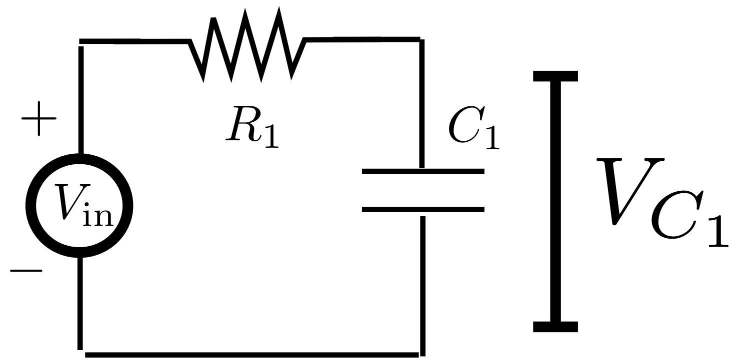
Diagrams | Dynamics | Mechanical Analogs

RC Circuits

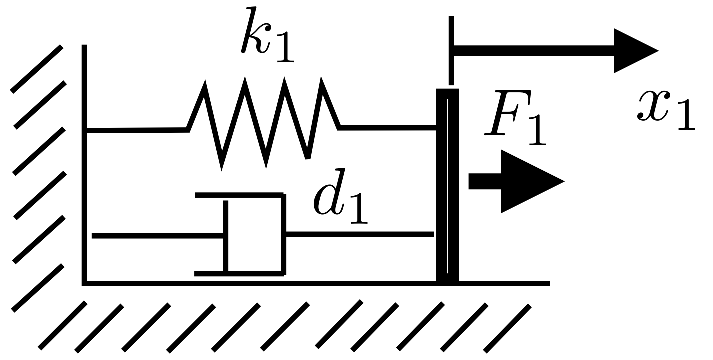
Capacitor state:

charge or voltage $V_C = CQ_C$

(proportional)



$$\dot{Q} = -\frac{C}{R}Q + \frac{1}{R}V_{\text{in}}$$

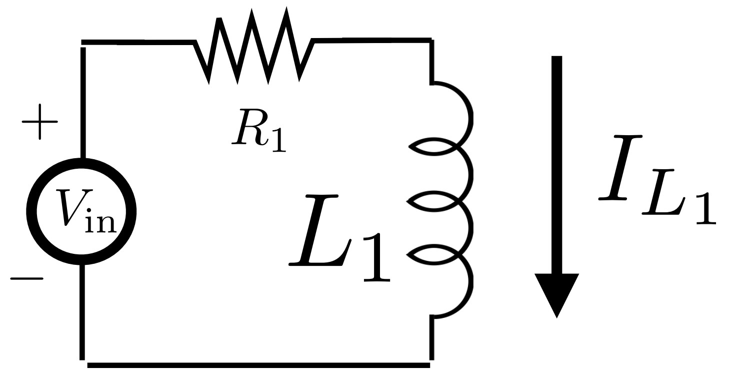


Compression spring (no mass, stores energy)

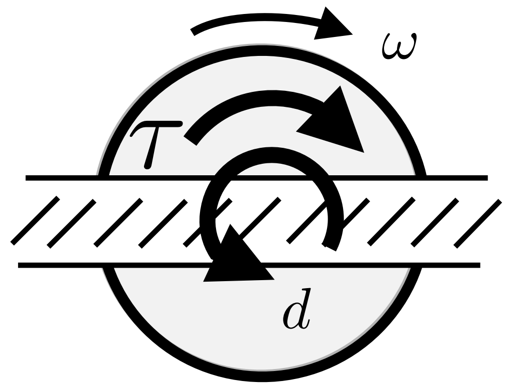
RL Circuits

Inductor state:

current I_L



$$\dot{I} = -\frac{R}{L}I + \frac{1}{L}V_{\text{in}}$$



Flywheel with angular momentum (no position, only angular velocity)

SECOND ORDER SYSTEMS

(2nd order derivatives)

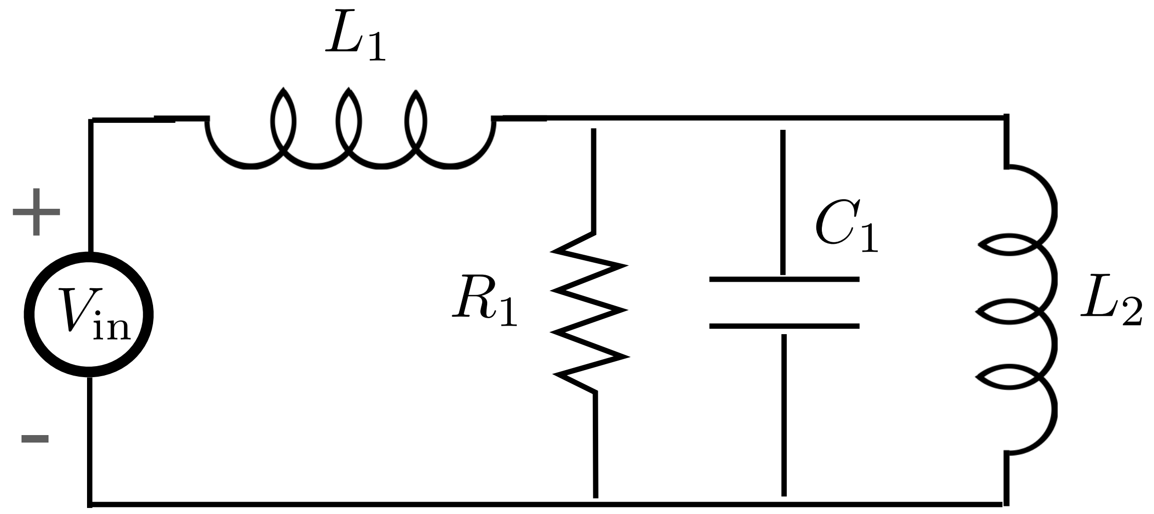
RLC Circuits

… one state per object with dynamics

States:

Capacitor state: charge or voltage **(proportional)** …analogous to position

Inductor state: current …analogous to velocity



States:

Q_{C_1}	Charge on capacitor 1
I_{L_1}	Current thru inductor 1
I_{L_2}	Current thru inductor 2

KVL/KCL Equations:

KVL: $V_{\text{in}} = V_{L_1} + V_{R_1}$

$V_{R_1} = V_{C_1} = V_{L_2}$

KCL: $I_{L_1} = I_{R_1} + I_{C_1} + I_{L_2}$

Dynamics:

$$\dot{Q}_C = I_C$$

$$L\dot{I}_L = V_L$$

Graph Relations:

KVL: **sum of voltage around a loop = 0**

KCL **sum of currents into a node = 0**

Elements:

$$V_C = CQ_C$$

$$V_R = RI_R$$

$$V_L = L\dot{I}_L$$

Derived in terms of states…

$$V_{C_1} = V_{R_1} = V_{L_2} = C_1 Q_{C_1}$$

$$V_{L_1} = V_{\text{in}} - C_1 Q_{C_1}$$

$$I_{R_1} = \frac{C_1}{R_1} Q_{C_1}$$

$$I_{C_1} = I_{L_1} - I_{L_2} - \frac{C_1}{R_1} Q_{C_1}$$

Dynamics:

$$\dot{Q}_{C_1} = I_{C_1} = I_{L_1} - I_{L_2} - \frac{C_1}{R_1} Q_{C_1}$$

$$L_1 \dot{I}_{L_1} = V_{L_1} = V_{\text{in}} - C_1 Q_{C_1}$$

$$L_2 \dot{I}_{L_2} = V_{L_2} = C_1 Q_{C_1}$$